Teaching quantum mechanics on an introductory level

Rainer Müller[a] and Hartmut Wiesner[b]
Lehrstuhl für Didaktik der Physik, University of Munich, Schellingstrasse 4, D-80799 München, Germany

(Received 25 July 2001; accepted 9 November 2001)

We present a new research-based course on quantum mechanics in which the conceptual issues of quantum mechanics are taught at an introductory level. In the context of virtual laboratories, the students discover from the very beginning how quantum phenomena deviate from our classical everyday experience. The results of the evaluation of the course show that most of the students acquired appropriate quantum mechanical conceptions, and that many of the common misconceptions encountered in traditional instruction have been avoided.
[DOI: 10.1119/1.1435346]

I. INTRODUCTION

Quantum mechanics has forever changed the physicists' picture of the world. At the beginning of the 20th century, the advent of relativity and quantum mechanics marked not only the discovery of just another new theory, but an entirely new framework for all of physics. Relativity changed our ideas of space and time, and quantum mechanics introduced indeterminism, probabilities, and nonlocality into the foundations of physics.

Quantum mechanics shapes our view of nature in a fundamentally new way. We think that not only physicists should have the privilege to understand how the world works. Educated citizens should at least have the possibility to become acquainted with the strangeness and beauty of quantum phenomena. However, most students who do not major in physics never have a chance to learn about the conceptual issues of quantum mechanics. For example, in the German Gymnasium (whose upper level is comparable to the first two years of college in the United States), atoms and quanta are standard parts of the curriculum. However, the emphasis is on aspects such as the photoelectric effect or Bohr's atomic model, which do not really probe the classical conceptions prevalent in the students' minds.

Our new course on quantum mechanics deals with the conceptual questions of quantum mechanics. It is addressed mainly to nonphysicists. Special emphasis is placed on qualitative reasoning. Physicists can rely on their knowledge of the quantum mechanical formalism to overcome conceptual difficulties, but nonphysicists do not possess such a strong supporting basis. Therefore, conceptual clarity is even more important. The strange and counterintuitive phenomena of quantum mechanics cannot be incorporated in a coherent cognitive picture without the aid of carefully chosen basic concepts that help to organize them. Our strategy is to let the students discover some of the exciting and bizarre quantum phenomena that deviate from our classical everyday experience. At the same time, we want to provide a conceptual framework within which a solid understanding can be constructed.

There are several books that follow a semiqualitative approach to quantum mechanics. First of all, there are the chapters on the double-slit experiment in Feynman's famous lectures,[1] which have had a great influence on most of the subsequent attempts to teach the conceptual aspects of quantum mechanics. From the more recent approaches we mention the books by Rae,[2] Albert,[3] and Silverman[4] which discuss the foundations of quantum mechanics at a moderate mathematical level. An example of a university course explicitly devoted to nonphysics majors is the Visual Quantum Mechanics project,[5] where a hands-on approach to the applications of quantum mechanics is pursued.

II. RESEARCH ON STUDENTS' CONCEPTIONS

It is known that after traditional instruction, students are likely to show classical misconceptions and to confuse classical and quantum notions. Given the counterintuitiveness of quantum mechanics, these misconceptions are not surprising. To avoid them and lead the students to a correct understanding of quantum mechanics, it is important to know the common misconceptions that traditional instruction is likely to promote.

There have been a number of investigations of students' misconceptions and their difficulties in understanding quantum mechanics. Much of the early work came from the Frankfurt, Bremen, and Berlin groups in Germany (for surveys in English see Refs. 6–9, where the courses developed by the German groups are also discussed). Further research has been carried out by Mashhadi,[10] Styer,[11] Johnston, Crawford, and Fletcher,[12] Bao, Redish, and Steinberg,[13] and Ireson.[14]

Here we want to report on the results of our own investigations which up to now have been published only in the German literature. In our first research project,[15,16] 523 Gymnasium students answered a questionnaire on their conceptions of quantum physics after instruction. In addition, 27 students were interviewed orally. The interviews lasted about 1 h. The questions ranged from fact reproduction ("How would you measure an atomic spectrum") to interpretational issues such as their view of determinism/indeterminism. In a second project,[17,18] 37 university students (future physics teachers) were interviewed in a similar manner. It was found that 52% of them had already heard about quantum physics in school, 79% had attended a theoretical quantum mechanics lecture. It is remarkable that both groups gave very similar answers. This similarity indicates that the results can be considered to be typical. In the following, we give an overview of the main misconceptions found in our investigations.

The first three questions (from Ref. 16) show how students distinguish classical and quantum objects.

(1) *What are the essential properties of classical objects?* The student responses can be categorized into the following items (multiple replies possible): (i) mass, weight (85%), (ii)

size, volume, shape (43%), (iii) velocity, movement (38%), (iv) momentum (27%), (v) position (15%), (vi) density (15%), (vii) energy (12%). It is remarkable that nearly all students mentioned mass, but only few mentioned position. The (dynamical) property velocity/momentum is considered more important than position or energy.

(2) *What are the essential properties of quantum objects?* (i) Mass (37%), (ii) charge (37%), (iii) velocity/momentum (37%), (iv) energy (26%), (v) spin (22%), (vi) energy levels/quanta (15%), (vii) position not exactly determined (11%), (viii) no absolute mass (11%), (ix) de Broglie wavelength (7%). For quantum objects, mass is not as dominant as for classical objects. Charge is mentioned often. Category (vii) indicates the conception of "smeared" quantum objects, which is discussed in more detail below.

(3) *What is the main difference between a classical and a quantum object?* (i) For many students (30%), there is a smooth transition between quantum and classical physics. As objects become smaller, quantum behavior shows up more clearly. (ii) 26% of the students argue in terms of dualism or the necessity of model descriptions. (iii) Quantization, especially energy quantization (19%). (iv) Large velocities are possible for quantum objects (15%). (v) Quantum effects such as the Compton effect, spin, interference. (vi) Quantum objects do not possess a position property (11%). (vii) Other answers (7%).

The following questions give an overview of the common conceptions and misconceptions of quantum objects such as photons and atoms. To give a more vivid illustration, we have included typical student answers to most categories.

(4) *What do you mean by "photon"* (from Ref. 16)? (i) One third of the students described a photon as particle of light that has wave as well as particle properties; 17% misinterpreted the wavy line that symbolizes a photon in many Compton effect diagrams as the trajectory of the photon. [Student P2: "Photon is denoted a light quantum, a particle, and it moves in the form of a wave." Interviewer: "The photon itself?" P2: "In the form of a wave forward (draws wavy arrow)."]. (ii) 25% of the students remark that photons do not have a rest mass ("they only have a mass when they move at the velocity of light"). (iii) 17% define a photon as an energy quantum, and (iv) 8% state that a photon is emitted in the transition of an atom from an excited state to the ground state.

