Multipath De-fragmentation: Achieving Better Spectral Efficiency in Elastic Optical Path Networks

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***Abstract*—In elastic optical networks, the spectrum consecutive and continuous constraints may cause the so-called *spectrum fragmentation* issue, degrading spectrum utilization, which is especially critical under dynamic traffic scenarios. In this paper, we propose a novel *multipath de-fragmentation* method which aggregates spectrum fragments instead of reconfiguring existing spectrum paths. We propose an optimization model based on Integer Linear Programming (ILP) and heuristic algorithms and discuss the practical feasibility of the proposed method. We show that multipath routing is an effective de-fragmentation method, as it improves spectral efficiency and reduces blocking under dynamic traffic conditions. We also show that the differential delay issue does not present an obstacle to the application of multipath de-fragmentation in elastic optical networks.**

I. Introduction

Elastic optical networks, also known as *"flexi-grid"*, have been proposed as an alternative to equidistant spectrum allocation in conventional optical networks [1]. With Orthogonal Frequency Division Multiplexing (OFDM) techniques, elastic optical networks can flexibly allocate optical spectrum by distributing high speed serial data into a group of parallel sub-carriers with low data rates. This flexibility comes at price however: the so-called *spectrum fragmentation* issue presents a major obstacle to the effectiveness of elastic spectrum allocation. Figure 1 illustrates the case in point, where traffic demand is denoted as $R(S, D, T_r)$. S, D and T_r are source, destination, and the number of required sub-carriers, respectively. The spectrum on each fiber link is divided into 16 sub-carriers and the size of guard-band is assumed to be 2 sub-carriers. After setting up six connections, i.e., $R1 - R6$, the spectrum on all the fiber links is fragmented, due to the spectrum continuous and consecutive constraints in Routing and Spectrum Assignment (RSA). It is not possible to allocate four consecutive sub-carriers, leading to the rejection of R_7. In fact, any connection requesting more than 3 sub-carriers will be blocked under the current network condition. This phenomena is particularly pronounced when spectrum paths are dynamically set-up and torn-down in an on-demand fashion.

In this context, the previously proposed methods for spectrum de-fragmentation have focused on rerouting the existing paths to maximize the consecutive spectrum range, see [2], [3]. In this paper, we propose a new method to effectively alleviate spectrum fragmentation issue by multipath routing, which we refer to as *multipath de-fragmentation*. The main idea is illustrated in the example shown in Figure 1, where R_7 can be served with two spectrum paths, i.e., $A - B - D$ and $A - C - D$, with two sub-carriers on each spectrum path.

To analyze the effectiveness of multipath de-fragmentation, we propose an Integer Linear Programming (ILP) based optimization model for multipath routing and spectrum assignment. We also propose heuristic algorithms that are applicable in large networks where the ILP model may become infeasible due to the complexity issue. All proposed algorithms have considered *differential delay issue* which is the major concern in multipath routing. The results show that differential delay issue is not a major obstacle, and can be managed to meet the requirements defined in the current standards, for instance, the ITU-T G.709 [4], or the commercially available products, such as Synchronous Dynamic Random Access Memory (SDRAM) [5] [6]. The results also show that the proposed algorithms can effectively aggregate spectrum fragments, thus reducing the blocking ratio of traffic demands. To the best of our knowledge, it is the first attempt to address spectrum fragmentation issue with multipath routing.

The rest of the paper is organized as follows. Section II presents the proposed ILP optimization model and heuristic algorithms. Section III and Section IV present the performance evaluation and conclusion, respectively.

II. Dynamic Multipath Routing and Spectrum Assignment Algorithm

Multipath de-fragmentation relies on optimal aggregation of unused spectrum fragments, while respecting routing and spectrum assignment (RSA) constraints in elastic optical networks. We first clarify the assumptions and notations before introducing the detailed algorithms.

Assumptions: We assume all sub-carriers have same transmission rate in this paper, which is a key enabler of implementing multipath de-fragmentation in practice, since it facilitates to utilize existing inverse multiplexing technologies. For instance, it simplifies the mapping between OTN frames and sub-carriers. In addition, we don't consider Physical Layer Impairment (PLI) in this paper. It is mainly because PLI is not expected to have significant impact on an OFDM based system due to the low symbol rate and coherent detection [7].

