Prof. Dr. Wolfram Brenig
Alexandros Metavitsiadis

Hand in until Tuesday, June 28, 09:45

Exercise sheet 4

Exercise 1 (10 Points)

Use the fluctuation dissipation theorem to check the equipartition law for the harmonic oscillator described by the Hamiltonian

$$H = \omega_0 b^\dagger b \, . \tag{1}$$

Here, ω_0 is the characteristic oscillation frequency, and $b(b^\dagger)$ lowering(raising) operators, obeying the bosonic commutation relations $[b, b^\dagger] = 1$.

Exercise 2 (20 Points)

A one–dimensional Heisenberg antferromagnet is described by the Hamiltonian

$$H = J \sum_{l=0}^{N-1} \mathbf{S}_l \cdot \mathbf{S}_{l+1} \, , \tag{2}$$

where J is the exchange interaction, $\mathbf{S}$ spin $1/2$ operators, and N the system size.

- Show that
$$\int_0^\infty d\omega (1 - e^{-\beta\omega}) \omega S^{zz}(q,\omega) = \frac{4\pi}{3} \left[\cos(q) - 1\right] \langle H \rangle \, , \tag{3}$$
with $S^{zz}(q,\omega)$ the autocorrelation function of the spin density operator
$$S^z_q = \sum e^{-iql} S^z_l \, . \tag{4}$$

- How is Eq. (3) modified at the zero temperature limit?