(5) *How do you conceive electrons in an atom* (from Ref. 17)? (i) Bohr's atomic model or planetary model (17%). (S31: "There are circles ... around the nucleus ... just orbits. They are circles. And the electrons are on different orbits. They move on them and they can jump from one orbit to another ... if they get more energy, they can jump to a higher orbit.") (ii) Bohr's model with cautionary remarks (24%). (S18: "The orbits ... I still have that picture when I think of an atom. One is told that it's not correct, but one is so used to it and, after all, it is employed again and again.") (iii) Concrete ideas of "clouds"/smeared charge (14%). (iv) "Orbitals" with probability distribution (38%). (S29: "It's the wave function that represents the particles, there is the theory of orbitals, the orbitals can be represented in space. Then you know where the electrons are approximately and the whole thing works with the probability interpretation").

The two dominant conceptions are the two variants of Bohr's model (together 41%) and the picture of orbitals (38%). It is remarkable that even if quantum mechanical ideas are mentioned, Bohr's model is almost always used as the starting point of the discussion. Because most of the students interviewed for this study had quantum physics courses in high school as well as in the university, it is legitimate to say that the Bohr model is a very dominant and stable conception.

Although it is not compatible with the quantum mechanical conception of the atom, the Bohr model may be inevitable as an intermediate step.[19] Possibly the lack of an easy visualization of the quantum mechanical model forces students to stick to this model. If this hypothesis is true, the goal of our instruction should not be to erase the Bohr model in the students' minds, but to convey the conscious use of physical models and let them have insight into the models' limitations.

(6) *Permanent localization* (from Ref. 17). The students were asked, "Does an electron in an atom have a definite position at each moment of time?" The answer categories were (i) the electron has a definite but unknown position (21%). (S1: "Yes, it has to be somewhere, but it isn't accessible through a measurement." S2: "I would say in principle it has a definite position, we just don't know it. That's how I imagine.") (ii) The electron has a position but no trajectory (due to insufficient knowledge of initial conditions) (7%). (iii) Localization in a region with some probability (25%). (S8: "It's like that, they have no definite position, to my mind, they are just located arbitrarily somewhere in a certain region." S32: "You cannot localize it that precisely, you can only give a probability of finding ...") (iv) No definite position because of the uncertainty relation (18%). (v) Other (11%), indifferent (18%).

(7) *Heisenberg's uncertainty relation* (from Ref. 17): Question: What is the meaning of Δx and Δp? (multiple responses allowed). (i) Measurement uncertainties (15%). (S15: "Suppose you know the error Δx. Then you can determine the minimum error you have done in the momentum measurement.") (ii) Disturbance during measurement: position measurement influences the particle's momentum (21%). (S13: "When I measure the position very precisely, I alter the momentum.") (iii) "Regions of localization" (18%), for example, spatial region where the particle is confined; width of the wave function. (iv) Interval within which the exact value lies with some probability (18%). (S18: "It is, so to speak, the probabilities of the momenta at this place. This is the most precise statement about the momentum. I can only say the momentum lies in the interval between $p \pm \Delta p$.") (v) Standard deviation of a statistical distribution (13%). (S21: "If I repeat an experiment several times and measure position and momentum, I don't get always the same, i.e., if I have identical initial conditions, I don't get always the same x and the same p, but it varies. If I graph it I get a standard deviation.")

III. OUTLINE OF THE COURSE

The results presented in Sec. II form the empirical basis for the development of our course. We want to avoid classical misconceptions and help our students to construct a correct quantum mechanical understanding. For these reasons we concentrate on those features of quantum mechanics that are radically different in comparison to classical mechanics. We therefore focus on the following aspects:

- *Born's probability interpretation* is introduced early and used throughout. In introductory courses, wave–particle duality is often characterized as the main mystery of quantum mechanics. In contrast, we point out that there is nothing mysterious about wave–particle duality once a proper understanding of the probability interpretation has been achieved.

- A major new feature of quantum mechanics is that *classically well-defined dynamic properties* such as position, momentum or energy cannot always be attributed to quantum objects.[20,21] If an electron is not in a momentum eigenstate, it does not possess the ''momentum'' property (at least according to the standard interpretation). Similarly, an electron in an atomic orbital (an energy eigenstate) does not possess the ''position'' property. We consider this notable feature as a central element of quantum mechanics. Its discussion therefore takes a prominent place in our course.
- The *measurement process* has perhaps led to more debate than any other topic in quantum mechanics. In contrast to classical mechanics, measurement can no longer be considered as a passive reading of pre-existing values. In quantum mechanics, measurement is an active process. There is a difference between ''to possess a property'' and ''to measure a property.'' The special role of the measurement process comes to light in the process of state reduction and is illustrated, for example, by Schrödinger's cat paradox.

Although we consider nonlocality, the Einstein–Podolsky–Rosen (EPR) paradox, and Bell's inequality as important as the subjects above, no attempt has been made to include them in the course. The primary reason is time limitation.

Conceptual clarity is a vital condition for the success of the course. We therefore base our course on the *ensemble interpretation* of quantum mechanics,[21–23] according to which the predictions of quantum mechanics apply to ensembles of identically prepared objects. In our view, this interpretation provides a clear and comprehensible way of talking about quantum phenomena. Similarly, the idea of *state preparation*[20–25] helps one to construct a conceptual framework that serves as a basis for a deeper discussion.

The course consists of two parts with different goals. The emphasis of the first part, or *basic course*, is on purely qualitative reasoning. Students explore the foundations of quantum mechanics without the difficulties introduced by the formalism. Simulated laboratories provide an environment for their experiences. They are confronted early with the strange behavior of quanta and the central aspects of interpretation are discussed. In the second part, the *advanced course*, an introduction to the quantum mechanical formalism is given. It is intimately linked with the discussion of the quantum mechanical interpretation given in the basic course.

The division into two parts allows the course to be easily adapted to various demands. For example, in a course for liberal arts majors, we would stop after the basic course. Engineering science students (or, in the German school system, the Leistungskurse) would obtain a first introduction into the more formal elements of quantum mechanics in the advanced course. The two parts of the course will be discussed in detail in the following sections.

The structure of the basic course is summarized in Fig. 1. We proceed in a spiral fashion. First, we introduce *photons* and give a qualitative discussion of wave–particle duality, the probability interpretation, and the nontrivial notion of a dynamic property in quantum mechanics. In the second turn of the spiral, we consider *electrons* in the double-slit experiment. The insight gained with photons is deepened at a higher level. The probability interpretation is formulated qualitatively with wave functions, and the concept of state superposition is introduced. The basic course ends with a discussion of more complex issues such as state reduction, complementarity, Schrödinger's cat, and decoherence.