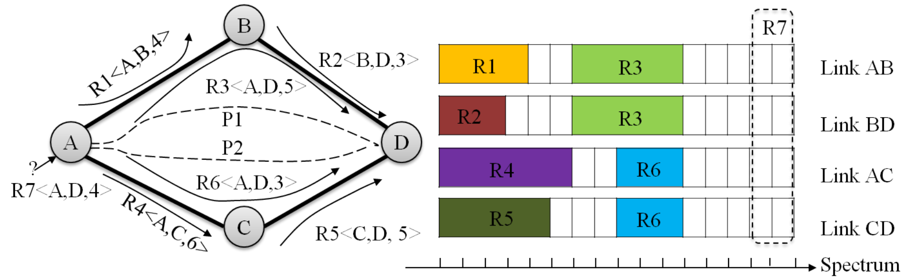

Fig. 1. An example of multipath de-fragmentation.

Notations: An elastic optical network is represented as $G(V,E)$, where V is the set of nodes and E is the set of fiber links. The spectrum on each fiber link is divided into $|F|$ slots (sub-carriers) and placed in an ordered set F. A sub-carrier with index i is denoted as f_i, $f_i \in F$. Available slots on a link e are placed in set F^e. Delay of link e is denoted as LD_e. Delay of a spectrum path is the sum of delays of all fiber links along the path, i.e., $pd_p = \sum_{e\in p} LD_e$. A connection demand is represented as $R(S,D,T_r)$, where S, D and T_r are source, destination, and the required number of sub-carriers, respectively. A spectrum path p is composed of one or multiple consecutive sub-carriers between source and destination, with consideration of spectrum continuity constraint, which is placed in a set denoted as $\mathcal{P}$. A fiber-level path fp is composed of a sequence of fiber links and can contain multiple spectrum paths, which is placed in a set denoted as $\mathcal{FP}$. The number of sub-carriers assigned to isolate adjacent spectrum paths, i.e., guard-band, is denoted as GB. The maximum acceptable differential delay is M, which can be $250us$ as suggested in ITU-T G.709 [4] or $128ms$ supported by a commercial framer device with off-chip memories [5] [6].

A. ILP Optimization Model

The variables used in the ILP model are summarized in Table I. We define the objective function as minimizing the

Variables	Description
x_p	Binary variable; it equals to 1 if a path p is used, otherwise it equals to 0
$x_{p,e}$	Binary variable; it equals to 1 if a path p uses e, otherwise it equals to 0
$y_{p,i}$	Binary variable; it equals to 1 if a path p uses sub-carrier $f_i \in F$, otherwise it equals to 0
$x_{p,e,i}$	Binary variable; it equals to 1 if a path p uses sub-carrier $f_i \in F$, otherwise it equals to 0
$o_{p,p'}$	Binary variable; it equals to 1 if path p and p' share at least one link, otherwise it equals to 0
pd_p	Integer variable; it denotes the delay of p
T_p	Integer variable; it denotes the number of sub-carriers allocated to p

TABLE I
VARIABLES

total number of sub-carriers assigned to the traffic demand, i.e.,

$$\textbf{Minimize} \sum_{p\in\mathcal{P}, f_i\in F, e\in E} x_{p,e,i} \tag{1}$$

subject to the constraints defined below.

Routing constraints: Eq.(2) ensures that traffic routed on a spectrum path does not get added or dropped in any node except for the source and destination. Eq.(3) and Eq.(4) define that a spectrum path can only start from source node and end at destination node.

$\forall p \in \mathcal{P}, \tilde{v}, v \in V, v \neq s,d$:

$$\sum_{e=(\tilde{v},v)\in E} x_{p,e} = \sum_{e=(v,\tilde{v})\in E} x_{p,e} \tag{2}$$

$$\forall p \in \mathcal{P}, \tilde{v} \in V: \sum_{e=(\tilde{v},d)\in E} x_{p,e} = x_p \tag{3}$$

$$\forall p \in \mathcal{P}, \tilde{v} \in V: \sum_{e=(s,\tilde{v})\in E} x_{p,e} = x_p \tag{4}$$

Spectrum continuity constraint: A spectrum path in the elastic optical network is an all-optical trail established between source and destination node [8]. Hence, the spectrum continuity has to be held during the path computation, i.e., $\forall f_i \in F, p \in \mathcal{P}, \tilde{v}, v \in V \setminus \{s,d\}$:

$$\sum_{e=(\tilde{v},v)\in E} x_{p,e,i} = \sum_{e=(v,\tilde{v})\in E} x_{p,e,i} \tag{5}$$

