Bayesian Parameter Identification in Plasticity

Ehsan Adeli, Bojana Rosić & Hermann G. Matthies

Institute of Scientific Computing, Technische Universität Braunschweig

e.adeli@tu-bs.de

GRK 2075

Abstract

To evaluate the cyclic behaviour under different loading conditions using the kinematic and isotropic hardening theory of steel a Chaboche visco-plastic material model is employed. The parameters of a constitutive model are usually identified by minimization of the distance between model response and experimental data. However, measurement errors and differences in the specimens lead to deviations in the determined parameters. In this work the Choboche model is used and a stochastic simulation technique is applied to generate artificial data which exhibit the same stochastic behaviour as experimental data. Then the model parameters are identified by applying a variaty of Bayes's theorem. Identified parameters are compared with the true parameters in the simulation and the efficiency of the identification method is discussed.

Introduction

In order to predict the behaviour of loaded metallic materials, constitutive models are applied, which present a mathematical frame for the description of elastic and inelastic deformation. All inelastic constitutive models contain parameters which have to be identified for a given material from experiments. In the literature only few investigations can be found, dealing with identification problems using stochastic approaches.

In this paper, a viscoplastic model of Chaboche [1] is studied. The model contains five material parameters which have to be determined from experimental data. It should be noted that virtual data are employed instead of real experimental data. In addition, a cyclic tension-compression test is applied in order to extract the virtual data.

Model problem

The mathematical description of metals under cyclic loading beyond the yield limit that includes the viscoplastic material behaviour as well as the characterization of compulsatory isotropic-kinematic hardening is here given in terms of a modified Chaboche model introduced by [2]. As we consider classical infinitesimal material behaviour, we assume an additive strain decomposition. The complete model is stated in the Table below.

Strain

$$\boldsymbol{\epsilon}(t) = \boldsymbol{\epsilon}_e(t) + \boldsymbol{\epsilon}_{vp}(t)$$

Hooke's Law

$$\boldsymbol{\sigma}(t) = \boldsymbol{E} : \boldsymbol{\epsilon}_e(t)$$

Flow Rule

$$\dot{\boldsymbol{\epsilon}}_{vp}(t) = \langle \frac{\sigma_{eq}(t) - \sigma_y - R(t)}{k} \rangle^n \frac{\partial \sigma_{eq}}{\partial \boldsymbol{\sigma}}$$

Hardening

$$\dot{R} = b_R(H_R - R)\dot{p}$$

$$\dot{\boldsymbol{\chi}} = b_\chi(\tfrac{2}{3} H_\chi \frac{\partial \sigma_{eq}}{\partial \boldsymbol{\sigma}} - \boldsymbol{\chi})\dot{p}$$

Initial Conditions

$$\epsilon_{vp}(0) = 0, \;\; R(0) = 0, \;\; \chi(0) = 0$$

Parameters

σ_y (Yield Stress)

k, n (Flow Rule)

b_R, H_R, b_χ, H_χ (Hardening)

By gathering all the desired material parameters to identify into the vector $\boldsymbol{q} = [\kappa \; G \; b_R \; b_\chi \; \sigma_y]$, where κ and G are bulk modulus and shear modulus, respectively, the goal is to estimate $\boldsymbol{q}$ given measurement displacement data, i.e.

$$u = Y(\boldsymbol{q}) + \varepsilon \tag{1}$$

in which $Y(\boldsymbol{q})$ represents the measurement operator and ε the measurement (also possibly the model) error. Being an ill-posed problem, the estimation of $\boldsymbol{q}$ given u is not an easy task and requires regularisation. This can be achieved either in a deterministic or probabilistic setting. Here, the latter one is taken into consideration as further described in the text.

Bayesian identification

By acquiring additional (prior) knowledge on the parameter set next to the observation data, the probabilistic approach regularise the problem of estimating $\boldsymbol{q}$ with the help of Bayes's theorem

$$\pi_{q|u}(\boldsymbol{q}|u) \propto L(\boldsymbol{q})\pi_q(\boldsymbol{q}) \tag{2}$$

in which the likelihood $L(\boldsymbol{q})$ describes how likely the measurement data are given prior knowledge $\pi_q(\boldsymbol{q})$. This in turn requires the reformulation of the deterministic model into the probabilistic one, and hence the propagation of material uncertainties through the model —the so-called forward problem— in order to obtain the likelihood [3]. An affine approximation of relation 2 is

$$q_a(\omega) = q_f(\omega) + k(z(\omega) - u_f(\omega)), \tag{3}$$

also known as a linear Bayesian posterior estimate. Here, q_f represents the prior random variable, q_a is the posterior approximation, u_f is the forecasted measurement and k represents the very well-known Kalman gain

$$k := C_{q_f u_f}(C_{u_f} + C_\varepsilon)^\dagger \tag{4}$$

which can be easily evaluated if the appropriate covariance matrices $C_{q_f u_f}$, C_{u_f} and C_ε are known. Namely, $q_a(\omega)$, $q_f(\omega)$, $z(\omega)$ and $u_f(\omega)$ denote the RVs used to model the posterior, prior, observation, and forecasted observation, respectively.

