Integraltransformationen

- Allgemeine Integraltransformation

$$f(t) \xrightarrow{\hat{T}} F(\omega)$$

$$\hat{T}f = F(\omega) = \int_a^b f(t)\mathcal{K}(\omega, t)dt$$

  - $\mathcal{K}(\omega, t)$ Kern
  - $F(\omega)$ Integraltransformierte von $f(t)$

- Fourier-Transformation

$$\hat{T}_F f = F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

  - Rücktransformation: $\hat{T}_F^{-1}F = f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega)e^{+i\omega t}d\omega$
  - Differentiationssatz: $\hat{T}_F d_t f = i\omega F$
  - Faltungssatz: $\hat{T}_F(f * g) = \hat{T}_F f \cdot \hat{T}_F g$
  - Delta-Funktion: $\delta(\omega - \omega_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(\omega-\omega_0)t}dt$

- Laplace-Transformation

$$\hat{T}_L f = F(\omega) = \int_0^{\infty} f(t)e^{-\omega t}dt$$

  - Rücktransformation: $\hat{T}_L^{-1}F = f(t) = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} F(\omega)e^{\omega t}d\omega$
    (Bromwich-Integral)
  - Differentiationssatz: $\hat{T}_L d_t f = \omega F(\omega) - f(0)$