Spectrum consecutive constraints: Eq.(6) determines the number of sub-carriers that are allocated to path p. When two sub-carriers with index f_i and f_j $(j \geq i)$ are used for p, the right-hand side of Eq.(7) equals to T_p. It ensures that the gap between two sub-carriers should be no larger than T_p. When f_i and f_j are not used at the same time, the right-hand side of Eq.(7) results in an infinite value, which keeps Eq.(7) true.

$$\forall p \in \mathcal{P}, e \in E, v \in V, f_i \in F : T_p = \sum_{e=(s,v)} \sum_{i} x_{p,e,i} \tag{6}$$

$\forall f_i, f_j \in F, j \geq i, p \in \mathcal{P}, e \in E$:

$$f_j \cdot x_{p,e,j} - f_i \cdot x_{p,e,i} + 1 \leq Tp + (2 - x_{p,e,i} - x_{p,e,j}) \cdot \infty \tag{7}$$

Non-overlapping constraints: The non-overlapping constraints enssure that a spectrum slot can not be simultaneously assigned to more than one spectrum path. The value of binary variable $o_{p,p'}$ which depicts the existence of common link(s) is determined by Eq.(8). When path p and p' share at least one fiber link, $o_{p,p'}$ is equal to 1. The constraint defined in Eq.(9) specifies that a spectrum slot f_i can not be assigned to p and

p' with shared links at the same time, i.e., either $y_{p,i}$ or $y_{p',i}$ can be equal to 1 when $o_{p,p'} = 1$. Finally, Eq.(10) defines that spectrum assignment only happens when p is used.

$$\forall p, p' \in \mathcal{P}, p \neq p', e \in E : x_{p,e} + x_{p',e} - o_{p,p'} \leq 1 \tag{8}$$

$$\forall p, p' \in \mathcal{P}, p \neq p', f_i \in F : y_{p,i} + y_{p',i} + o_{p,p'} \leq 2 \tag{9}$$

$$\forall p \in P, f_i \in F : x_p - y_{p,i} \geq 0 \tag{10}$$

Guard-band constraint: Eq.(11) specifies that the spectrum assignment only happens when the available sub-carriers are sufficient to meet the guard-band requirement. When a spectrum slot f_i is allocated to a spectrum path p, the spectrum slots within the range $\{f_i, f_i + GB\}$ cannot be allocated to other spectrum paths. Eq.(12) defines that two spectrum paths p and p' with shared link(s) should have a gap no less than guard-band GB. When p and p' do not have any common links, $o_{p,p'}$ is equal to 0, which keeps Eq.(12) always true.

$$\forall p \in \mathcal{P}, e \in E, \{f_i + GB\} \in F \setminus F^e : x_{p,e} + y_{p,i} \leq 1 \tag{11}$$

$$\forall p, p' \in \mathcal{P}, e \in E, f_i, f_j \in F : |f_j \cdot x_{p,e,j} - f_i \cdot x_{p,e,i}| \geq GB \cdot o_{p,p'} \tag{12}$$

Bandwidth constraint: This constraint ensures that the number of spectrum slots (sub-carriers) assigned for the connection demand R are equal to T_r, i.e.,

$$\sum_{p \in \mathcal{P}, f_i \in F} y_{p,i} = T_r \tag{13}$$

Differential delay constraint: This constraint specifies that the maximum differential delay of the paths used for a single connection does not exceed the compensation capability of the upper layer, i.e., the differential delay between any two paths used for a connection can not exceed M, i.e.,

$$\forall p, p' \in \mathcal{P} : |pd_p - pd_{p'}| \leq M \tag{14}$$

The complexity of an ILP formulation is known to be exponential, i..e., $O(2^n)$, where n is the number of variables. Thus the proposed ILP model has an exponential complexity with n in $O(|P| \cdot (|P| + |E| \cdot |F|))$, which makes it computationally expensive and infeasible in practice. The problem size can be reduced by pruning the variables, for instance, computing a set of fiber-level paths in advance as input to the ILP model. However, it limits the solutions in the pre-computed path set.