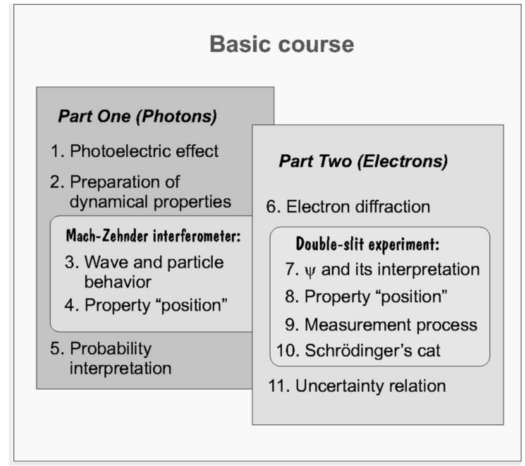

Fig. 1. Structure of the basic course.

IV. PHOTONS

We now discuss the contents of the course in more detail.

A. Photoelectric effect

The course starts with the photoelectric effect, its explanation in terms of photons, and the determination of Planck's constant. This topic is fairly standard and needs no further explanation.

B. Preparation of dynamical properties

As stated earlier, the notion of preparation plays a major role in our course. Already in classical physics, the preparation of initial conditions is important. For example, to test the law of projectile motion, one needs to prepare definite values of position and velocity for the projectile. In this stage, we define preparation as the systematic production of a dynamical property of a classical or a quantum object (this definition is extended later to *state* preparation).

Preparation of properties can be illustrated nicely with the polarization of light. A horizontally oriented polarization filter is a device that produces light with the ''horizontal polarization'' property. A second polarization filter can serve as a test for this property. If it also is oriented horizontally, nearly all of the light passes through it showing that the light possessed the property horizontal polarization. In contrast, a test with a vertically oriented polarization filter shows a negative result: no light passes through.

C. Wave and particle behavior in a Mach–Zehnder interferometer

The Mach–Zehnder interferometer sketched in Fig. 2 is the playground for the exploration of the quantum properties of light. We have developed a program that simulates a virtual laboratory in which all the experiments needed in the course can be performed by the students.[26]

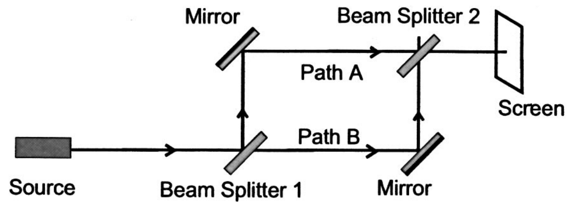

Fig. 2. Mach–Zehnder interferometer.

The path lengths in the two arms are slightly different so that a circular interference pattern appears on the screen when laser light is sent through the interferometer. The students know that interference is a characteristic of wave behavior. They are next confronted with an experiment where single photons are sent in. The interference pattern gradually builds up from the particle-like detection events of single photons. For the first time, the students see wave and particle behavior in the same experiment. The result shows that neither wave nor particle model suffices alone to explain the experimental results. A satisfying model must incorporate aspects of both.

The photon is detected as a localized object on the screen. It is natural to ask whether it is similarly localized within the interferometer. Or, formulated in the language of dynamical properties, does a photon in the interferometer possess the position property?

To answer this question, we place a polarization filter in each of the interferometer arms. If both are oriented horizontally, the interference pattern gradually builds up as in the previous experiment (see Fig. 3). The same is true if both are oriented vertically.

However, if the filters are oriented in orthogonal directions, a different situation emerges (see Figs. 3 and 4). As before, each photon leaves a localized trace at the screen. But from these traces, no interference pattern emerges. A structureless distribution develops instead. Photons are found in places where there were interference minima in the previous experiments.

What does this result mean for our conception of the photon? Imagine that a photon was a localized object traveling on exactly one of the two arms through the interferometer. If this idea were correct, the photon could interact with just one of the polarization filters. But to determine whether it is entitled to land on the position of an interference minimum, it needs information about the orientation of *both* filters. The photon has to "know" whether they are parallel or orthogonal.

If we exclude action-at-a-distance arguments, we have to give up the picture of a photon as a localized object traversing the interferometer. In the interferometer, the photon does not possess the position property.

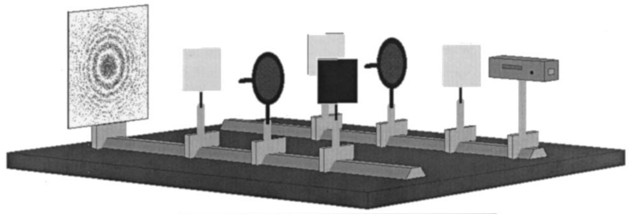

Fig. 3. Parallel polarization filters do not prevent the emergence of the interference pattern.

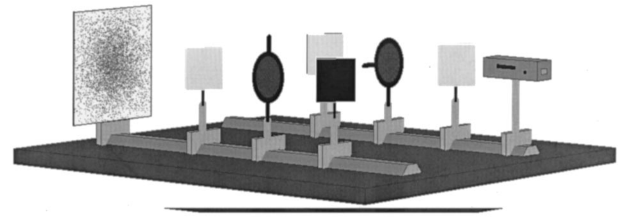

Fig. 4. No interference pattern emerges with orthogonal polarization filters.

The next experiment shows that the situation is even more weird. An obvious objection is that a photon could split into two parts at the beam splitter, go through the interferometer separately, and then recombine somehow at the end. That there is no such splitting can be shown if the polarization filters are replaced by photon detectors. If single photons are incident on the first beam splitter, it turns out that the detectors *never* click simultaneously. A photon is always detected as a single entity; parts of a photon are never found. It is remarkable that this experiment has been carried out in the laboratory.[27] Although its outcome sounds fairly obvious from a photon point of view, it is one of the few experiments that clearly contradict semiclassical theory (quantized atoms plus classical light).

Together, these experiments show that a single photon is as strange an object as one can imagine. The results cannot be explained by any classical model. They hint to the necessity of exploring the quantum mechanical measurement process in more detail.

D. The probability interpretation of quantum mechanics

As we have mentioned, a basic observation in an interference experiment with single photons is that the pattern on the screen builds up from the "hits" of single photons. It is legitimate to ask whether these positions are predetermined as in classical physics and can be predicted from the initial conditions. In this stage of the course, the students learn that one cannot predict the position of a single hit, but that it is nevertheless possible to make accurate predictions for the statistical distribution of *many* hits. This observation is generalized to the following important statement: *Quantum mechanics makes statistical predictions about the results of repeated measurements on an ensemble of identically prepared quantum objects*. This preliminary version of the probability interpretation is later, in the context of electrons, formulated more precisely in terms of the wave function.

V. ELECTRONS

The second part of the basic course is devoted to electrons. Here, the insights gained with photons are broadened and deepened at a higher conceptual level.

A. Wave behavior of electrons

The demonstration of electron wave behavior with an electron diffraction tube is a standard experiment that does not need to be described here. The students obtain the de Broglie relation from the analogous relation for photons and confirm it experimentally with the diffraction tube.

