Model Reduction for CFD Models

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General Model Reduction Problem

Given a dynamic system

$$\Sigma = (f,h) : \begin{cases} \dot{x} & = f(x(t),u(t)) \\ y(t) & = h(x(t),u(t)) \end{cases}$$

where

- $u(t) \in \mathbb{R}^i$ is the *input* function
- $y(t) \in \mathbb{R}^o$ is the *output* function
- $x(t) \in \mathbb{R}^n$ is the *state* of the system

approximate the system Σ (of order n) with a system

$$\hat{\Sigma} = (\hat{f},\hat{h})$$

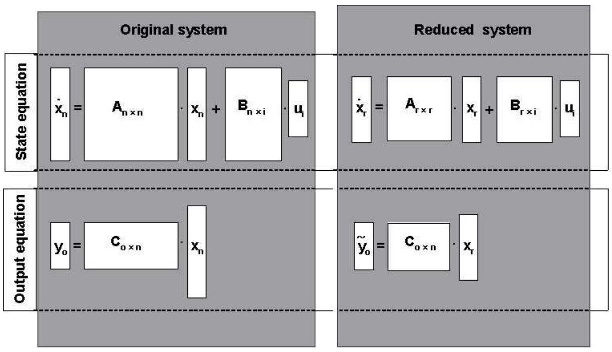
of *lower order* $k \ll n$, where $u(t) \in \mathbb{R}^i$, $\hat{y}(t) \in \mathbb{R}^o$ and $\hat{x}(t) \in \mathbb{R}^k$.

MR Problem for Linear Systems

Assume f,h to be linear functions:

$$\Sigma = (f,h) : \begin{cases} \dot{x} & = Ax+Bu \\ y(t) & = Cx \end{cases}$$

The model reduction can be depicted as



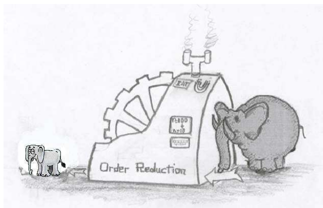
MR Methods

A classification from Antoulas and Sorensen[a] divides the MR methods in

- SVD-based methods
  - for non-linear systems: POD, Empirical grammians
  - for linear systems: balanced truncation, Henkel approximation
- Krylov-base methods: realization, interpolation, Lanczos, Arnoldi
- SVD-Krylov base methods

SVD	Krylov
- $O(n^3)$ operations	+ $O(k^2n)$ operations
- factorizations	+ only matrix-vector multiplications
+ global error estimators	- no global error estimator
+ preserve stability	- no guarantee for stability

The third class contains methods that combines the best feature of the first two classes. They represent an actual topic of research.



A pictorial representation of MR, by B. Lohmann, B. Salimbahrami and A. Yousefi.

[a] A. Antoulas and D. Sorensen - *Approximation of large-scale dynamical systems: An overview*, Technical report, 2001

CFD Model

A short description of the physical problem will be given in a couple of weeks.