Phase-field modeling of fracture in variably saturated porous materials

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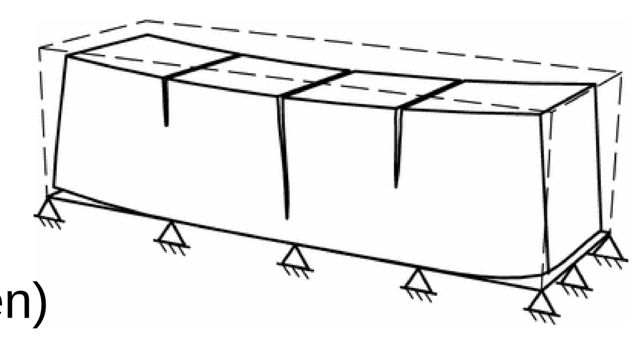
Research scope

Development of a mechanical and computational model to describe the coupled problem of poromechanics and cracking in variably saturated porous media. A classical poromechanical formulation is adopted and coupled with a phase-field formulation for the fracture problem. The latter has the advantage of being able to reproduce arbitrarily complex crack paths without introducing discontinuities on a fixed mesh.

The model requires several material and hygral properties:

- α Biot parameter
- n porosity
- $\rho_{s,w}$ density of the solid, water
- $K_{s,w}$ bulk modulus of the solid, water
- G_c fracture energy
- ℓ length scale parameter
- S_w degree of saturation (van Genuchten)
- k_{rw} relative permeability (van Genuchten)
- C_s moisture content
- k_i intrinsic permeability.



Computational Framework

The model contains three primary variables: displacement $\boldsymbol{u}$, water pressure p_w and crack phase-field d (1).

- Equilibrium equation

$$\nabla \cdot ((1-d^2)\boldsymbol{\sigma}'^{+} + \boldsymbol{\sigma}'^{-} - \alpha S_w p_w I) + n S_w \rho_w g + (1-n)\rho_s \boldsymbol{g} = 0$$

with the effective stress $\boldsymbol{\sigma}'^{\pm} = \mathbf{D}^{\pm} : \nabla^s \boldsymbol{u}$ (linear-elastic law)

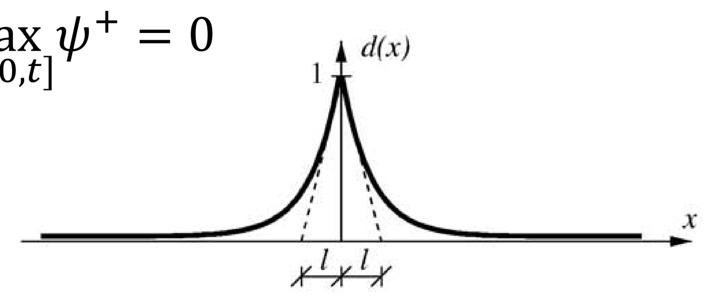
- Mass balance equation of liquid water

$$\nabla \cdot \left(-k_{rw}\frac{k_i \mathbf{I}}{\mu_w}(\nabla p_w - \rho_w \boldsymbol{g})\right) + \frac{S_w}{K_w}\frac{\partial p_w}{\partial t} + C_s \frac{\partial p_w}{\partial t} + S_w \frac{\alpha - n}{K_s}\left(S_w + \frac{C_s}{n} p_w\right)\frac{\partial p_w}{\partial t} + S_w \alpha \frac{\partial \mathrm{tr}(\nabla^s \boldsymbol{u})}{\partial t} = 0$$

- Phase-field evolution equation

$$\frac{G_c}{l}(d - l^2 \Delta d) \max_{\tau \in [0,t]} \psi^+ = 0$$

where ψ^+ is the positive energy.