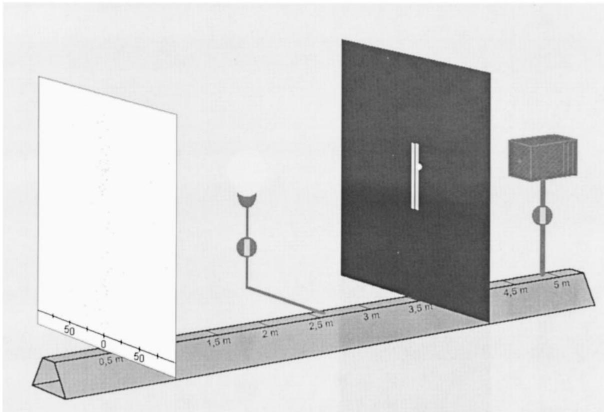

Fig. 5. Simulation program for the double-slit experiment.

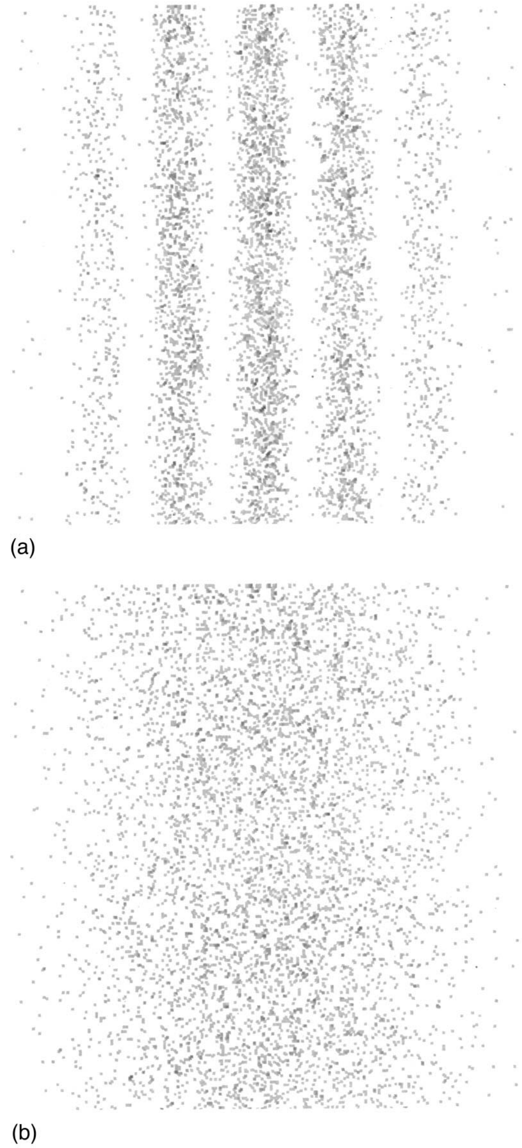

Fig. 6. (a) Double-slit interference; (b) superposition of two single-slit patterns.

B. Double-slit experiment and probability interpretation

Since Feynman's lectures,[1] the double-slit experiment is a basic ingredient in many quantum mechanics courses. We have written a computer program to interactively simulate the double-slit experiment (see Fig. 5).[28] Electrons (or other objects) are emitted by the source on the right, pass through the double slit, and are detected on a screen (the light bulb shown in Fig. 5 is needed later in the discussion of the measurement process). Many of the experimental parameters can be varied by the students.

In our course, we first use the double-slit experiment to resolve the problem of wave–particle duality in terms of the probability interpretation. We consider single electrons passing through the apparatus. The students see that in complete analogy with the photon case, the interference pattern gradually emerges from single electron detection events. Again, only statistical predictions are possible.

Next we go one step further. We introduce the wave function to describe the state of electrons in a completely qualitative way in analogy with water or sound waves. For example, the wave function behind the double slit can be visualized as a superposition of two cylindrical waves emerging from the slits: $\psi=(\psi_A+\psi_B)/\sqrt{2}$, where A and B label the slits.

An essential point for the interpretation of quantum mechanics is Born's interpretation of $|\psi(x)|^2$ as the *probability density* of finding an electron at the position x in a measurement. With this interpretation, the duality of wave and particle, which is often so much emphasized in popular accounts, is no longer a mystery: The wave function spreads in space much like a classical wave and shows typical wave phenomena such as superposition and interference. Mathematically, interference arises from the cross terms in the square of $(\psi_A+\psi_B)$. However, when an electron is detected, that is, when a position measurement is made, it is always found localized at a certain position. The statistical distribution of detection events can be calculated if the wave function is known. With the Born interpretation, wave and particle behavior which seemed to be incompatible are captured in a single picture.

C. Position property of electrons

In the interferometer experiment discussed in Sec. IV we discussed how the conception of a photon as a localized object with a well-defined position leads to conflicts with experiment. This conflict is very counterintuitive, and we cannot expect the students to accept such a far-reaching conclusion at once. Therefore we present analogous reasoning for electrons as well. The argument has been published in similar form many times in the literature so we will give only a brief sketch. Suppose the electron goes as a localized entity through one definite slit. Then it cannot "know" whether the other slit is open or closed. Its final position on the screen should not be influenced by this fact. Therefore, the same pattern on the screen should appear if (a) both slits are opened together for a time t, or (b) first one slit is opened for the time t, then the other one. However, experiment shows that it does matter whether (a) or (b) is realized. Double-slit interference is observed in case (a), whereas a superposition of two single slit patterns is observed in case (b) (see Fig. 6). This result demonstrates that the initial assumption was wrong. In this experiment, electrons do not behave like localized objects. They do not possess the position property.

D. Position measurement and the measurement postulate

The next major discussion is on the quantum mechanical measurement process. We start directly from the previous experiment and ask the seemingly innocent question: What is the result of a position measurement if the electrons actually do not possess the property position? The experiment we use to answer this question has been conceived by Feynman, although we interpret it slightly differently in our course.

The light bulb in the double slit simulation program symbolizes a position measurement device. It emits light that is scattered by the electrons that pass the slit. Eventually, the scattered light ends in our eye or a detection apparatus, and we can infer from the direction of incidence where it was scattered.

If we turn on the light bulb, we see little light flashes behind one of the slits (visible in Fig. 5). They give the result of the position measurement. The electron is always found behind exactly one of the slits. For a single electron, we never see the light flashes at two or more positions simultaneously (see the analogous result for photons in Sec. IV). This result seems to contradict our previous finding that an electron does not possess the position property. But this apparent contradiction once more underlines the special status of a measurement in quantum mechanics. It is resolved by the fundamental *measurement postulate*: In each measurement, a definite value for the measured observable is found. Later, in the advanced part of the course, we make a connection to the idea of an eigenvalue: The possible results of a measurement are the eigenvalues of the measured observable. The reason why eigenvalues are interesting in quantum mechanics is that they are the possible results of measurements.

On the other hand, this experiment shows that there is a difference between "to possess a property" and "to measure a property." If we find an electron behind one of the slits, we cannot assume that it did possess the position property already before the measurement. A definite position value is only realized in the measurement.

