TECHNISCHE UNIVERSITÄT DRESDEN

GRK 2075

Institute for Structural Analysis

Modelling of Wood: Multiphysics and Polymorphic Uncertainty

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Institute for Structural Analysis

Modelling of Wood: Multiphysics

Motivation





Timber Engineering



hygro-mechanical behaviour

- coupled hygro-mechanical properties
- moisture transport

long-term behaviour

- viscous creeping and creep failure
- mechano-sorptive creeping

inhomogeneities

- material structure
- uncertainty modelling

outlook

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outlook

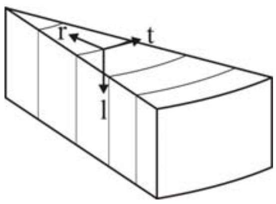
Influence of Moisture Content

elastic potential

$$\Psi = \frac{1}{2}\,\underline{\varepsilon} : \underline{\underline{C}}^{el}(m) : \underline{\varepsilon}$$

HOOKE's law

$$\underline{\sigma} = \underline{\underline{C}}^{el}(m) : \underline{\varepsilon}$$



elasticity tensor in local material coordinates

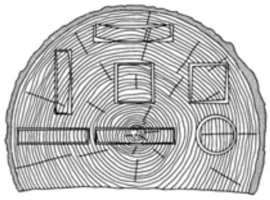
$$\left(\underline{\underline{C}}^{el}\right)^{-1} = \begin{bmatrix} \frac{1}{E_r(m)} & -\frac{v_{rt}}{E_t(m)} & -\frac{v_{rl}}{E_l(m)} & 0 & 0 & 0 \\ -\frac{v_{rt}}{E_t(m)} & \frac{1}{E_t(m)} & -\frac{v_{tl}}{E_l(m)} & 0 & 0 & 0 \\ -\frac{v_{rl}}{E_l(m)} & -\frac{v_{tl}}{E_l(m)} & \frac{1}{E_l(m)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{rt}(m)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{tl}(m)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{rl}(m)} \end{bmatrix}$$

Influence of Moisture Content

swelling and shrinkage

volume change due to inclusion/elimination of water molecules into/from the cell wall

- constrained free swelling/shrinkage → mechanical stresses
- influence of local material directions



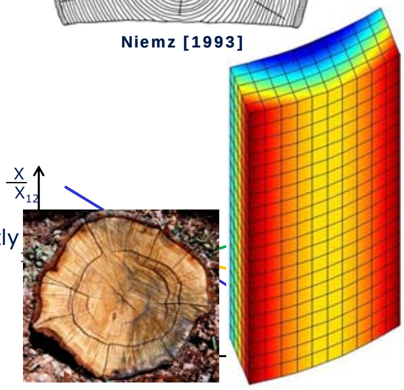
Niemz [1993]

in $\underline{\varepsilon}(m) = \underline{\beta} \cdot (m - m_{ref})$ **moisture content on mechanical properties**

- $$\underline{\beta} = \begin{bmatrix} \beta_r \\ \beta_t \\ \beta_l \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
- $\beta_t \approx 2 \cdot \beta_r$ s
- $\beta_l \approx 0.1 \cdot \beta_r$ s significantly

ergy increases

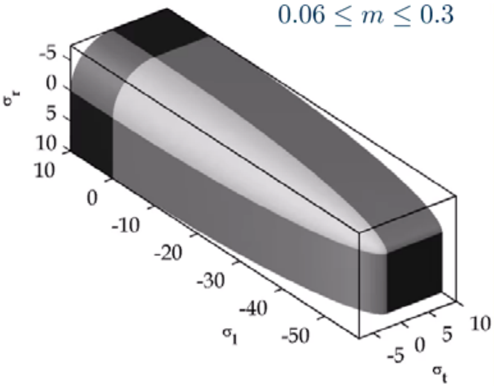
→ **saturation point**

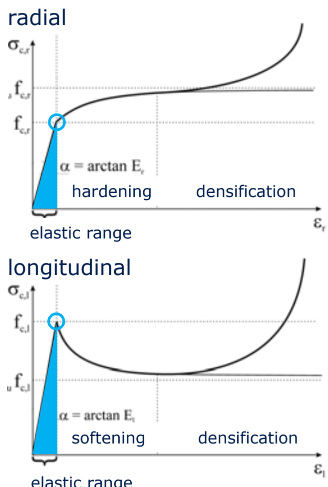


Coupled Multi-Surface Plasticity Model

moisture-dependent strength properties

moisture-dependent yield surface





SCHMIDT [2009]

Coupled Multi-Surface Plasticity Model

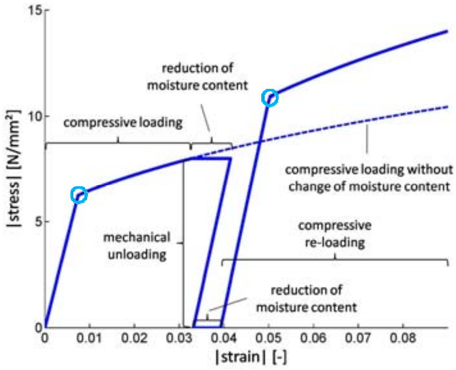
C_1-continuous, anisotropic, moisture-dependent yield criterion

$$f = \underline{\boldsymbol{\sigma}} : \underline{\underline{\boldsymbol{b}}}(m) : \underline{\boldsymbol{\sigma}} + q - 1 \leq 0$$

$$\underline{\underline{\boldsymbol{b}}}(m) = \underline{\boldsymbol{b}}_s \otimes \underline{\boldsymbol{b}}_f(m)$$

$$\underline{\boldsymbol{b}}_s = \frac{1}{2} \cdot \underline{\mathbf{1}} - \frac{1}{2} \cdot sign\left(\underline{\boldsymbol{\sigma}} \circ \underline{\mathbf{1}}\right)$$

$$\underline{\boldsymbol{b}}_f(m) = \begin{bmatrix} \frac{1}{f_{c,r}(m)^2} \\ \frac{1}{f_{c,t}(m)^2} \\ \frac{1}{f_{c,l}(m)^2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



SAFT, KALISKE [2011]

