ADVECTION OPERATOR

$$u_{,t} + a u_{,x} = 0$$

a : Advection Velocity (=const.)

Exact Solution:

$$u(x,t) = u_0(x - at)$$

- Propagation With **Finite** Velocity a
- No Damping
- Model For: Euler Eqns.

TRANSIENT DIFFUSION OPERATOR

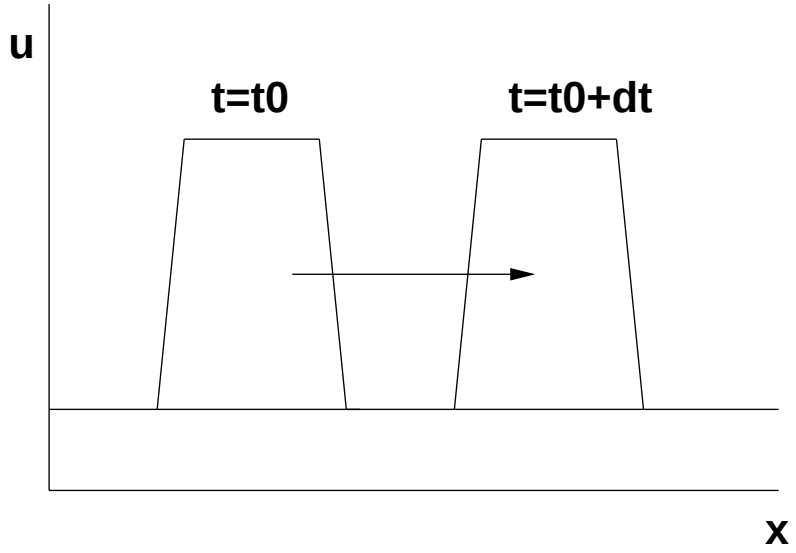
$$u_{,t} = k u_{,xx}$$

Exact Solution:

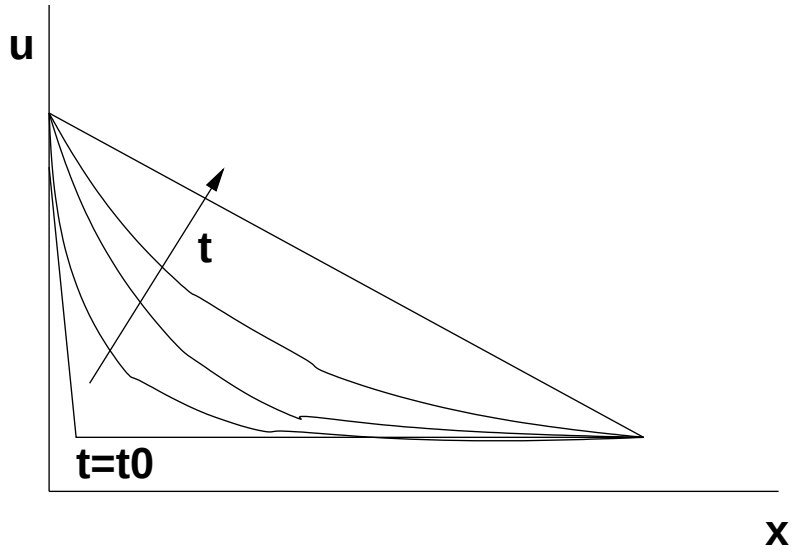
$$u = u_0 e^{-k\omega^2 t} e^{i\omega x}$$

- Propagation With **Infinite** Velocity
- Decay In Time With Factor ω^2
  Higher Spatial Frequencies Decay Faster
- $\Rightarrow$ Damping
- Model For: Viscous Part of NavSto

SOLUTIONS OF MODEL EQUATIONS



Advection Operator



Diffusion Operator

HELMHOLTZ OPERATOR

$$u_{,xx} + \lambda u = 0$$

Exact Solution:

$$u = u_0 e^{i\omega x} \quad , \quad \omega = \sqrt{\lambda}$$

- Oscillatory In Space
- Model For: Transient Small Disturbance Components of Euler/NavSto

HELMHOLTZ OPERATOR (Cont)

$$\nabla^2 \tilde{p} + \left(\frac{\omega}{c}\right)^2 \tilde{p} = 0$$

2-D Point Source:

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial \tilde{p}}{\partial r} + \left(\frac{\omega}{c}\right)^2 \tilde{p} = 0$$

$\Rightarrow$ Hankel Functions

- Wave Character
- Spatial Damping (Energy Considerations)
- Similar and More Pronounced in 3-D

LAPLACE OPERATOR

$$\nabla^2 \Phi = 0$$

Closed-Form Solutions:

- Uniform Flow:

$$\Phi = (U_\infty x, V_\infty y, W_\infty z)$$

$$u = U_\infty \quad , \quad v = V_\infty \quad , \quad w = W_\infty$$

- Source/Sink (3-D):

$$\Phi = -\frac{E}{4\pi r} = -\frac{E}{4\pi\sqrt{x^2+y^2+z^2}}$$

$$u = \frac{E}{4\pi}\frac{x}{r^3} \quad , \quad v = \frac{E}{4\pi}\frac{y}{r^3} \quad , \quad w = \frac{E}{4\pi}\frac{z}{r^3}$$

LAPLACE OPERATOR (Cont)

- Vortex:

$$\Phi = \frac{\Gamma}{2\pi}\Theta = \frac{\Gamma}{2\pi} artg(\frac{y}{x})$$

$$u = -\frac{\Gamma}{2\pi}\frac{y}{x^2+y^2} \quad , \quad v = \frac{\Gamma}{2\pi}\frac{x}{x^2+y^2}$$

$$v_r = 0 \quad , \quad v_\Theta = \frac{\Gamma}{2\pi r}$$

- From Superposition Principle:
  - Semi-Infinite Body
  - Dipole
  - Cylinder/Sphere
  - General Bodies
  - Wings/Wakes
- Model For: External Subsonic Flow