E. Advanced topics in the interpretation of quantum mechanics

With the double slit experiment, some other important aspects of quantum mechanics can be easily demonstrated, such as state reduction and complementarity. State reduction is the phenomenological way of describing the influence of a measurement on the subsequent state of the measured object. In our example, state reduction takes place after the detection of the electron behind one of the slits. As a result of state reduction, instead of the usual double-slit pattern, a pattern such as the one shown in Fig. 6(b) builds up behind the screen.

Complementarity has been a central element in Bohr's philosophy of quantum mechanics. A simple form of it is the complementarity between interference pattern and path information. It can be demonstrated by a series of experiments. In each one the intensity of the detection light is decreased so that more and more electrons escape undetected. Accordingly, the visibility of the interference pattern increases in each experiment.

The discussion of superposition states provides a basis for the discussion of Schrödinger's cat paradox where a superposition of macroscopically different states is considered. The paradox is resolved by the mechanism of decoherence. The interaction with its natural environment (for example, photon or air molecule scattering) renders the cat effectively classical. That photon scattering may inhibit interference has already been observed in the double slit experiment with the light bulb. This experience can be used to make the decoherence mechanism plausible.

F. Heisenberg's uncertainty relation

It has been mentioned that there are many misconceptions of the Heisenberg uncertainty relation. In our view, its clearest formulation is as a statement about the preparation of quantum objects. It restricts the possibility of simultaneously preparing certain pairs of observables on an ensemble of quantum objects.[23] We start from an arbitrarily prepared ensemble of electrons described by the wave function ψ. We now take a subensemble of this ensemble and perform many position measurements on its members. We obtain a statistical distribution of measurement results and call the standard deviation of this distribution Δx. Now we take another subensemble and perform a large number of momentum measurements to obtain Δp. The uncertainty relation states that it is not possible to prepare a state ψ such that the product of Δx and Δp is smaller than $\hbar/2$. This way of reasoning shows that the uncertainty relation is not a statement about simultaneous measurements or the mutual disturbance of two measurements. Quantitatively, the uncertainty relation can be illustrated by the well-known example of electrons incident on a single slit.

VI. AN INTRODUCTION TO THE QUANTUM MECHANICAL FORMALISM

In the above we presented the first part of our course, which is addressed to students who are not likely to choose physics at the university. The advanced course is aimed at students with a special interest in physics (Leistungskurse). In this part of the course we give an introduction to the formalism of quantum mechanics and its applications at a mathematically very basic level. The discussion is based on the qualitative understanding gained in the basic course and takes up the ideas introduced there (such as preparation or quantum mechanical properties).

A. Wave functions and operators

The wave function has already been introduced qualitatively to describe the state of electrons. We now discuss mathematically the wave function of free electrons $\Psi_{E_{\text{kin}}}$. It describes an ensemble of electrons prepared to have a fixed value of the property "kinetic energy" (or, likewise, momentum).

Next we introduce the concept of an operator. We look for a mathematical operation that extracts the value of the kinetic energy from the wave function of free electrons. By trial and error, the students find that the operation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

applied to $\Psi_{E_{\text{kin}}}$, leads to the desired result. It leaves the wave function unchanged and pulls a factor E_{kin} out of it. They have found the operator of kinetic energy.

B. Eigenvalue equation

Now we ask the opposite question. Given a wave function Ψ, how can we decide whether it describes an ensemble with the property kinetic energy? The answer is to apply again the operator of kinetic energy. If the wave function is reproduced,

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E_{\mathrm{kin}}\psi,$$

the corresponding ensemble possesses the kinetic energy property, and the constant E_{kin} gives the value of the kinetic energy. The wave function is an eigenfunction of kinetic energy and E_{kin} is the eigenvalue. If the eigenvalue equation is not fulfilled, the ensemble described by Ψ does not possess the kinetic energy property. If a series of measurements on the corresponding electrons is made, the measured values will have a distribution.

C. Schrödinger equation

With the concept of an eigenvalue equation at hand, it is only a small step to the stationary Schrödinger equation, which is the eigenvalue equation of total energy. Solving the Schrödinger equation means to search for states with the "total energy" property. These are called stationary states.

The Schrödinger equation is the basic equation of quantum mechanics, and we can solve it for some cases. The simplest example is the infinite potential well where the quantization of energy appears for the first time. The three-dimensional potential well is used to illustrate probability distributions in three-dimensional space and to introduce the idea of orbitals.

D. Atoms

The final part of the course is devoted to atoms. The students observe line spectra of atoms and see the quantization of energy in the Franck–Hertz experiment. As mentioned, the Bohr model is rooted deeply in the students' minds. We therefore discuss the Bohr model critically and emphasize that it conflicts with some of the basic results of quantum mechanics covered so far (for example, Heisenberg relation and the impossibility of trajectories). As an alternative, we discuss the orbital model of the atom.

The mathematics necessary to solve the Schrödinger equation for the hydrogen atom is beyond the scope of our students. We therefore use an approach proposed in Ref. 29 and model the Coulomb potential by an infinite potential well with appropriately chosen parameters. We obtain a self-consistent equation for the possible energy values and find the $-1/n^2$ behavior of the energy eigenvalues. This important result completes our course.

VII. EVALUATION OF THE COURSE

An important part in the development of a new course is its evaluation. In a pilot study, preliminary versions of the courses were taught in five classes by P. Engelhardt and the authors. In the actual evaluation, we tested our course in five other Gymnasium school classes with about 60 students. Two of the classes were nonspecialized physics classes (Grundkurse) with 3 hours a week, and three classes were for students with a special interest in physics (Leistungskurse) with 5 hours per week.

Table I. Some of the statements the students had to rate in the questionnaire.

An atom has a similar structure as the solar system (planets that orbit the sun)
Electrons in an atom are like a smeared charge cloud that surrounds the nucleus
Electrons move around the nucleus in definite orbits with a high velocity
Nobody accurately knows the position of an electron in orbit around the nucleus because it is very small and moves very fast
An electron that goes from the source to the screen in the double-slit experiment takes a definite path, even if it cannot be determined
If we knew the initial conditions precisely enough we could predict where the next electron is found on the screen
The wave function determines the distribution of electrons on the screen
In principle, quantum objects can simultaneously possess position and momentum
The uncertainty relation sets a limit on how good the momentum of an electron can be determined
If a precise position measurement is carried out on an electron, it is only possible to make an imprecise momentum measurement afterwards

The design of the study was as follows: The students were instructed by their regular teachers. Instead of a textbook they were provided with a text (approximately 100 pages) containing the contents of the course. The simulation programs developed for the course were used in the classroom and, in part, also at home.

We used several instruments to measure the success of the course.