Hygro-Mechanical Coupling

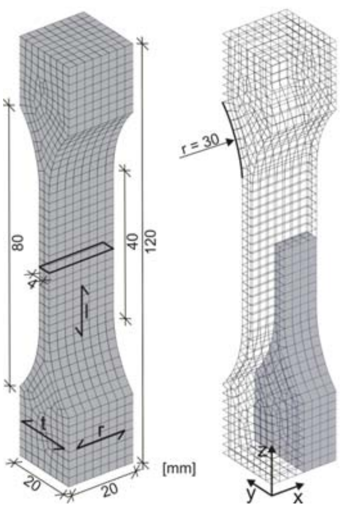
bone shaped spruce sample

KRAUSS [1988]

- swelling pressure in longitudinal direction
- increase of ambient relative humidity

$$RH_{init} = 0.001 \qquad RH_{fin} = 0.99$$

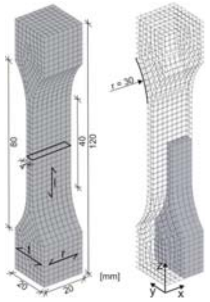
- clamped top and bottom

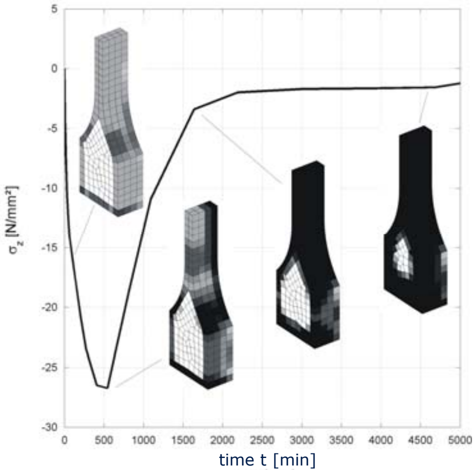


SAFT, KALISKE [2011]

Hygro-Mechanical Coupling

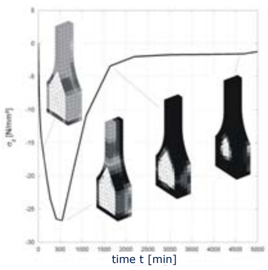
longitudinal stresses
and plastic zones

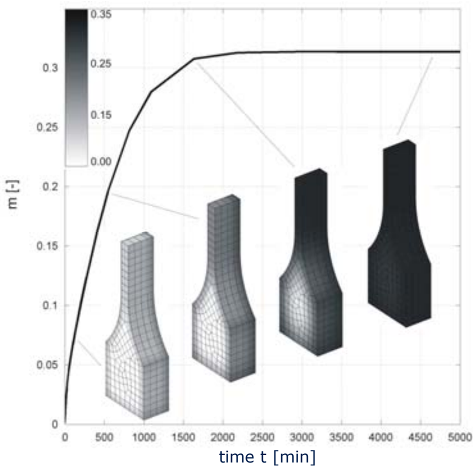




Hygro-Mechanical Coupling

moisture distribution

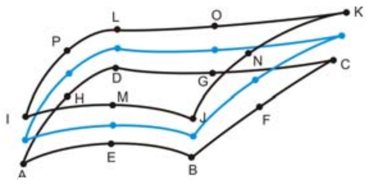


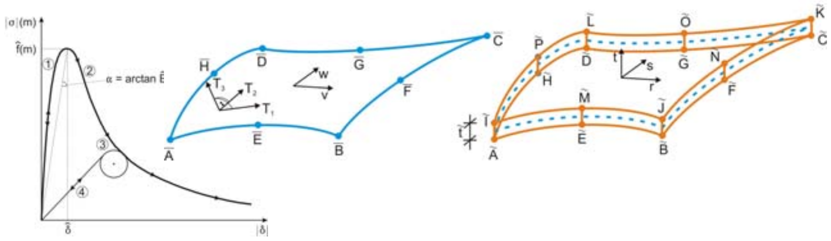


Hybrid Interface-Element

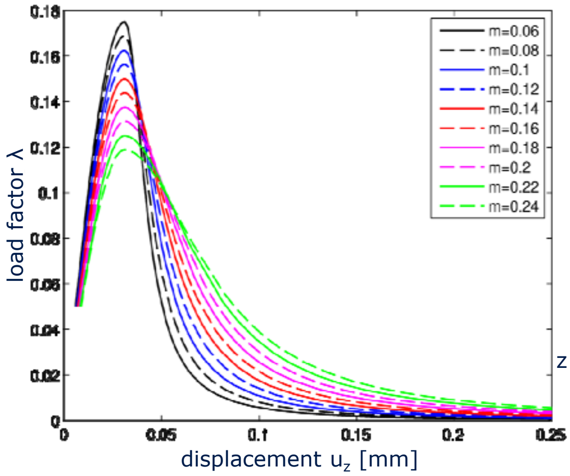
$$\boldsymbol{\delta} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \boldsymbol{B_u} \cdot \boldsymbol{u}$$

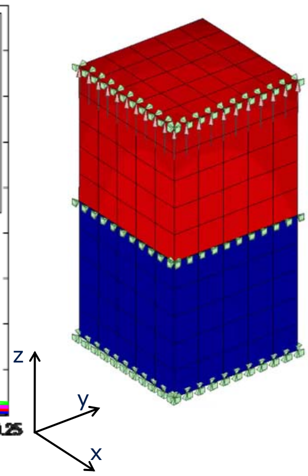
$$\nabla m = \boldsymbol{B_m} \cdot \boldsymbol{m}$$



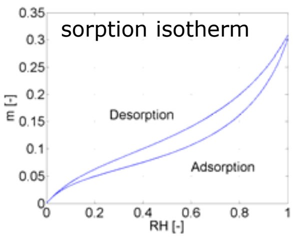


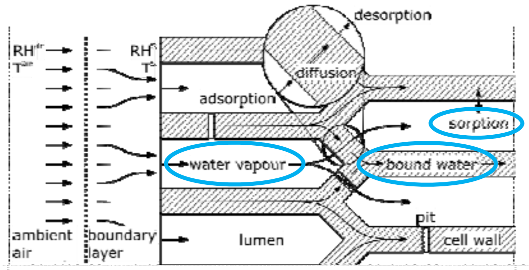
Moisture Induced Tensile Failure





Water Transport





surface emission

boundary conditions (3D)

