Fluid-Structure Interaction with Domain Decomposition using Finite and Fast Boundary Element Methods

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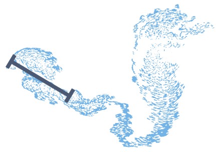
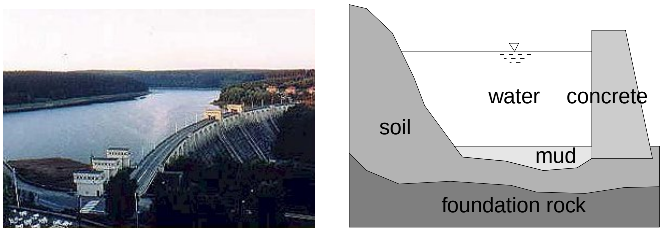


Motivation

The analysis of multi-field systems requires an appropriate consideration of each component. A dam-water-foundation system, for example, consists of a fluid domain, the concrete dam, and various layers of soils. The numerical simulation of waves caused by an earthquake excitation running through this complex system is challenging not only concerning a consistent physical model which describes well the constituents but also in terms of an efficient use of computing resources due the large scale inherent to the model.



The Weser-dam near Eupen, Belgium, and a schematic sketch of its cross-section

In this context, it is important to make use of the advantages of different numerical methods, such as the finite element method (FEM) and the boundary element method (BEM). Whereas the former has proven to cope with even highly non-linear problems, it is restricted to finite domains (e.g., the concrete dam). The boundary element method, on the other hand, is restricted to linear problems but can be easily applied to domains of infinite extension (e.g., the soil half-space).

Therefore, one major goal of this work will be the development of a domain decomposition method which allows for the coupling of differently and optimally discretized subsystems treated by either of the above methods.

Another important goal will be the development of a Fast BEM, which is necessary due to the high computational cost of the standard BEM with its fully populated matrices. The use of hierarchical or $\mathcal{H}$-matrices seems to be promising in this context.

Mathematical Models

- elastodynamic field equation

$$\boldsymbol{\nabla} \cdot [\mathbf{C} : \boldsymbol{\epsilon}(\mathbf{u})] + \mathbf{f} = \rho \mathbf{u}_{,tt}$$

- poroelastic field equations (Biot's theory)

$$\boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \mathbf{f} = \rho \mathbf{u}_{,tt} + \phi \rho_f \mathbf{v}_{,tt} \qquad \zeta_{,t} + \boldsymbol{\nabla} \cdot \mathbf{q} = a$$

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{\epsilon} + \lambda \mathrm{Tr}(\boldsymbol{\epsilon})\mathbf{1} - \alpha p \mathbf{1} \qquad \zeta = \alpha \mathrm{Tr}(\boldsymbol{\epsilon}) + \frac{\Phi^2}{R} p$$

$$\mathbf{q} = -\kappa \left(\boldsymbol{\nabla} p + \rho_f \mathbf{u}_{,tt} + \frac{\rho_a + \Phi \rho_f}{\Phi} \mathbf{v}_{,tt} \right)$$

- scalar wave equation

$$\Delta p - \frac{1}{c} p_{,tt} = g$$

Numerical Methods

- Finite Element Method

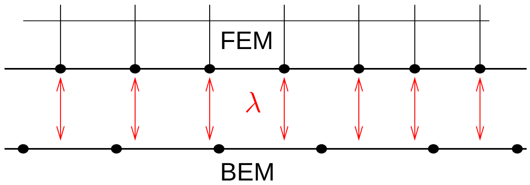
$$\mathbf{M}\mathbf{u}_{,tt} + \mathbf{K}\mathbf{u} = \mathbf{F}$$

- Boundary Element Method

$$\mathbf{c}\mathbf{u} = \int_\Gamma \mathbf{U} \phi_q ds *_t \mathbf{q} - \fint_\Gamma \mathbf{Q} \phi_u ds *_t \mathbf{u}$$

Domain Decomposition Method

- independent discretizations and different numerical methods



- Lagrange multipliers for weak transmission conditions

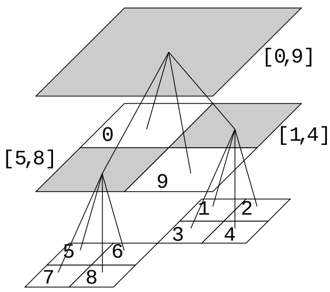
$$\int_\Gamma \lambda (u_1 - u_2) ds \rightarrow \mathrm{stat}\,.$$

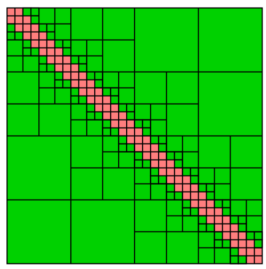
- preconditioner and fast solver for saddle point problem

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{0} & \mathbf{B}_1^T \\ \mathbf{0} & \mathbf{A}_2 & \mathbf{B}_2^T \\ \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{0} \end{bmatrix}$$

- distributed treatment of the sub-domains and parallel solution algorithm for each sub-domain

$\mathcal{H}$-Matrices





- construction of block cluster tree
- admissibility condition

$$\min\{\mathrm{diam}(Q_t), \mathrm{diam}(Q_s)\} \leq \eta \,\mathrm{dist}(Q_t, Q_s)$$

- near-field with *full-matrix* format and far-field with *low-rank* approximation
- storage requirements and matrix operations reduced from $\mathcal{O}(n^2)$ to $\mathcal{O}(n \log^k n)$