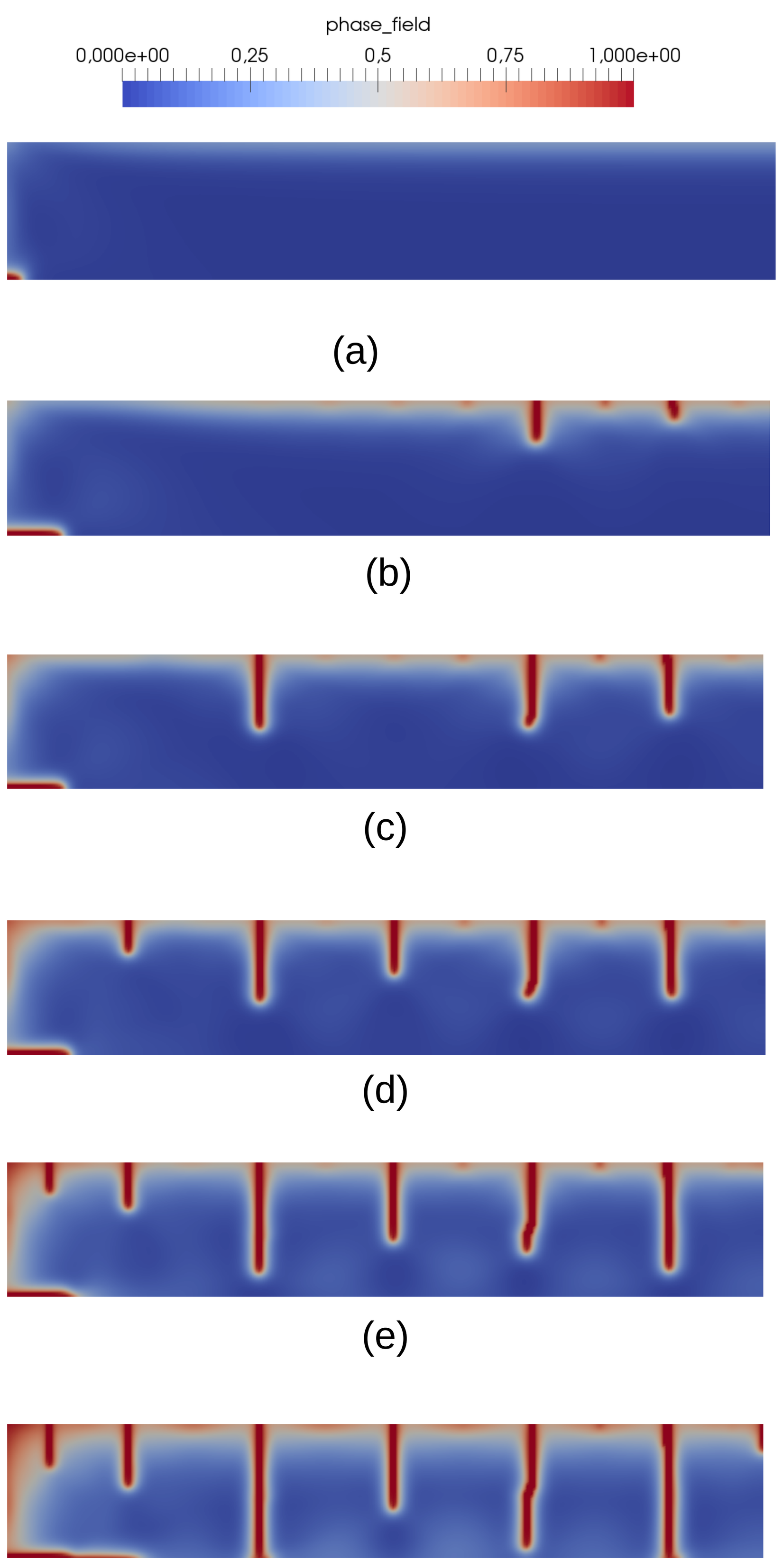


Numerical results

Desiccation in porous materials

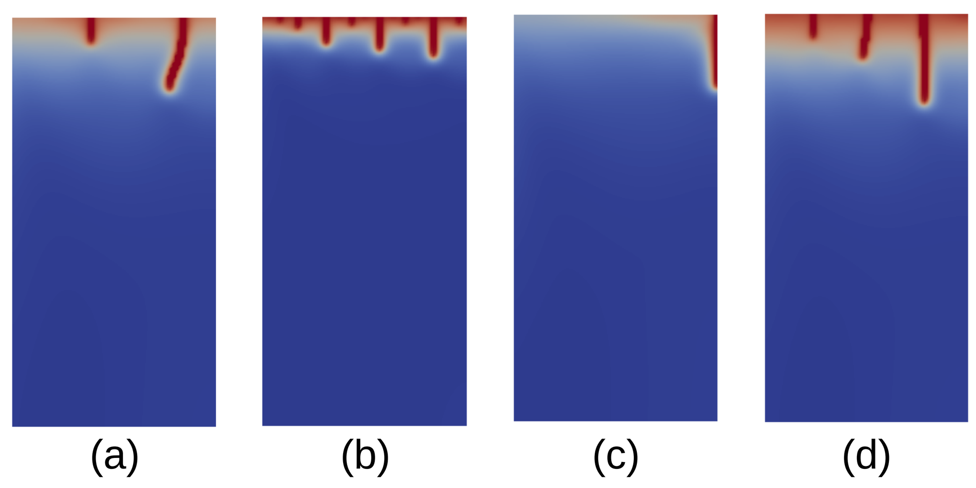
Desiccation is the result of drying, shrinkage and cracking in porous materials, such as soils. The computational framework is used in a compacted clay based material.

A drying flux is applied on the top and sides of a symmetric, rectangular sample. The sample is mechanically restrained at the bottom.
The results show the phase-field evolution in half specimen with intrinsic permeability $k_i = 0.5 \times 10^{-15} m^2$ and drying flux $6 \times 10^{-7}\, m/s$ at **a** t = 15 min, **b** 30 min onset of first upper crack, **c** 45 min, **d** 60 min, **e** 90 min and **f** 120 min. The crack at the corner of the specimen is limited in this test case. These and further results are presented and discussed in (1).
The computational results show good qualitative agreement with experiments.



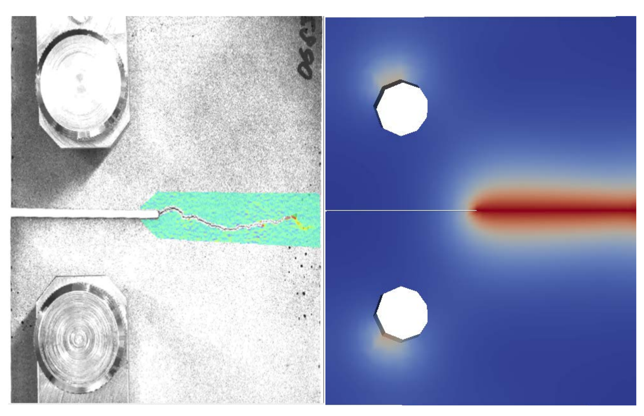
Shrinkage in cementitious materials and calibration of material parameters

The framework is applied to a cementitious material and a sensitivity analysis of the material parameters and drying flux is conducted (2).



The crack evolution at early age (2 hours) for **a** a reference set of parameters, **b** intrinsic permeability reduced by a factor of 10, **c** half of the reference flux and **d** double of the reference flux.