$\boldsymbol{n} \cdot \mathbf{J}_b = 0$

$\boldsymbol{n} \cdot \mathbf{J}_v = k_v \cdot (p_v^s - p_v^{air})$

$\boldsymbol{n}$	normal vector
k_v	surface emissivity [mm/s]
p_v^s, p_v^{air}	water vapour pressures (surface, ambient air) [Pa]

diffusion

FICK's Law (transient)

- cell wall (bound water)

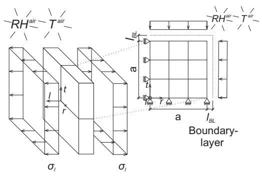
$$\frac{\partial c_b}{\partial t} = \nabla \cdot (D_b \cdot \nabla c_b) + \dot{c}$$

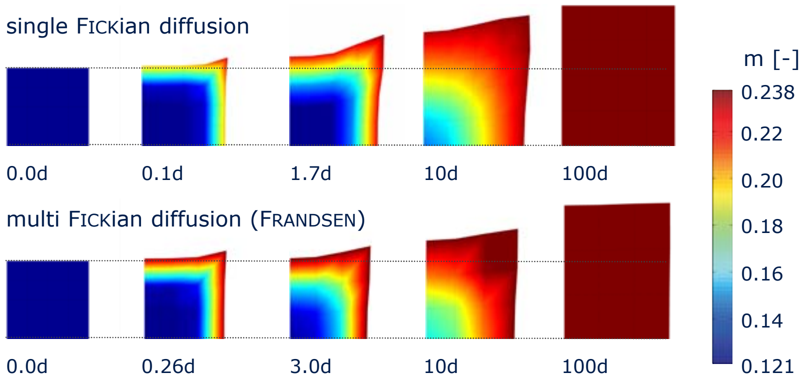
- lumen (water vapour)

$$\frac{\partial c_v}{\partial t} = \nabla \cdot (D_v \cdot \nabla c_v) - \dot{c}$$

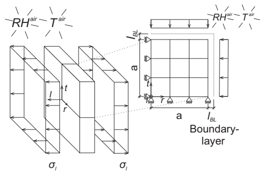
Comparison of Diffusion Models

displacements and moisture distribution





Comparison of Diffusion Models



moisture transport inside the sample

single FICKian diffusion

multi FICKian diffusion (FRANDSEN)

moisture content m [-]   stress σ_{rt} [N/mm²]

Introduction

history

- Lucas CRANACH the El
- made 1506
- pa

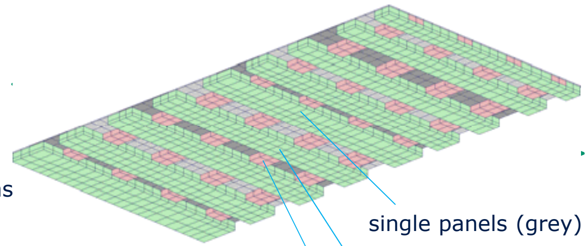
cur

- pa
- cr
- la
- frame is currently

Realisation

discretisation

- 2247 elements
- quadratic shape functions
- transient moisture transport simulation
- varying grain directions



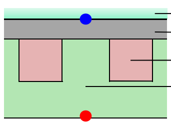
setup

- 636 x 1214 x 14 mm
- panel depth: 3 mm
- used material formulations:
  - frame: spruce
  - panels: cherry
- very low permeability of painting surface
- loadcases: change of relative humidity (RH) 55% → 35%

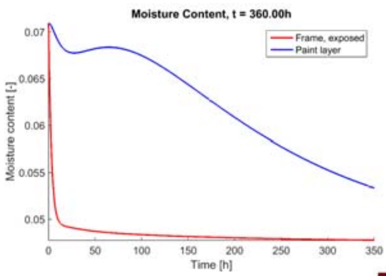
Results

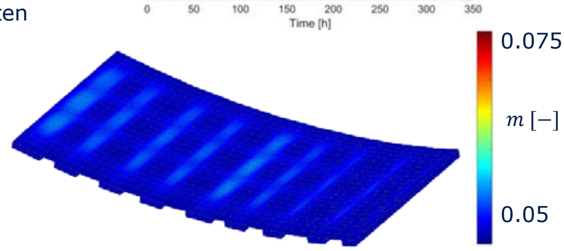
decrease of relative humidity

- more than 15 days until EMC
- concave bending
- realistic distribution of MC



evaluation points





Moisture Influence

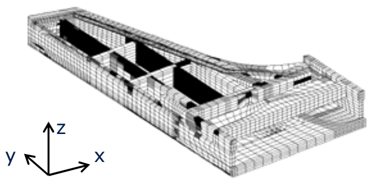
pianoforte

mechanical load: tensioned strings (ca. 52 kN)

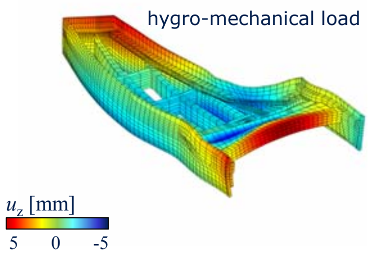
hygrical load: increase of relative humidity

$RH_{init} = 0.4 \qquad RH_{fin} = 0.7$

(thunderstorm)



displacement u_z [mm]



Timber Engineering

hygro-mechanical behaviour

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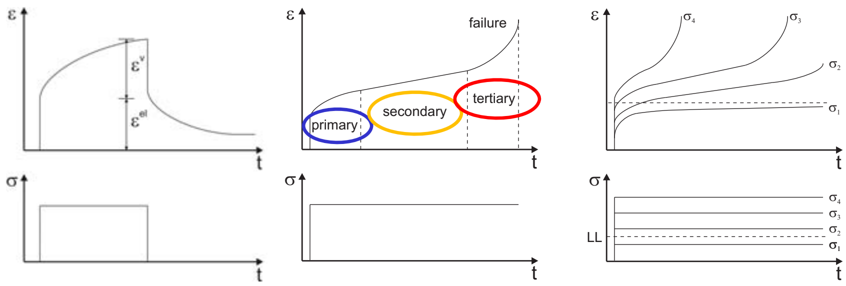
inhomogeneities