B. Heuristic Algorithms for Multipath RSA

Given the complexity of multipath RSA, we decompose the problem into two sub-problems, i.e., multipath computation and spectrum assignment. We propose heuristic algorithms for each sub-problem, as shown in Alg.1 and Alg.2, respectively. The output from multipath computation is used as input to the spectrum assignment. The heuristic algorithms respect the same constraints and objective as the ILP model. The maximum allowable differential delay is assumed to be M and maximum K fiber-level paths are computed.

1) Multipath Computation: Alg.1 starts from collecting all paths originated from source node S and places these paths in an increasing order of delay in the set $\mathcal{S}$. Afterwards, the shortest path in $\mathcal{S}$ is selected, denoted as fp which is then extended to all the nodes that are connected to the sink node of fp, denoted as $destination(fp)$. The path set $\mathcal{S}$ is updated with new paths. The algorithm stops till the shortest path in $\mathcal{S}$, i.e., fp reaches destination node D. The computed fiber-level path fp is placed into the set $\mathcal{FP}$ and removed from $\mathcal{S}$. The algorithm then checks the shortest path in current $\mathcal{S}$ and repeats the same procedure. It stops when there are no more available paths or K paths have been found. In the worst case scenario, the algorithm has to visit all the nodes in the network to find a fiber-level path fp between S and D. Assume the maximum node degree in the network is $Deg(V)$, the complexity of Alg.1 is in $O(|V^2| \cdot Deg(V) \cdot K)$.

Algorithm 1: Multipath Computation

Input: $G(V, E), K, R(S, D, T_r)$
Output: One or multiple spectrum paths for R
Parameters: $\mathcal{S}$ is an ordered (via delay) set of paths starting from source S;
while *($|\mathcal{FP}| \leq K$)* **do**
  while $destination(fp) \neq D$ **do**
    Select min-delay path fp from $\mathcal{S}$
    for *all nodes v' connected to $destination(fp)$* **do**
      if *(v' not traversed in fp)* **then**
        create fp' by extending p to v'
        add fp' to $\mathcal{S}$
      end
      Put fp into $\mathcal{FP}$
      Remove fp from $\mathcal{S}$
    end
  end
end
Return $\mathcal{FP}$

2) Spectrum Assignment: Alg.2 takes $\mathcal{FP}$ as input and tries to find a single path solution first. In the first step, the algorithm identifies the spectrum path with maximum available bandwidth in $\mathcal{FP}$. It starts from the shortest fiber-level path and stops when a single spectrum path p_k on a fiber level path fp_k is found. When it fails to find a single path solution, the algorithm resorts to aggregate spectrum fragments from multiple spectrum paths. All spectrum paths are sorted in the increasing order of delay in the set P. Afterwards, the differential delay and bandwidth constraints are checked. If the differential delay between a spectrum path $p_k \in \mathcal{P}$ and the shortest path $p_1 \in \mathcal{P}$ is no larger than M, i.e., $pd_{p_k} - pd_{p_1} \leq M$, p_k is included in the solution. Finally, the algorithm outputs a solution when bandwidth requirement is satisfied. In the worst case scenario, Alg.2 has to check all the sub-carriers over all fiber links. Hence, the computational complexity of Alg.2 is in $O(|K| \cdot |F| \cdot |E|)$.

III. Performance Evaluation

We evaluate the proposed algorithms in Janos-US network (26 nodes and 84 links) [9]. We first compare the performance of the ILP model and heuristic algorithms. Afterwards, we only study the heuristic algorithms in the same network with

Algorithm 2: Spectrum Assignment

Input: $G(V, E)$, $R(S, D, T_r)$ and $\mathcal{FP}$
Output: One or multiple spectrum path(s) for R

//Step 1: Single spectrum path first;
for $k = 1$ *to* K, $fp_k \in \mathcal{FP}$ **do**
  identify the spectrum path with maximum consecutive sub-carriers, i.e., p_k;
  if $F(p_k) \geq T_r$ **then**
    A single spectrum path found; break;
  end
end
//Step2: Multiple Spectrum Paths;
for $k = 1$ *to* K, $fp_k \in \mathcal{FP}$ **do**
  for *all* $e_i \in fp_k$ **do**
    Find spectrum paths on the fiber-level path fp_k and put in the path set $\mathcal{P}_k$
  end
  for $k = 1$ *to* K, $fp_k \in \mathcal{FP}$ **do**
    Sort all available spectrum paths in the increasing order of delay; and put them in path set $\mathcal{P}$
    $N = |\mathcal{P}|$
  end
  for $k = 1$ *to* N **do**
    if $pd_{p_k} - pd_{p_1} \leq M$ **then**
      $F+ = F_{p_k}$
      if $F \geq T_r$ **then**
        Return spectrum paths and break;
      end
    end
  end
end

higher number of sub-carriers per fiber link, where ILP model becomes intractable and thus of little practical relevance.