In this light the linear Bayesian procedure can be reduced to a simple algebraic method. Starting from the functional representation of the prior

$$\hat{q}_f = \sum_\alpha q_f^{(\alpha)} \psi_\alpha(\omega) \tag{5}$$

where ψ_α is the Hermite function, one may discretise 3 as:

$$\hat{q}_a = \hat{q}_f + k(\hat{z} - \hat{u}_f), \tag{6}$$

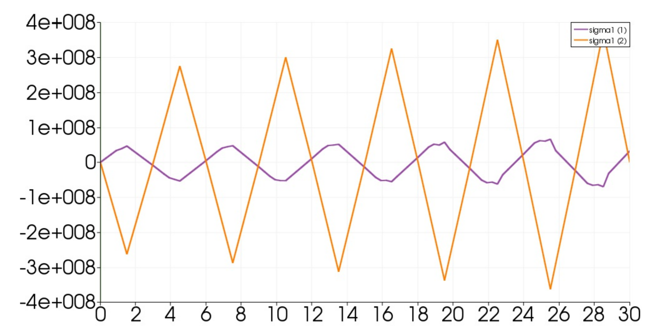
where $\hat{z} \in \mathbb{R}^{L \times Z}$ are the PCE coefficient of the measurement. Here, k in equation 6 is the Kalman gain evaluated in an algebraic way knowing that

$$C_{q_f, u_f} = \sum_{\alpha > 0} \alpha! \; q_f^{(\alpha)} (\boldsymbol{u}_f^{(\alpha)})^T. \tag{7}$$

Note that in the numerical computation $\hat{q}_f := [q_f(\omega_1), ..., q_f(\omega_Z)]$ is the PCE coefficient of the prior and $\hat{q}_a := [q_a(\omega_1), ..., q_a(\omega_Z)]$ is the PCE coefficient of the posterior with cardinality z determined by $(L+1)$ RVs and polynomial order p [4].

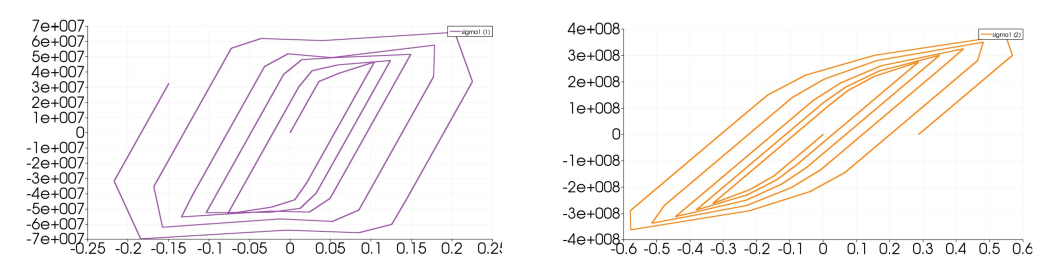
Numerical results

Preliminary study is on a regular cube, modelled with one 8 node element, completely restrained on the back face, and with normal traction on the opposite (front) face. The magnitude of the normal traction and a stress in the plane of the front face is plotted in the Figure below. Purple and orange colours represent the stress value in normal and in plane directions, respectively.