(1) *Questionnaire on students' conceptions*. The questionnaire consisted of two parts. In the first part the students had to rate statements on a five-point scale from 1 ("strongly agree") to 5 ("strongly disagree"). They had to judge 44 items from four different subfields of the course: Conception of the atom, determinism/indeterminism, quantum mechanical properties, and the uncertainty relation. Some typical questions are given in Table I. The questions were in part adapted from Ref. 14. The second part of the questionnaire contained questions with the possibility of answers. Here, students were asked to explain the uncertainty relation or to draw their visual image of an atom (including a commentary whether there are features that cannot be drawn).

(2) *Student interviews*. In two of the classes ($N=22$), semi-structured interviews were conducted. Each lasted approximately 1 h. They were taped, transcribed, and analyzed. Topics again included the conception of the atom and the uncertainty relation. In addition, we investigated in more detail the understanding of more complex issues such as determinism, probabilistic laws, state superposition, and the insights gained in the double slit experiment.

(3) *Questionnaire on physics interest*. We also used the questionnaire to compare the students' interest in the physics of waves and in quantum mechanics.

To express the success of the course by a single number, we calculated a statistical index C from 29 items of the questionnaire on students conceptions. An index value of $C=+100$ corresponds to fully quantum mechanical conceptions, $C=-100$ means conceptions that contradict strongly to the quantum mechanical ones, and $C=0$ corresponds to an indifferent attitude (for example, rating each statement with a 3).

Figure 7(a) shows the distribution of the index value C for the students that were taught our course. All of the students have a positive value of C which means that quantum mechanical conceptions dominate. The average is $+55.8$ with a

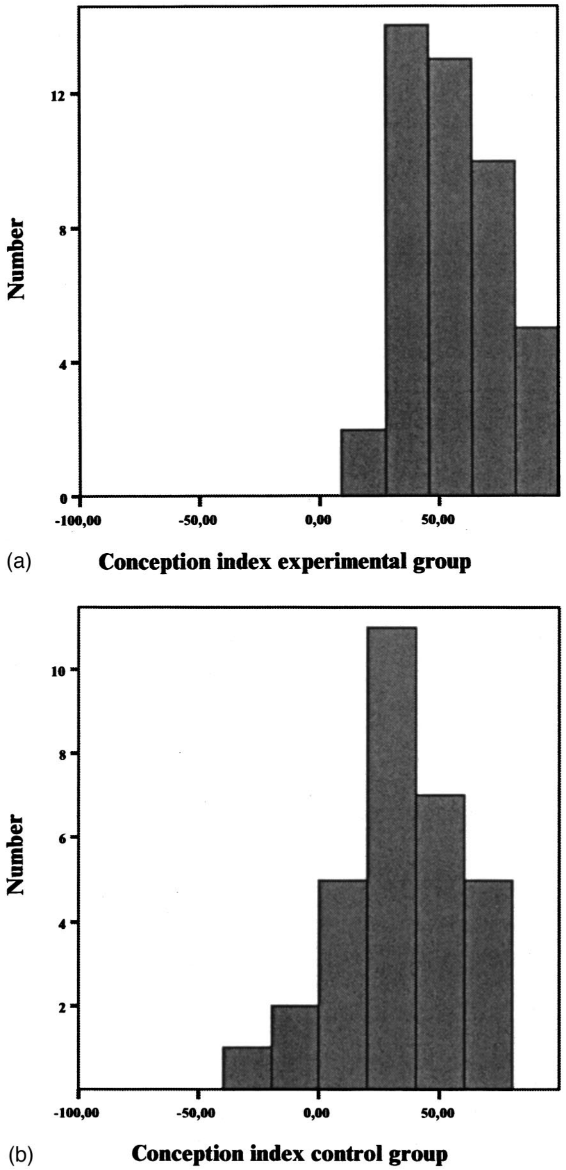

Fig. 7. (a) Distribution of the conception index C in the experimental group; (b) distribution of C in the control group.

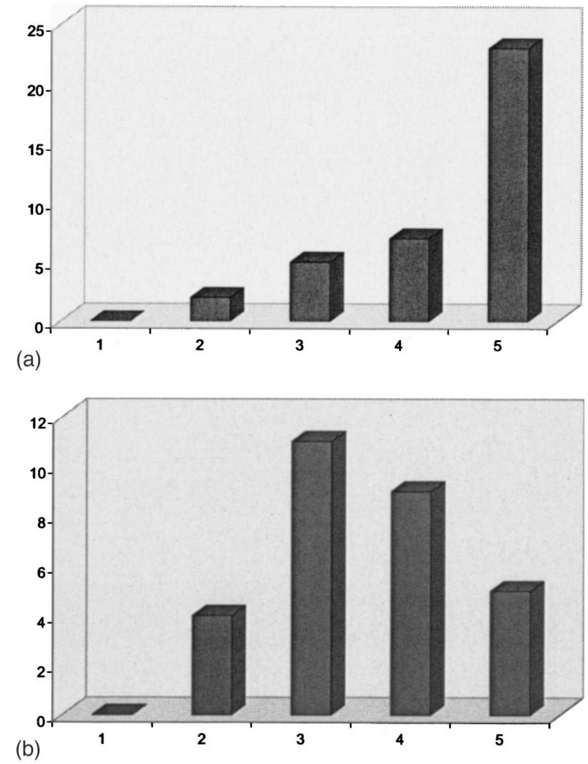

Fig. 8. Response to the statement: "An atom has a structure similar to the solar system (planets that orbit the sun);" (a) experimental group, (b) control group (1="strongly agree," 5="strongly disagree").

standard deviation of 19.5. We consider this result as an indication that the students successfully learned quantum mechanical conceptions.

To compare the results with a group of traditionally instructed students, we gave the questionnaire on student conceptions to a group of 35 first-year university students who had been taught quantum physics during their time in Gymnasium. We took care to include in the definition of C only those questions that we considered to be fair to the control group. The distribution of the index C in this control group is shown in Fig. 7(b). The average is +35.2 with a standard deviation of 23.7. A comparison of the two diagrams shows that the experimental group has developed more pronounced quantum mechanical conceptions than the control group. The difference is highly significant (significance level 0.1%). To appreciate this result one has to take into account that the control group is positively selected because these students decided to major in physics at the university. We have calculated similar indices for the four subfields mentioned above. In all of them, the experimental group was superior.

Figures 8–10 show the responses of both groups to some individual statements in the questionnaire. In Fig. 8, the following statement had to be rated: "An atom has a similar structure to the solar system (planets that orbit the sun)." The distribution of the experimental group [Fig. 8(a)] is peaked toward complete rejection (average 4.38) in accordance with the quantum mechanical model. The control group is much more indifferent and not so critical of the planetary model (average 3.52).

In Fig. 9 the question of determinism/indeterminism is addressed. The following statement was given in the context of the double slit experiment: "If we knew the initial conditions precisely enough, we could predict where the next electron is found on the screen." The experimental group [Fig. 9(a)] clearly denies the determinism of classical mechanics in the quantum domain (average 4.75). The control group is much less certain (average 3.88).