- material structure
- uncertainty modelling

outlook

Creeping



elastic and viscous strains due to constant stress

creeping phases

SL-dependent creeping

- SL: stress level
- LL: limit of linearity

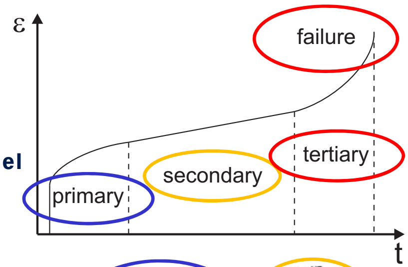
Stress Level-Dependent Creeping

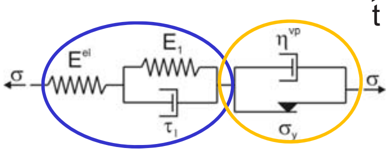
linear viscoelastic $\boldsymbol{SL \leq LL}$

nonlinear viscoelastic-viscoplastic $\boldsymbol{SL > LL}$

extended standard-solid body-model

- spring: instantaneous, elastic deformations
- KELVIN: time-dependent, reversible creeping
- BINGHAM: time-dependent, irreversible deformations for $\sigma > \sigma_y$





tertiary creeping and creep failure

- concept of strain-energy density

$$e(t) = \int_0^t \sigma \cdot \dot{\varepsilon}\, dt \qquad e(t) \geq e_{crit}$$

Consideration of Moisture Influence

moisture dependent material parameter

primary

$\varphi_\infty \quad \tau_1$

$E_1(m) = \varphi_\infty \cdot E^{el}(m)$

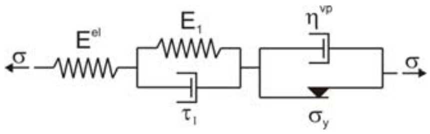
secondary

$\eta^{vp}(SL)$

$\sigma_y(m) = f(m) \cdot LL(m)$

tertiary

$e_{crit}(m)$



hygro-expansion and mechano-sorptive creeping

- loading $\quad \varepsilon = \varepsilon^{u} - \varepsilon^{he} - \varepsilon^{ms}$
- structural response $\quad \varepsilon = \varepsilon^{el} + \varepsilon^{ve} + \varepsilon^{vp}$

Consideration of Moisture Influence

deformation dependent hygro-expansion

$$\varepsilon^{he} = \beta_{mod} \cdot (m - m_{ref})$$

$$\beta_{mod} = \beta \cdot (1 + d\beta\,(\varepsilon + \varepsilon^{ms}))$$

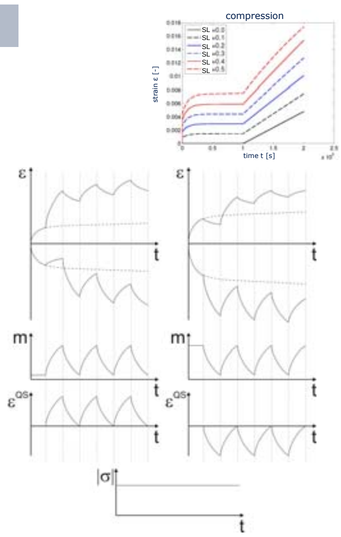
mechano-sorptive creeping

massive increase in time-dependent deformations during adsorption and desorption

$$\dot{\underline{\varepsilon}}^{ms} = \underline{\underline{\kappa}}(\underline{\varepsilon}) : \dot{\underline{\varepsilon}} \cdot |\dot{m}|$$

- tensor of mechano-sorptive parameters $\underline{\underline{\kappa}}$

$$\underline{\underline{\kappa}}(\underline{\varepsilon}) = \mathrm{diag}(\kappa_{ii})$$



strain development due to moisture changes

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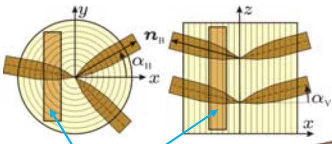
Geometrical Model

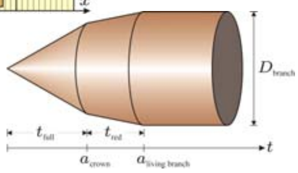
branches in spruce

- geometrical shapes
- position branches and pith by *tree model*



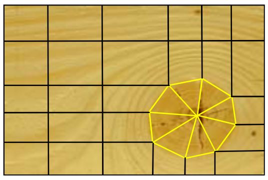
timber piece

- position in tree
- dimensions

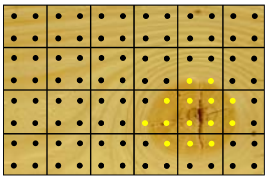


FE-model

direct



indirect



Modelling of Fibre Course

fibre orientation
- undisturbed in direction of stem axis
- deviation in area of branches

function - model
- load transfer – principal stresses
- transport – fluid mechanics

stream line approach

potential flow
- velocity potential $\Phi\,(x,y)$
- stream function $\Psi\,(x,y)$

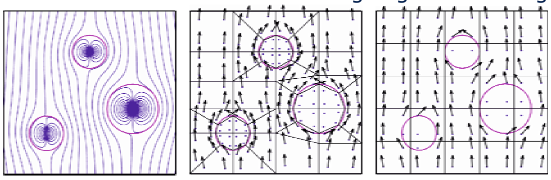
$$v_x = \frac{\partial \Psi}{\partial y},\ -v_y = \frac{\partial \Psi}{\partial x}$$

- stream lines by

$\Psi\,(x,y) = \text{const}$

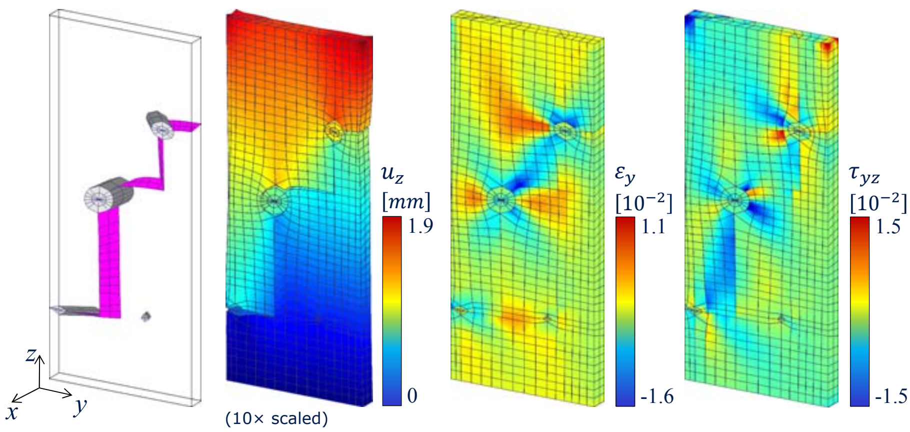
→ fibre course

streamlines　discrete meshing　regular meshing



Brittle Failure Analysis

segment of board No. 3 (SM, knot type 3)