The performance of proposed algorithms is evaluated against multiple factors, including the network load ($Erlang$), maximum number of fiber-level paths (K) and different differential delay (MP-1 with $128ms$ and MP-2 with $250us$). The ILP model is implemented in Gurobi Optimizer [10] and the heuristic algorithms are evaluated in an event-driven simulator implemented in Java. The network load ($Erlang$) is defined as $u * h$, where u is connection arrival rate and h is the mean connection holding time. Blocking ratio is defined as the percentage of the blocked connections out of total incoming demands. The confidence interval of all results is 95%.

A. Comparison of ILP and Heuristics

This study aims to show the effectiveness of the proposed ILP model in a reduced problem space, i.e., very small number of sub-carriers per fiber link; and compare the performance with proposed heuristic algorithms. The number of sub-carriers per fiber link are 16 and guard-band is 1 sub-carrier. Network load is generated with uniformly arrived requests with bandwidth requirement between 1 and 4 sub-carriers. When network is stable at the certain network load, a connection demand requesting between 4 and 6 sub-carriers is sent to a randomly selected source and destination pair. In each experiment, average 2000 connection requests are tested and the same experiment is repeated over 50 times for each routing scheme. We compare the performance of the ILP model and heuristics in terms of average blocking ratio. Maximum 4 fiber-level paths are used in the ILP model, i.e., K= max.$|\mathcal{FP}|$ = 4. In order to show the impact of the pre-computed paths, the maximum number of fiber-level paths in heuristics are set to be 10 and 40, i.e., K=10 and 40, respectively. The maximum differential delay in this study is $128ms$ [5] [6].

TABLE II
BLOCKING RATIO OF THE ILP OPTIMIZATION AND MULTIPATH (MP) HEURISTICS (JANOS-US)

Load ($Erlang$)	MP-heuristic (K= 10)	MP-heuristic (K= 40)	ILP (K= 4)
30	16%	8%	0%
35	24%	10%	6%
40	34%	18%	14%
45	40%	22%	20%

Table II shows the percentage of blocked connections out of all connection requests at each given network load. It can be seen that ILP model always outperforms the heuristic algorithms even within a small set of fiber-level paths, i.e. $K = 4$, when the problem is tractable. The high blocking of heuristics is caused by the limit number of pre-computed fiber-level paths. Despite of reduction in complexity, the solutions found by heuristics are limited in the pre-computed paths. With increasing of K, the blocking probability is reduced significantly. For instance, 22% connections are blocked at network load of $45Erlang$ with K = 40, while 40% connections are blocked with K = 10. Especially when the network load is high, the performance of heuristic with larger path set is getting close to the ILP model. For instance, 20% connections are blocked at network load of $45Erlang$ with ILP optimization model, while 22% blocking ratio is observed using heuristic with K =40. Unlike the ILP model, multipath heuristics can always obtain solutions in a reasonable time, whereas we have observed that the ILP failed to obtain a feasible solution in a larger network or with increasing network load where more paths are needed. Hence, we will only show the performance of heuristics in the following section.

B. Performance of the Heuristics

In this section, we only study the heuristic algorithms in the same network with 48 sub-carriers per fiber link. Single path routing(SP) based on the shortest-path-first algorithm is studied as performance benchmark. The connection demands arrive in a Poisson process and are uniformly distributed among all node pairs. The number of sub-carriers required by the connections varies between one and five sub-carriers. For each network load, average over 10,000 connections are generated in order to obtain a statistically relevant value. Finally, MP-1 and MP-2 denote multipath routing with maximum allowable differential delay $128ms$ and $250us$, respectively.