Figure 10 shows the response to a statement on the uncertainty relation. The statement was, "In principle, quantum objects can possess simultaneously position and momentum." Again, the experimental group definitely rejected the statement (average 4.89), whereas there are mixed opinions in the control group (average 2.92). In all of the examples mentioned, the difference between the groups is highly significant.

The response to a question with an open reply possibility is shown in Fig. 11. The students were asked to draw an atom according to their conceptions. We categorized the results according to the scheme of Ref. 30: (i) Bohr (nucleus with electrons on orbits), 13% in the experimental group

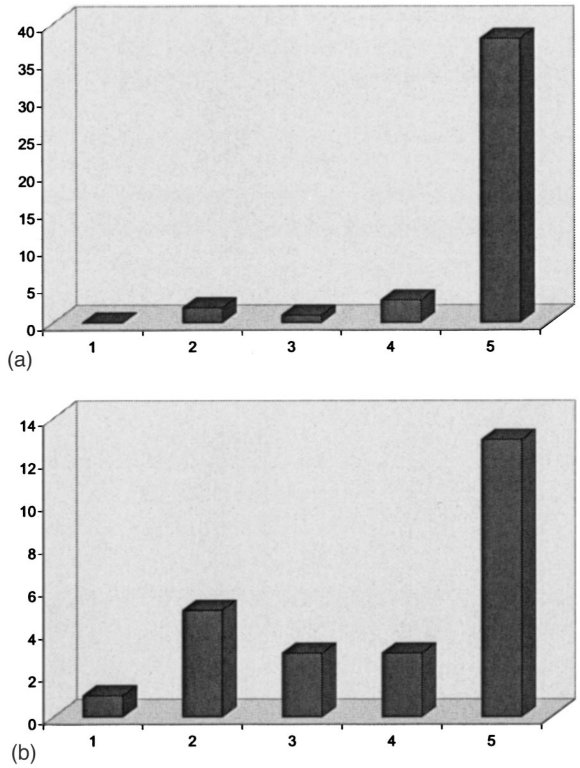

Fig. 9. Response to the statement: "If we knew the initial conditions precisely enough, we could predict where the next electron is found on the screen." (a) Experimental group, (b) control group.

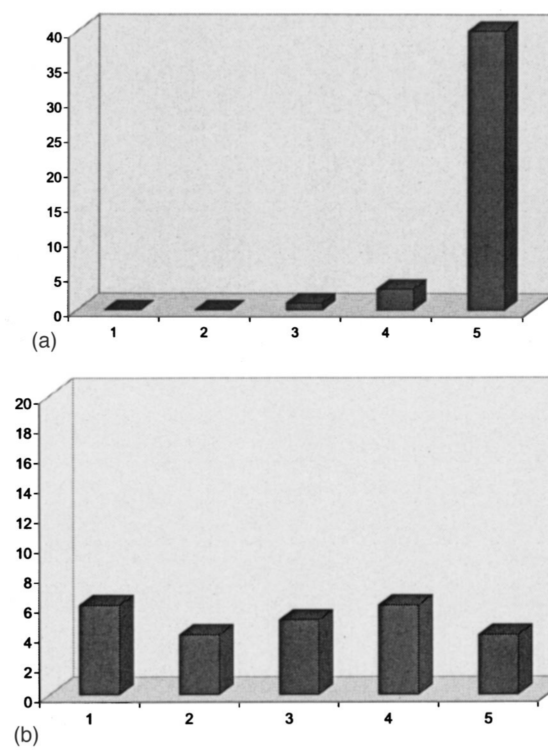

Fig. 10. Response to the statement: "In principle, quantum objects can simultaneously possess position and momentum." (a) Experimental group, (b) control group.

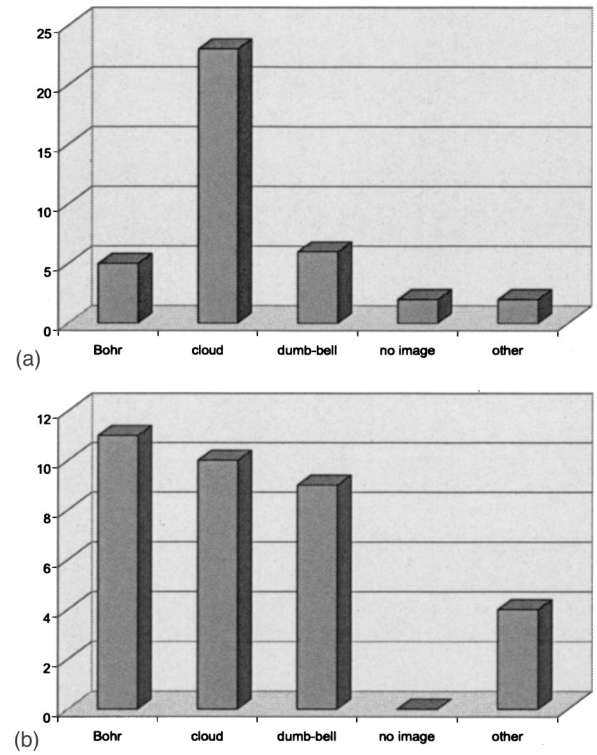

Fig. 11. Student drawings of an atom (classification as in Ref. 28); (a) experimental group, (b) control group.

versus 32% in the control group; (ii) cloud (distributed cloud around the nucleus): 61% vs 29%; (iii) dumb-bell (reminiscent of a *p*- or *d*-orbital): 16% vs 27%, and (iv) "no image possible": 5% vs 11%. In accordance with the results discussed earlier, the Bohr model played a minor role in the experimental group. Nearly all of the students drew a "cloud" image. In the control group the classical Bohr model, the quantum mechanical cloud, and the dumb-bell image were approximately evenly distributed.

Another interesting question is whether the students have developed the competence to argue freely and unaided within the new conceptual framework. This ability was tested in the interviews. Students were asked to comment in their own words on several questions and statements related to the contents of the course. The results are instructive in particular for the physically more complex topics. For example, an important aim of the course was to realize that electrons do not necessarily possess the position property. To what extent were the students able to justify their statements instead of merely memorizing the correct answer?