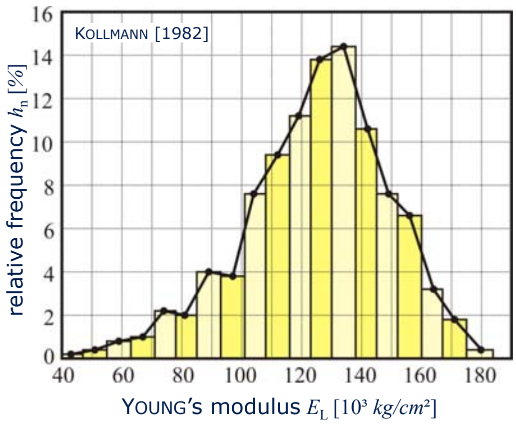


(10× scaled)

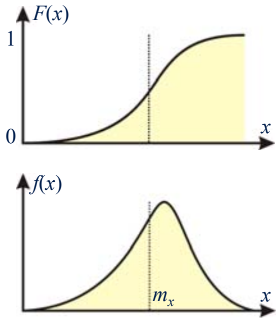
Uncertain Material Parameters

relative frequency distribution

YOUNG's modulus of ash wood (1546 samples)



randomness



e.g. WEIBULL distribution

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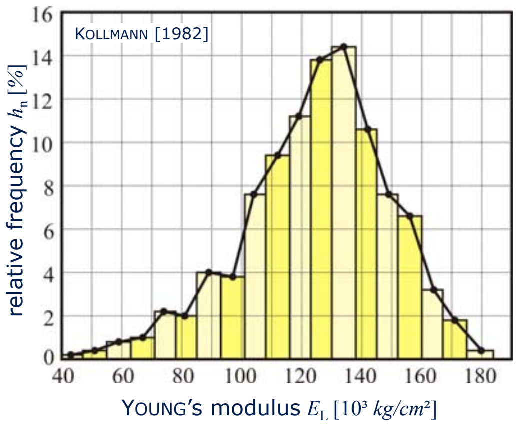
Material Inhomogeneities

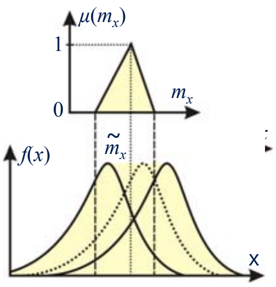
Uncertain Material Parameters

relative frequency distribution

YOUNG's modulus of ash wood (1546 samples)

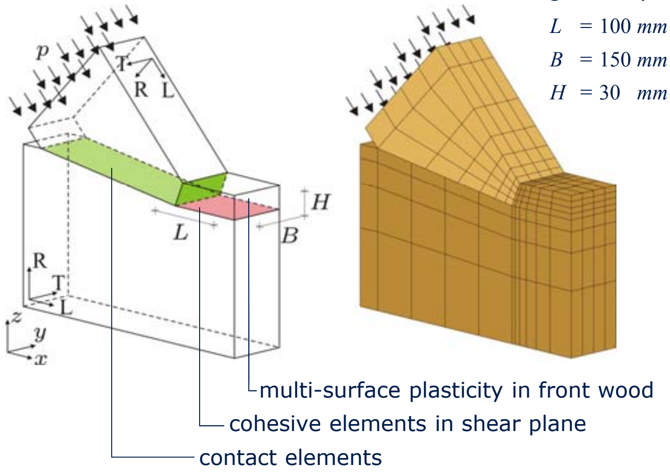


Fuzzy randomness



Face Staggered Joint

system data



geometry

$L = 100\ mm$
$B = 150\ mm$
$H = 30\ mm$

material

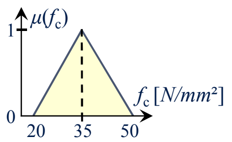
$[N/mm^2]$
$E_R = 820$
$E_T = 430$
$E_L = 13200$
$G_{RT} = 40$
$G_{TL} = 730$
$G_{RL} = 660$
[-]
$\nu_{RT} = 0.24$
$\nu_{TL} = 0.45$
$\nu_{RL} = 0.45$

Face Staggered Joint

fuzziness

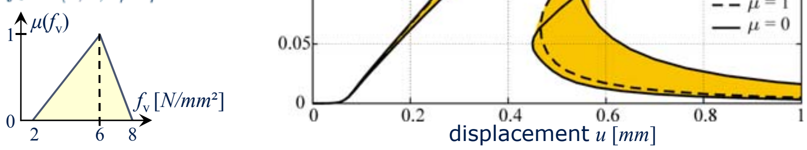
compression strength

$f_c = \langle 20, 35, 50 \rangle\ N/mm^2$



shear strength

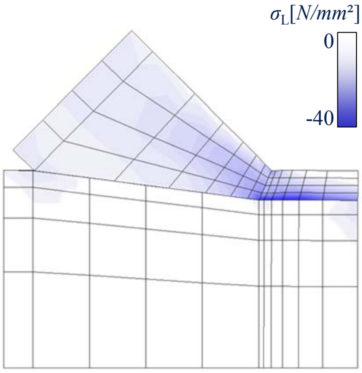
$f_v = \langle 2, 6, 8 \rangle\ N/mm^2$



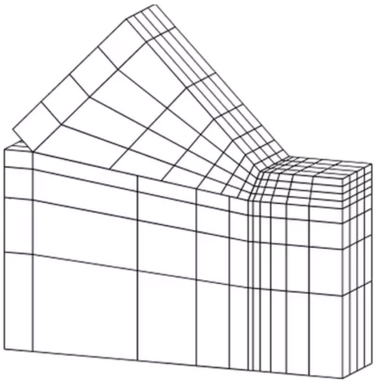
Face Staggered Joint

failure $\mu(\lambda) = 1$

local compressive stresses in fibre direction



shear failure front wood



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hygro-mechanical behaviour

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inhomogeneities

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outlook

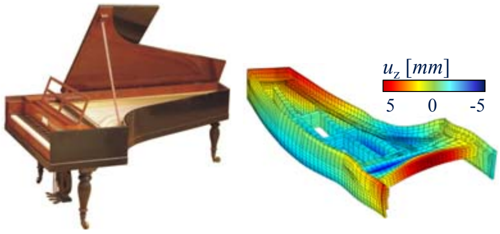
Outlook

material models

- hygro-mechanical coupling
- plasticity formulation large strains
- adaptive meshing of cohesive elements
- determination of crack path