Figure 2 shows that multipath routing can effectively reduce the number of blocked connection requests, especially when

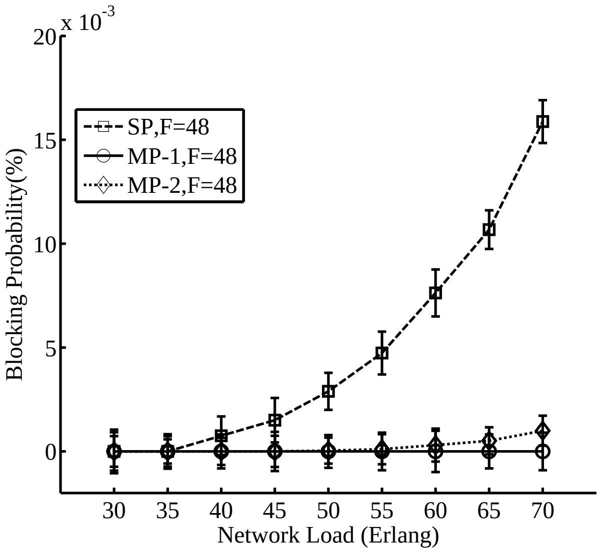

Fig. 2. Blocking probability with GB=1

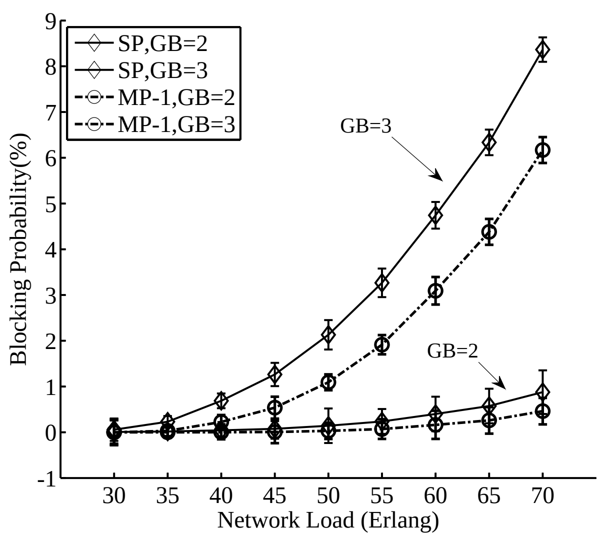

Fig. 3. Impact of guard-band size

network load is high, e.g., at $70Erlang$. When network is well-utilized, single path routing can cause significant amount of "wasted" spectrum fragments, leading to the increased blocking ratio. On the contrary, multipath routing can aggregate spectrum fragments to serve the incoming connections. It should be noted that only one sub-carrier is assigned as guard-band in Figure 2, i.e., $GB = 1$, which explains the relatively low blocking. The impact of guard-band size is shown in Figure 3, where only MP-1 is shown as an representative. It can be seen that the increasing of guard-band size leads to the increasing of blocking probability. However, multipath routing outperforms single path routing regardless of guard-band size.

Finally, the impact of maximum allowable differential delay is studied. Table III shows the results obtained at $70Erlang$.

TABLE III
IMPACT OF MAXIMUM ALLOWABLE DIFFERENTIAL DELAY IN JANOS-US NETWORK (48 SUB-CARRIERS PER FIBER LINK, $70Erlang$)

Guard-band	SP	MP-1 ($128ms$)	MP-2 ($250us$)
GB=1	0.0169%	0.0000%	0.0011%
GB=2	1.0718%	0.4620%	0.6739%
GB=3	8.3639%	6.1692%	6.9047 %

It can be seen that blocking probability slightly increases when the electronic layer has a smaller buffer to compensate the differential delay ($250us$ with MP-2). However, multipath routing has lower blocking ratio with all guard-band size (GB= 2 and 3) in both cases ($128ms$ with MP-1 and $250us$ with MP-2), comparing with the single path approach.

IV. CONCLUSION

In this paper, we presented a novel multipath de-fragmentation method to aggregate spectrum fragments instead of reconfiguring existing spectrum paths. We proposed an ILP optimization model and heuristic algorithms to study the effectiveness of multipath de-fragmentation in elastic optical networks. The numerical results showed that multipath routing is an effective de-fragmentation method as it improved spectrum efficiency and reduced blocking ratio. We also showed that differential delay issue of multipath routing is not an obstacle to applying multipath de-fragmentation in elastic optical networks. In our future work, we plan to study adaptive guard-band management in the proposed architecture, where the size of guardband is adapted to the network conditions and the level of parallelism in the optical multipath transmission.

V. ACKNOWLEDGMENT

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