Due to the sensitivity of the results and the necessity of several material parameters in the model, an initial calibration of the fracture parameters such as the fracture energy was carried out in (3). The results show the crack propagation in a compact tension specimen.



Calibration of fracture parameters with the compact tension test

The improvement and calibration of further material and hygral parameters for quantitative validation of the framework are the main objectives of ongoing research.

Ongoing research

- Study on the change of effective properties and flow due to cracking. Tandem project, IRMB, Technische Universität Braunschweig.
- Experimental tests for numerical validation of cracking due to autogenous and drying shrinkage. Collaboration with Empa and ETH, Switzerland (Visiting researcher).
- Study on preconditioners and iterative solvers. Collaboration with University of Padova, Italy.

Publications

(1) Cajuhi, T., Sanavia, L. & De Lorenzis, L. (2017). Phase-field modeling of fracture in variably saturated porous media. Computational Mechanics, in print.
(2) Cajuhi, T., Lura, P. & De Lorenzis, L. (2017). Phase-field modeling of shrinkage cracks in cement-based materials. Proceedings of the 2 nd Intertional RILEM/COST Conference on Early Age cracking
(3) Cajuhi, T., Lura, P. & De Lorenzis, L. (2017). Preliminary calibration of a phase-field model for cracks due to shrinkage in cement-based materials. Proceedings of the 7th GACM.