structural investigations

- experimental validation
- timber connections
- molded wooden tubes
- dynamic analysis



analysis of historical keyboard instruments

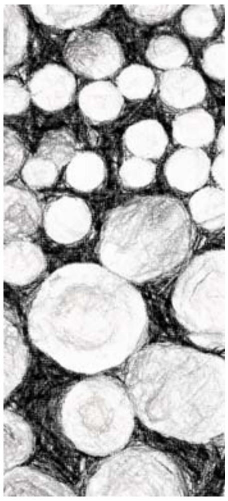
TU Dresden, ISH [2015]

analysis of formed wooden structures

Modelling of Wood: Polymorphic Uncertainty

Outline

Content



Outline

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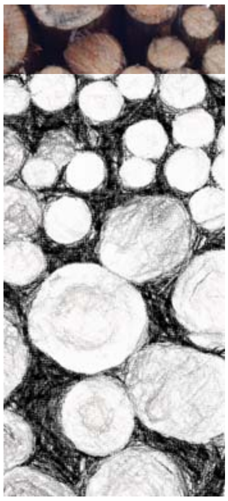


Uncertainties in Wood

wood construction

- design rules
- inhomogeneities indirectly captured
- complex material models

structural inhomogeneities

- growth inhomogeneities
- modelling of knots

material inhomogeneities

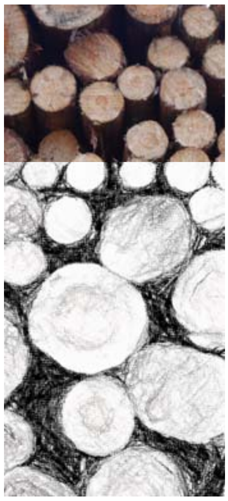
- variation of material parameters
- spatially dependent uncertainties

→ numerical simulation of wooden structures considering uncertainty



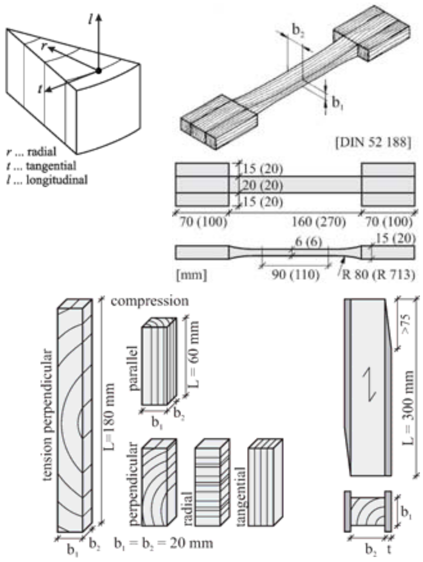


Content



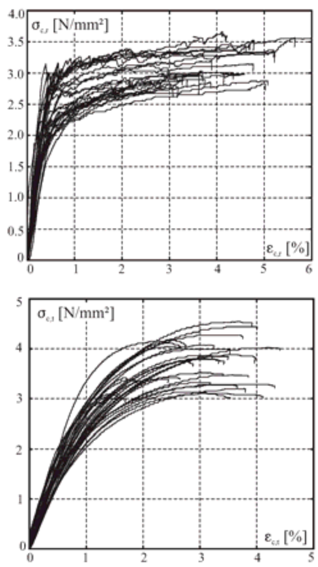
Material Specifications

- Norway spruce specimen
- material strengths
  $f_{t,90}$, $f_{t,l}$, $f_{c,r}$, $f_{c,t}$, $f_{c,l}$, f_v
- elasticity moduli
  E_r, E_t, E_l
- all tests according to DIN and DIN EN
- 45/30 samples for each test mode
- standard climate conditions
  $T = 20 \pm 2°\mathrm{C}$, $RH = 65 \pm \%$



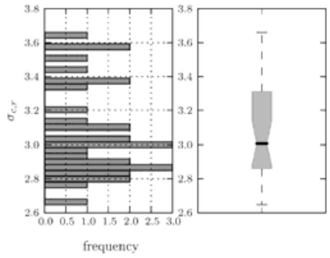
Evaluation of Test Results

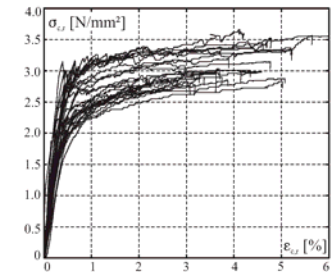
- stable experimental conditions
  - → constant temperature & humidity
- no strong correlation between geometric/climate conditions and test results



Evaluation of Test Results

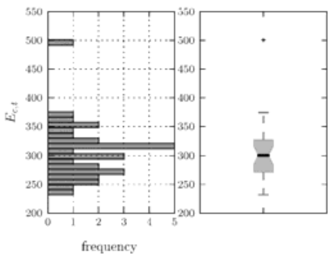
- stable experimental conditions
  - ➞ constant temperature & humidity
- no strong correlation between geometric/climate conditions and test results
- experimental data for material parameters with outliers





Utilized Data Base

- considerably reliable circumstances
  - ➞ constant temperature & humidity
- no strong correlation between geometric/climate conditions and test results
- experimental data for material parameters with outliers



parameter	E_r	E_t	E_l	$f_{t,90}$	$f_{t,l}$	$f_{c,r}$	$f_{c,t}$	$f_{c,l}$	f_v
standard	DIN 52192	DIN 52192	DIN 52185	EN 408	DIN 52188	DIN 52192	DIN 52192	DIN 52185	EN 408
samples	41	44	28	30	30	45	45	30	30
$\bar{m}$ $[N/mm^2]$	656	298	17132	2.64	121.64	3.09	3.64	43.60	5.77
$\bar{\sigma}$ $[N/mm^2]$	107	36	2211	0.33	18.20	0.23	0.43	2.07	0.73

