[thm]Definition

Mesh Generation Techniques
Course Notes

Hang Si

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1 Introduction

Mesh generation refers to a growing topic concerned with discretizing geometric objects into small simple elements. Primary elements include triangle, quads, tetrahedra, hexahedra, prisms, etc. It finds numerous applications in scientific computing, computer graphics, solid modeling, computer aided design, geographic information system, and medical imaging.

Mesh generation is an essential pre-requsite for numerical methods such as finite element methods (FEM) or finite volume methods (FVM). However, it's importance is frequently neglected in the study of FEM or FVM. Some typical words about meshes in a FEM textbook would like the following (where, meshes are usually referred to as triangulations):
"... Let us assume the domain Ω has been subdivided into an admissible triangulation $\mathcal{T}_h$, ...", and "... Let $\mathcal{T}_h$ be a triangulation where h tends to 0, then the convergence holds",
the above $\mathcal{T}_h$ being provided in some way or other (with no further details given on this point). The construction of $\mathcal{T}_h$ is no longer a theoretical problem of interest.

Practically, people actually involved in mesh generation face a different problem. Many issues need to be addressed in order to get a computable mesh. In addition, the mesh size parameter h does not tend towards 0. The mesh construction is usually the most complicated, time-cosuming task in the process of FEM.

In most of advanced applications, mesh generation has become a bottleneck for the applying of numerical methods. Despite its importance, our understanding of mesh generation is rather superficial. A systemic study of this field is very necessary.

Acutally, mesh generation involves several different fields of study. These include mathematics (geometry and topology), computer science (algorithms and data structures), and engineering (numerical simulation). Mesh generation is a topic of a meaningful combination of these different approaches to problem solving is inevitable.

Purpose and scope of this course This course is about mesh generation techniques. Despite its big title, it would not be a comprehensive survey of the mesh technologies which have being developed so far. Instead of that, the practical aspects of the mesh technologies is emphasized. I will introduce some recent developments in mesh generation including new theorems, mesh algorithms, and some softwares.

The material I selected to present is aimed to help students or people who want to solve their meshing problems without much background of the mesh techniques. Although no special knowledge is required, a little knowledge of basic geometry, computational geometry and numerical analysis will be much helpful in understanding the course.

In this course, I restrict to the techniques to generate meshes with simplical elements, i.e., triangles in 2D and tetrahedra in 3D.

2 Basic Conceptions

Let S be a finite set of points in $\mathbf{R}^d$ ($d = 2$ or $d = 3$), the *convex hull* of S, denoted as $\mathrm{Conv}(S)$ is the smallest convex set in $\mathbf{R}^d$ that contains S.

Theorem 2.1 (Triangulation)**.** *A triangulation is ...*