Vielteilchenphysik: Magnetismus und Korrelationen - SS 16
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Hand in until Tuesday, July 12, 09:45

Exercise sheet 5

Exercise 1 (5 Points)

If an operator A is an eigenvector of the Liouville operator, i.e. $LA = aA$, show that the projection operator

$$P = |A\rangle \frac{1}{\langle A|A\rangle} \langle A| \tag{1}$$

commutes with the Liouville operator.

Exercise 2 (40 Points)

Consider the relaxation of non–interacting spins in a magnetic field due to lattice modulations. This problem can be formulated as spins interacting with a magnetic field and a bosonic bath which represents the phononic excitations. The Hamiltonian that describes this problem is given by

$$\begin{aligned} H &= H_s + H_b + H_{sb} \\ &= -g\mu_B B S^z + \sum_{\mathbf{k}} \omega_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{k}} \left(\lambda_{\mathbf{k}} b_{\mathbf{k}} + \lambda^*_{\mathbf{k}} b^\dagger_{\mathbf{k}}\right) S^x \,. \end{aligned} \tag{2}$$

In the first term, g is the Landé factor, μ_B the Bohr magneton, B the magnetic field, and S^z the z component of the spin–1/2 operator. In the second term, $\omega_{\mathbf{k}}$ is the dispersion of the phonons, and $b^\dagger, b$ are bosonic creation, annihilation operators obeying the commutation relations

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}] = 0 = [b^\dagger_{\mathbf{k}}, b^\dagger_{\mathbf{k}'}] \quad \text{and} \quad [b_{\mathbf{k}}, b^\dagger_{\mathbf{k}'}] = \delta_{\mathbf{k},\mathbf{k}'} \,.$$

Lastly, in the third term, $\lambda_{\mathbf{k}}$ is the coupling strength of the bosonic bath to the spins, and S^x is the x component of the spin operator which can be written in terms of raising(S^+), lowering(S^-) operators as $S^x = \frac{1}{2}(S^+ + S^-)$. In the rotating wave approximation, the interaction term between spin and phonons reduces to

$$H_{sb} \approx \sum_{\mathbf{k}} \left(\lambda_{\mathbf{k}} b_{\mathbf{k}} S^- + \lambda^*_{\mathbf{k}} b^\dagger_{\mathbf{k}} S^+\right) \,. \tag{3}$$

Furthermore, for weak $\lambda_{\mathbf{k}}$, H_{sb} can be considered as a perturbation to the non interacting system, viz.

$$H = H_0 + H_1 \,, \quad \text{with} \quad H_0 = H_s + H_b, \quad \text{and} \quad H_1 = H_{sb} \,.$$

- Show that in the unperturbed system ($\lambda_{\mathbf{k}} = 0$), the spin operators $S^z, S^\pm$ as well as, the bosonic $b_{\mathbf{k}}, b^\dagger_{\mathbf{k}}$ are eigenvectors of the Liouville operator L_0.
- In the unperturbed system, calculate the static susceptibility χ^{zz}_0 for the S^z operator, and the imaginary part of the dynamical susceptibility $\Im[\chi^{bb}_0(\omega)]$ with
$$\Im[\chi^{bb}_0(\omega)] = \int_{-\infty}^{\infty} dt e^{i\omega t} \langle [b_{\mathbf{k}}(t), b^\dagger_{\mathbf{k}'}]\rangle_0 \,. \tag{4}$$
- Show that the operator $L_1 S^z$ is orthogonal to S^z with respect to the unperturbed system.
- Show that in second order of the coupling λ, the memory function for the S^z operator is given by
$$M^{zz}(z) \simeq \langle QL_1S^z | \frac{1}{z - L_0} QL_1 S^z\rangle_0 + \mathcal{O}(\lambda^3) \,. \tag{5}$$
- Calculate the memory function $M^{zz}(z)$ to second order in the interaction coupling λ.

🎁 Exercise 3 (10 Points)

In the long time approximation, an observable A is considered to be slowly varying in some characteristic time scale τ. In that case, the memory function $M_{AA}(z)$ can be considered as frequency independent at the origin of the complex plane. Describe the relaxation function $\Phi_{AA}(t)$ for the observable A.