Data Models for Uncertainty

p-box	variability, imprecision, unknown cdf many imprecise unvalued data	imprecise probability models
fuzzy randomness	variability, imprecision, unknown cdf many imprecise assessed data	
fuzzy probability based randomness	variability, incompleteness, known cdf small amount of precise data	

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Spruce Specimen Tests

Data Models for Uncertainty

p-box	variability, imprecision, unknown cdf many imprecise unvalued data	imprecise probability models
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properties of experimental data

- average sample size: 32.3
- "precise" data measurements
- probably random
  - → *fuzzy probability based randomness*
    (known distribution type with fuzzy parameters)

Outline

Content

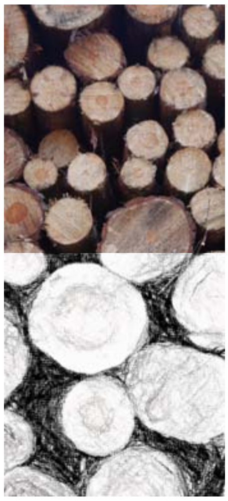
motivation

spruce specimen tests

uncertain structural analysis

numerical example

conclusion and outlook

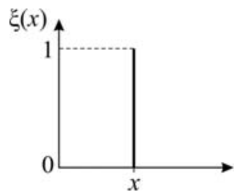


Uncertain Structural Analysis

Fuzziness

– characterizing function $\xi(\cdot)$ for precise set $A \subseteq \mathbb{R}$

$$\xi_A : \mathbb{R} \to \{0;1\}, x \mapsto \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$



Fuzziness

– characterizing function $\xi(\cdot)$ for precise set $A \subseteq \mathbb{R}$

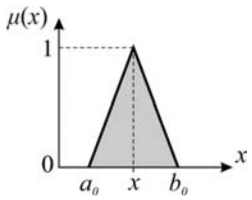
$$\xi_A : \mathbb{R} \to \{0;1\}, x \mapsto \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

– extension to non-precise data set

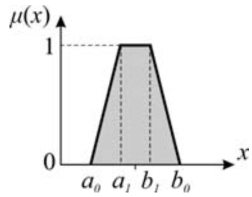
$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X, \mu_{\tilde{A}}(x) \geq 0\}$$

→ membership function $\mu(\cdot)$

$$\mu : \mathbb{R} \to [0,1]$$
$$\exists\, x_0 \in \mathbb{R} : \mu(x_0) = 1$$
$$A_\alpha := \{x \in \mathbb{R} \mid \mu_{\tilde{A}}(x) \geq \alpha\} = [a_\alpha, b_\alpha]$$



$$\tilde{x} = \langle a_0, x_0, b_0 \rangle$$



$$\tilde{x} = \langle a_0, a_1, b_1, b_0 \rangle$$

Fuzzy Randomness

– random variable X unsuitable for statistical population

– introduction of fuzzy probability

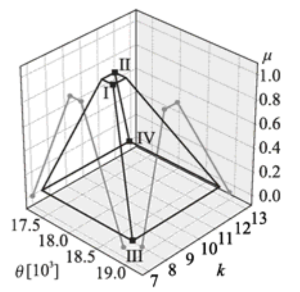
$$\tilde{P} = (P_\alpha)_{\alpha \in (0;1]}$$

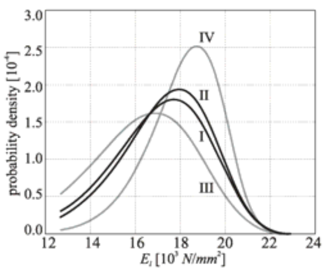
and fuzzy probability distribution

$$\tilde{P}_X = ((P_X)_\alpha)_{\alpha \in (0;1]}$$

→ fuzzy cdf with uncertain parameters $\tilde{\theta}_i$

$$\tilde{F}_X = ((F_X)_\alpha)_{\alpha \in (0,1]}$$
$$\tilde{F}_X = (\{F_{\theta_1 \times \theta_2} \mid \theta_1 \in \tilde{\theta}_{1,\alpha}, \theta_2 \in \tilde{\theta}_{2,\alpha}\})_{\alpha \in (0,1]}$$

