Introduction to Scientific Computing

Exercise 1: Lyapunov direct method

(a) Compute the derivative of the function $V(x,y) = x^2 + y^2$ with respect to the following system of differential equations (4 points)

$$\dot{x} = y + x(1 - x^2 - y^2),$$

 $\dot{y} = -x + y(1 - x^2 - y^2).$

(b) Using Lyapunov direct method investigate for stability the zero solution (10 points) of the nonlinear system

$$\dot{x} = -xy^2, \qquad \dot{y} = 3yx^2$$

compare the result with the eigenvalue method.

14 points