Fluid-Structure Interaction with Space-Time Finite Elements
I. Elastodynamics and Stabilization Techniques

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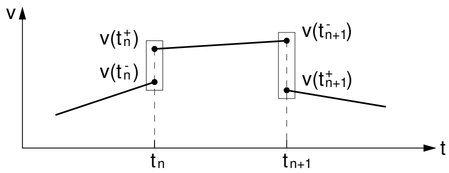
Motivation

In case of long-span bridges, high-rise buildings and light-weight roof structures the interaction of wind flow and structural motion may induce the collapse of the structure. The fluid-structure interaction must be considered for liquid-filled tanks due to dynamic excitation as well.



Concept for a carbon fiber bridge over the strait of Gibraltar

Goal of this work is the development and application of numerical methods for solving fluid-structure interaction problems with strong coupling in a single system of equations, what allows stability analyses of the coupled system.

Therefore, a consistent model of the whole system composed of elastic structure, turbulent flow field and coupling conditions is developed. For the discretization of all fields the time-discontinuous stabilized space-time finite element method is used.

Nonlinear elastodynamics

- Weak form of momentum conservation

$$\int_{Q_0} \delta\mathbf{v}\cdot\rho_0\,\dot{\mathbf{v}}\,\mathrm{d}Q_0 \;+\; \int_{Q_0} \delta\dot{\mathbf{E}}\cdot\cdot\,\mathbf{S}\,\mathrm{d}Q_0 \;-\; \int_{Q_0} \delta\mathbf{v}\cdot\mathbf{f}_0\,\mathrm{d}Q_0$$

- Rate formulation of the constitutive law

$$\int_{Q_0} \delta\mathbf{S}\cdot\cdot\left(\underline{\mathbf{C}}^{-1}\cdot\cdot\,\dot{\mathbf{S}} - \dot{\mathbf{E}}\right)\mathrm{d}Q_0$$

- Nonlinear kinematics with Green's strain tensor

$$\mathbf{E} \;=\; \tfrac{1}{2}\left(\mathrm{grad}_0\mathbf{u} + (\mathrm{grad}_0\mathbf{u})^{\mathrm{T}} + (\mathrm{grad}_0\mathbf{u})^{\mathrm{T}}\,\mathrm{grad}_0\mathbf{u}\right)$$

- Mixed/hybrid formulation with global velocity variables and elemination of stress variables on element level

Time-discontinuous Galerkin

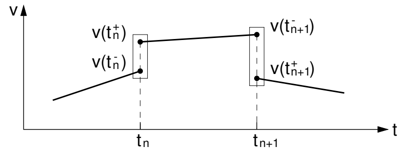
- Partitioning the time interval into space-time slabs

$$Q_{0,n} = \Omega_0 \times (t_n, t_{n+1})$$

- Fulfillment of initial conditions in integral form with a jump term for variables derived with respect to time

$$\int_{\Omega_0} \delta\mathbf{v}(t_n^+)\cdot\rho_0\left(\mathbf{v}(t_n^+) - \mathbf{v}(t_n^-)\right)\mathrm{d}\Omega_0$$

- Independent degrees of freedom in adjoining time slabs at discrete time levels t_n



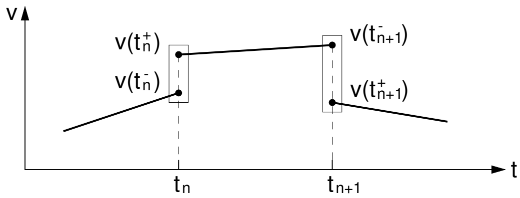
- High order implicit A-stable time integration scheme (third order for linear interpolation functions)

Galerkin/least-squares stabilization

- Stabilization term for the momentum equation

$$\sum_e \int_{Q_0^e} \tau\,\frac{1}{\rho_0}\,\mathscr{L}(\delta\mathbf{v},\delta\mathbf{P})\cdot\left[\mathscr{L}(\mathbf{v},\mathbf{P}) - \mathbf{f}_0\right]\mathrm{d}Q_0$$

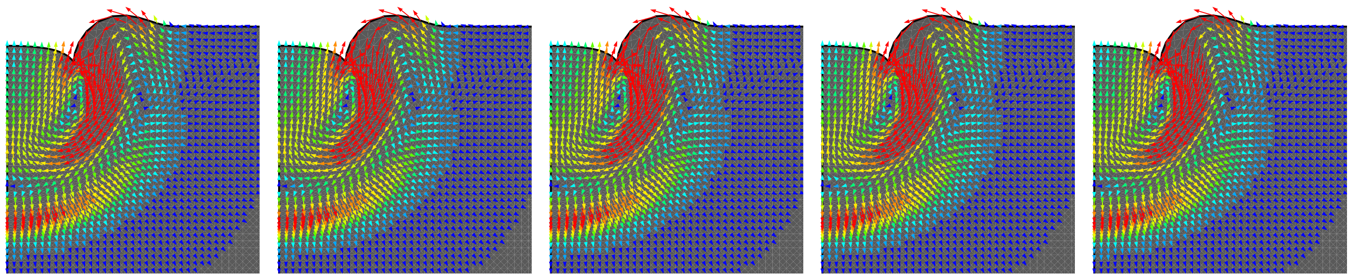
- Prevents oszillating solutions of hyperbolic PDEs like convection dominated flows or wave propagation



Time history of a shock wave in an elastic bar

- Allows equal order interpolation functions for velocity and pressure in case of mixed finite elements

Numerical example of elastic wave propagation



An impact load in the upper left corner of a bounded elastic continuum induces pressure, shear and Rayleigh waves. The figure shows the velocity field and a sequence of absolute velocity values visualized onto the deformed geometry.