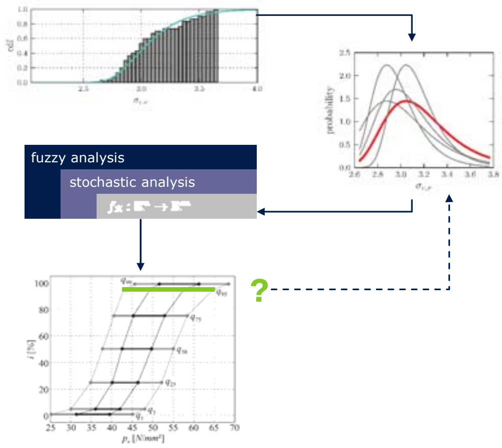




Computation with Uncertain Inputs

workflow for polymorphic uncertainty analysis

1. data analysis
2. uncertainty modelling
3. uncertainty analysis
4. decisions



Computational Procedure

– three loop computational model

fuzzy analysis

$\tilde{\theta}_i = (\bar{\theta}_{1,i} \times \bar{\theta}_{2,i})$

stochastic analysis

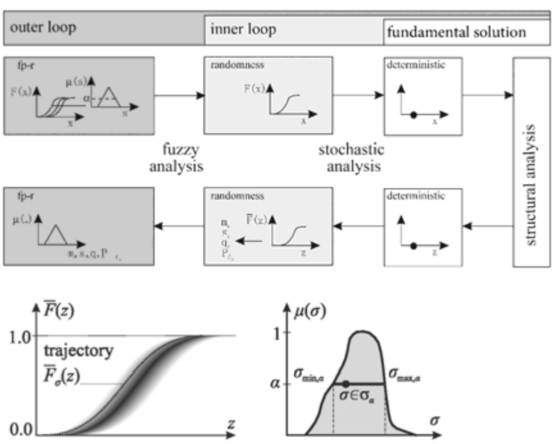
$F_{\theta_i}(x)$

fundamental solution

$f_Z : \mathbb{R}^n \rightarrow \mathbb{R}^m$

$\bar{F}_j(z)$

$\tilde{\sigma} = \langle \sigma_{\min,\alpha}, \sigma_{\max,\alpha} \rangle$



Content

Timber Board

– timber board with 4 knotholes

$$t \times b \times l = 18 \times 150 \times 350\,mm$$

– computation of ultimate load

$$p_u = \max(p)$$

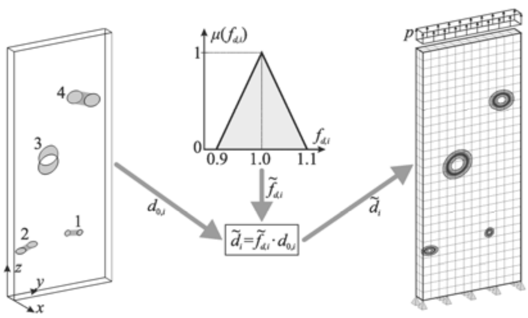
– uncertain knothole size

$$\tilde{f}_{d,i} = \langle 0.9, 1.0, 1.1 \rangle$$
$$\tilde{d}_i = \tilde{f}_{d,i} \cdot d_{0,i}$$
$$i = \{1, 2, 3, 4\}$$

regular mesh: $2 \times 12 \times 30$

empty knotholes: $k_{type} = 1$

knotholes filled with branches: $k_{type} = 2$



Modelling of Material Behaviour

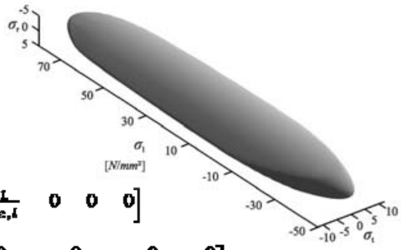
– multi-surface plasticity with hardening and softening rules

– TSAI-WU criterion for failure

$$f(\underline{\sigma}) = \underline{a} : \underline{\sigma} + \underline{\sigma} : \underline{\underline{b}} : \underline{\sigma} - 1$$

yield condition

$$f(\underline{\sigma}) \leq 0$$



TSAI & WU [1971]

$$[\underline{a}]^V = \begin{bmatrix} \frac{1}{f_{t,r}} + \frac{1}{f_{c,r}} & \frac{1}{f_{t,t}} + \frac{1}{f_{c,t}} & \frac{1}{f_{t,l}} + \frac{1}{f_{c,l}} & 0 & 0 & 0 \end{bmatrix}$$

$$[\underline{\underline{b}}]^V = \begin{bmatrix} \frac{-1}{f_{t,r} \cdot f_{c,r}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{f_{t,t} \cdot f_{c,t}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{f_{t,l} \cdot f_{c,l}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{f_{v,tl}^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{f_{v,rl}^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{f_{v,rt}^2} & 0 \end{bmatrix}$$

Modelling of Material Behaviour

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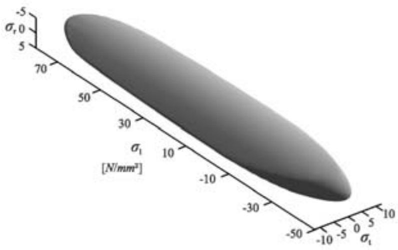


TSAI & WU [1971]

– seven failure modes
  - *tension* in radial direction
  - *pressure* in radial direction
  - *tension* in tangential direction
  - *pressure* in tangential direction
  - *tension* in longitudinal direction
  - *pressure* in longitudinal direction
  - *shear* failure

Sensitivity Analysis

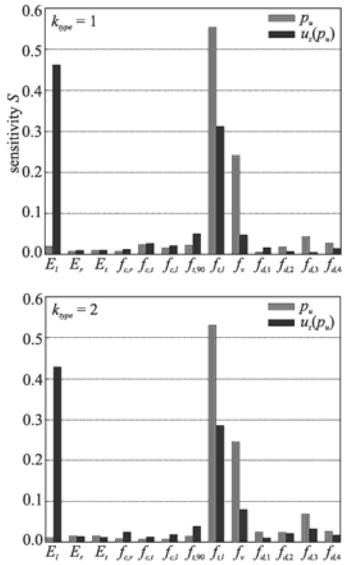
preliminary investigation

- preselection of input parameters based on sensitivity analysis
- evidently no significant influence of knothole size $\tilde{d}_i$

→ trapezoidal interval distribution parameters E_l, $f_{t,l}$, f_v

→ triangular distribution parameters $f_{t,90}$, $f_{c,r}$, $f_{c,l}$, $f_{d,1}$, $f_{d,2}$, $f_{d,3}$, $f_{d,4}$

→ mean values E_r, E_t, $f_{c,t}$



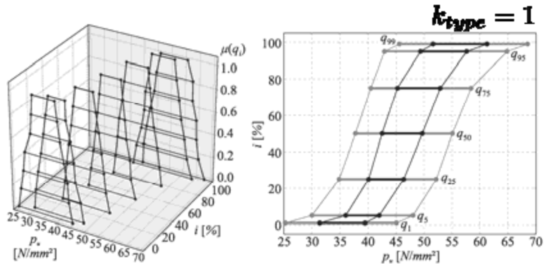
Results

- 7α- levels
  $\alpha = \{0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1\}$
- uncertain ultimate load
  $\tilde{p}_u$
- multiple level values q_i
  $i = \{1, 5, 25, 50, 75, 95, 99\}$

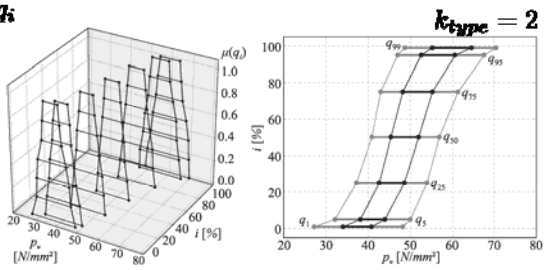
→ apparently filling type minor effect to $\tilde{p}_u$

→ further information reduction necessary

$k_{type} = 1$



$k_{type} = 2$



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Outline

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Conclusion and Outlook

Summary

- fuzzy probability based randomness
  utilized for structural analysis considering uncertainty
- material properties determined by physical experiments
- anisotropic material model for homogenous timber
- information extension compared to deterministic design value
- result strongly dependent on provided data

→ further interpretation of polymorphic uncertain result variables necessary

outlook

- spatial distributed uncertain material parameters (fuzzy fields)
- correlation consideration

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GRK 2075

Institute for Structural Analysis

Modelling of Wood: Multiphysics and Polymorphic Uncertainty

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