Extremwertaufgaben

1. **Ohne Nebenbedingung**

$$f(x_1, \ldots, x_n) \qquad \mathbb{R}^n \mapsto \mathbb{R}^1$$

$$df = \sum_{i=1}^{n} \partial_{x_i} f dx_i = 0$$

$$\Rightarrow \quad \forall i : \partial_{x_i} f = 0$$

$$\text{Minimum:} \quad \det\left(\partial_{x_i}\partial_{x_j} f\right) \text{ positiv definit}$$

2. **Mit Nebenbedingungen**

$$\text{Nebenbedingungen: } g_k(x_1, \ldots, x_n) = 0; k = 1, \ldots, s; s < n$$

$$F(x_1, \ldots, x_n, \lambda_1, \ldots, \lambda_s) = f(x_1, \ldots, x_n) + \sum_{k=1}^{s} \lambda_k g_k(x_1, \ldots, x_n)$$

$$dF = \sum_{i=1}^{n} \left(\partial_{x_i} f + \sum_{k=1}^{s} \lambda_k \partial_{x_i} g_k \right) dx_i + \sum_{k=1}^{s} g_k d\lambda_k = 0$$

$$\Rightarrow \forall i = 1, \ldots, n : \partial_{x_i} F = \partial_{x_i} f + \sum_{k=1}^{s} \lambda_k \partial_{x_i} g_k = 0$$

$$\forall k = 1, \ldots, s : \partial_{\lambda_k} F = g_k = 0$$