Model Reduction for CFD Models

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General Model Reduction Problem

Given a dynamical system

$$\Sigma = (f,h) : \begin{cases} \dot{x} & = f(x(t), u(t)) \\ y(t) & = h(x(t), u(t)) \end{cases}$$

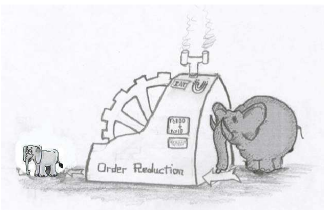
where

- $u(t) \in \mathbb{R}^i$ is the *input* function
- $y(t) \in \mathbb{R}^o$ is the *output* function
- $x(t) \in \mathbb{R}^n$ is the *state* of the system

approximate the system Σ (of order n) with a system

$$\hat{\Sigma} = (\hat{f}, \hat{h})$$

of *lower order* $k \ll n$, where $u(t) \in \mathbb{R}^i$, $\hat{y}(t) \in \mathbb{R}^o$ and $\hat{x}(t) \in \mathbb{R}^k$.



A pictorial representation of MR, by B. Lohmann, B. Salimbahrami and A. Yousefi.

Model Reduction Methods

A classification from Antoulas and Sorensen[a] divides the MR methods in

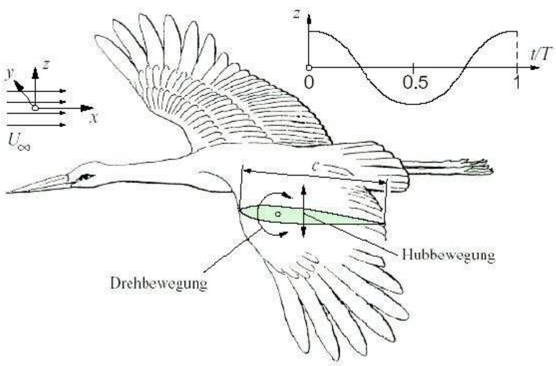
- SVD-based methods
  - for non-linear systems: POD, Empirical grammi-ans
  - for linear systems: balanced truncation, Hankel approximation
- Krylov-base methods: realization, interpolation, Lan-czos, Arnoldi
- SVD-Krylov base methods

SVD	Krylov
- $O(n^3)$ operations	+ $O(k^2 n)$ operations
- factorizations	+ only matrix-vector multiplications
+ global error estimators	- no global error estima-tor
+ preserve stability	- no guarantee for stabi-lity

The third class contains methods that combines the best feature of the first two classes. They represent an actual topic of research.

[a] A. Antoulas - *Approximation of Large-Scale Dynamical Systems* SIAM, 2005

CFD Model



Flapping wing propulsion can be described by Navier-Stokes equations

The governing equations are

$$\frac{d}{dt}\int_V w dV + \int_S (F^c - F^v) n dS - \int_S (w v_b) n dS + \int_V g dV = 0 \quad (1)$$

where

- w is the vector of conservative variables $w = (\rho, \rho v, \rho E)^T$
- F^c and F^v are the convective and viscous fluxes

$$F^c = \begin{pmatrix} \rho v \\ \rho v v^T + pI \\ \rho E v + p v \end{pmatrix}, \; F^v = \begin{pmatrix} 0 \\ T \\ T v - q \end{pmatrix}$$

- g is the additional source term $g = (0, \rho \dot{\varphi} \times v, 0)^T$

The first two integrals in (1) are the same as in the Navier-Stokes equations for fixed control volumes. The third integral results from the deformation of the control volumes in the fixed Cartesian coordinate system, with v_b as the velocity vector of the moving cell boundaries. The fourth integral is generated in the transformation from the fixed to the moving coordinate system. (See M.F.Neef - *Analyse des Schlagflugs durch numerische Strömungsberechnung*, Dissertation TU Braunschweig, 2002)

Finite Volume Method will be used to discretize the PDEs. Different method for generating reduced-order model will be considered and compared concerning computational efficiency and errors in the model.