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Hand in until Tuesday, April 26, 09:45

Exercise sheet 1

Exercise 1 (10 Points)

Show that the Heaviside step function defined as

$$\theta(t-t') = \begin{cases} 1 & \text{for} \quad t > t' \\ \\ 0 & \text{for} \quad t < t' \end{cases} , \tag{1}$$

acquires the integral representation

$$\theta(t-t') = -\int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \frac{e^{-i\omega(t-t')}}{\omega + i\eta} , \quad \eta \to 0^+ . \tag{2}$$

Exercise 2 (10 Points)

The imaginary part of a function is given by

$$f''(\omega) = 2\sqrt{1-\omega^2}\,\theta(1-\omega^2) , \tag{3}$$

with θ the Heaviside step function.

- Calculate $f(z)$.
- Make a graphical representation of $f(z)$.

Exercise 3 (10 Points)

- What is the behavior under inversion of:
  a) the density $n(\mathbf{r}) = \sum_l \delta(\mathbf{r} - \mathbf{R}_l)$,
  b) the momentum density $\mathbf{p}(\mathbf{r}) = \sum_l \delta(\mathbf{r} - \mathbf{R}_l) \frac{\partial}{i\mathbf{R}_l}$,
  c) the spin density $\mathbf{s}(\mathbf{r}) = \sum_l \delta(\mathbf{r} - \mathbf{R}_l) \mathbf{S}_l$?
- What are the consequences for χ_{np}^{iso} and χ_{sp}^{iso} in a system with inversion symmetric statistical operator ρ?

Exercise 4 (5 Points)

- Show the identity

$$\lim_{\eta \to 0^+} \frac{1}{\omega \pm i\eta} = \mathcal{P}\left(\frac{1}{\omega}\right) \mp i\pi\delta(\omega) , \tag{4}$$

  where $\mathcal{P}$ denotes the principal value, and δ is the Dirac δ–function.*

- What is the representation of the principal value, and the δ–function for a finite η?

* *Hint*: Show that it holds for the integral $\int d\omega \frac{f(\omega)}{\omega \pm i\eta}$, with $f(\omega)$ a smooth function in the vicinity of $\omega = 0$.