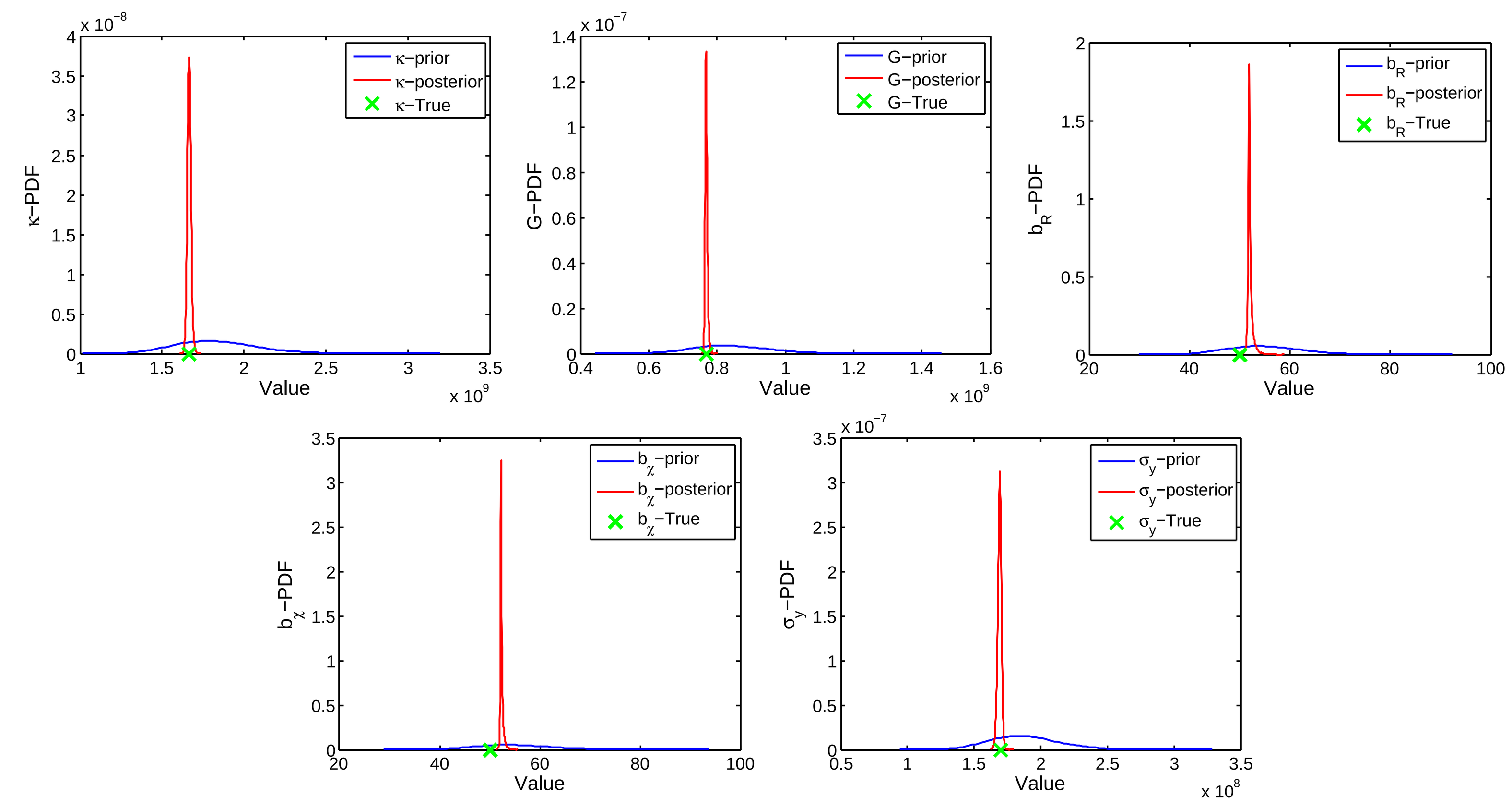


Considering the parameters listed in the Table below, the related σ-ϵ hysteretic graph obtained which can be seen in the Figure below.

κ	G	σ_y	n	k	b_R	H_R	b_χ	H_χ
1.66e9	7.69e8	1.7e8	1	1.5e8	50	0.5e8	50	0.5e8



The displacements of a node on the front surface in normal and in plane directions are observed as the virtual data in this study. Applying stochastic identification and introducing likelihood in such a way that 10 percent of mean values are equal to the variance of the related parameter, the probability density function of prior and posterior of the identified parameters can be seen in the Figure below.



Summarising the results, the true values and the mean and variance of the estimated parameters are compared in the Table below.

Parameters	q_{true}	q_{est}(mean)	q_{est}(standard deviation)
κ	1.66e9	1.66e9	2.59e6
G	7.69e8	7.68e8	6.39e5
b_R	50	52.38	0.29
b_χ	50	52.05	0.53
σ_y	1.7e8	1.69e8	1.52e5

Conclusions

Using the stochastic methods explained to identify the model parameters of the Choboche model indicates that it is possible to identify the model parameters using Gauss-Markov Kalman filter. The parameters are well estimated and the uncertainty of the parameters is reduced while the probability density function of the parameters are updated during the process.

Forthcoming Research

The model is going to be developed by adding a damage model and then the efficiency of the methods used and their developments will also be studied in the near future. Also, the design of experiments in the Bayesian setting will be investigated.

Acknowledgements

This work is partially supported by the DFG through GRK 2075.

References

[1] J. L. Chaboche, G. Rousselier. *On the plastic and viscoplastic constitutive equations part 1: rules developed with internal variable concept*. J. Press. Vessel Technol., 1983, 105, 153158.

[2] J. Velde. *3D Nonlocal Damage Modeling for Steel Structures under Earthquake Loading*. Department of Architecture, Civil Engineering and Environmental Sciences University of Braunschweig - Institute of Technology (2010).

[3] H. G. Matthies. *Uncertainty Quantification with Stochastic Finite Elements*. Encyclopedia of computational mechanics, pages 1-70, 2007.

[4] H. G. Matthies, E. Zander, B. V. Rosić, A. Litvinenko. *Parameter Estimation via Conditional Expectation — A Bayesian Inversion*. Advanced Modeling and Simulation in Engineering Sciences, 2016.