One of the interview questions was, "Someone claims that an electron in the double-slit experiment goes either through the left or through the right slit. How can you disprove this?" To evaluate the student answers quantitatively, we graded the replies from 1 (physically correct, clear and careful reasoning) to 5 (insufficient or totally confused answer). The results are shown in Fig. 12. Most of the students (55%) were able to argue adequately (mark 1 or 2) whereas 32% apparently did not develop a deeper understanding (mark 4 or 5). A typical example of an (oral) student reply marked with 1 is the following: "Well, it's actually this point in physics that is most fascinating to me, in quantum physics. Because if it would go through the left or the right slit, then the electron would have to end up in the same region if both slits were

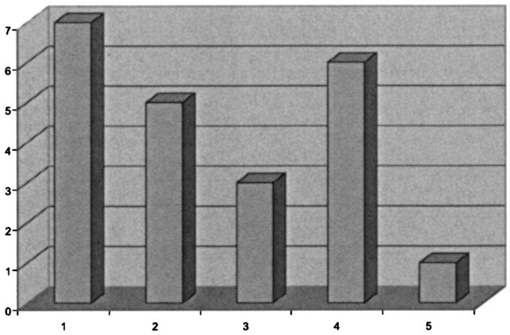

Fig. 12. Why do electrons not necessarily possess a position property? Distribution of marks on the free replies in the interviews.

open or if only one slit was open. But this is not true. That is, if I open only the left slit, then I get a single-slit distribution behind the left slit. If I open them both, I get the interference pattern; and on the location of the maximum of the single slit pattern there is a minimum of the interference pattern. This means: it cannot go through just one of the slits, there must be something else (S16).''

To summarize the results of the evaluation, we can say that the correct quantum mechanical conceptions were successfully imparted to most of the students of the experimental group. This understanding was achieved in spite of the large conceptual difficulties inherent in the subject. In addition, many of the common misconceptions encountered in traditional instruction, e.g., in the uncertainty relation, the determinism/indeterminism problem, or the atom conception, have been avoided.

VIII. TEACHER'S TRAINING ON THE WEB

Finally, we briefly mention our Web-project milq (Munich Internet Project for teacher's training on quantum mechanics).[31] In this project we want to use the Internet as a new medium for teacher's training. Traditional courses have a capacity of 20-40 persons. With the Internet, there is no upper limit to the number of participants so that the potential impact is much larger. In addition, we can investigate the changes and limitations offered by new possibilities such as the use of multimedia, simulations, and hypertext. Up to now, milq has been available in German only. However, in collaboration with the Visual Quantum Mechanics Project[5] at Kansas State University, an English version is planned.

There are two main content areas in milq. We present the ideas of the quantum mechanics course described in this article, together with background information and teaching material. Interested teachers can acquaint themselves with the new approach of teaching quantum mechanics. We also provide additional information relevant (not only) to quantum mechanics classes in school. This information includes current results of research in quantum mechanics (e.g., quantum information) and also material on the ever-recurring questions of quantum mechanics such as the discussion of the EPR paradox and Bell's inequalitites.

[a]Electronic mail: rainer.mueller@physik.uni-muenchen.de
[b]Electronic mail: hartmut.wiesner@physik.uni-muenchen.de

[1]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics, Vol. III* (Addison–Wesley, Reading, 1966).
[2]A. Rae, *Quantum Physics: Illusion or Reality* (Cambridge U.P., Cambridge, 1986).
[3]M. P. Silverman, *More Than One Mystery. Explorations in Quantum Interference* (Springer, New York, 1995).
[4]D. Z. Albert, *Quantum Mechanics and Experience* (Harvard U.P., Cambridge, 1992).
[5]Available online at http://www.phys.ksu.edu/perg/vqm.
[6]H. Niedderer, T. Bethge, and H. Cassens, ''A simplified quantum model: A teaching approach and evaluation of understanding,'' in *Relating Macroscopic Phenomena to Microscopic Particles: A Central Problem in Secondary Science Education*, edited by P. L. Lijnse *et al.* (CD-β Press, Utrecht, 1990).
[7]J. Petri and H. Niedderer, ''A learning pathway in high-school level atomic physics,'' Int. J. Sci. Educ. **20**, 1075–1088 (1998).
[8]H. Fischler and M. Lichtfeldt, ''Learning quantum mechanics,'' in *Research in Physics Learning: Theoretical Issues and Empirical Studies*, edited by R. Duit, F. Goldberg, and H. Niedderer (IPN, Kiel, 1991).
[9]H. Fischler and M. Lichtfeldt, ''Modern physics and students' conceptions,'' Int. J. Sci. Educ. **14**, 181–190 (1992).
[10]A. Mashhadi, ''Students' conceptions of quantum physics,'' in *Research in Science Education in Europe: Current Issues and Themes*, edited by G. Welford, J. Osborne, and P. Scott (Falmere, London, 1996), pp. 254–265.
[11]D. F. Styer, ''Common misconceptions regarding quantum mechanics,'' Am. J. Phys. **64**, 31–34 (1996).
[12]I. D. Johnston, K. Crawford, and P. R. Fletcher, ''Student difficulties in learning quantum mechanics,'' Int. J. Sci. Educ. **20**, 427–446 (1998).
[13]L. Bao, E. F. Redish, and R. N. Steinberg, ''Student misunderstandings of the quantum wavefunction'' (unpublished).
[14]G. Ireson, ''The quantum understanding of pre-university physics students,'' Phys. Educ. **35**, 15–21 (2000).
[15]H. Wiesner, *Beiträge zur Didaktik der Unterrichts über Quantenphysik in der Oberstufe* (Westarp, Essen, 1993).
[16]H. Wiesner, ''Verständnisse von Leistungskursschülern über Quantenphysik. Ergebnisse mündlicher Befragungen,'' Phys. in der Schule **34**, 95–99 (1996); **34**, 136–140 (1996).
[17]R. Müller and H. Wiesner, ''Vorstellungen von Lehramtsstudenten zu begrifflichen Problemen der Quantenmechanik,'' *Didaktik der Physik*, Beiträge zur Vol. 62 (Physikertagung, Regensburg, 1998), pp. S. 458–463.
[18]R. Müller and H. Wiesner, ''Students' conceptions on quantum physics,'' presented at the annual meeting of the National Association for Research in Science Teaching, 1999; available online at http://www.phys.ksu.edu/perg/papers/narst/QM_papers.pdf.
[19]H. Niedderer, ''Physiklernen als kognitive Entwicklung,'' *Didaktik der Physik*, Beiträge zur (Physikertagung Ludwigsburg, 1999), pp. 49–66.
[20]L. Eisenbud, *The Conceptual Foundations of Quantum Mechanics* (van Nostrand Reinhold, New York, 1971).
[21]L. Ballentine, *Quantum Mechanics, A Modern Development* (World Scientific, Singapore, 1998).
[22]D. I. Blokhintsev, *Quantum Mechanics* (Reidel, Dordrecht, 1964).
[23]L. Ballentine, ''The statistical interpretation of quantum mechanics,'' Rev. Mod. Phys. **42**, 358–380 (1970).
[24]W. E. Lamb, ''An operational interpretation of nonrelativistic quantum mechanics,'' Phys. Today **22**, 23–28 (1969).
[25]G. Ludwig, *Foundations of Quantum Mechanics* (Springer, Berlin, 1983).
[26]It can be downloaded from www.physik.uni-muenchen.de/didaktik/Computer/interfer/interfere.html (the German and English languages are supported).
[27]P. Grangier, G. Roger, and A. Aspect, ''Experimental evidence for a photon anticorrelation effect on a beamsplitter,'' Europhys. Lett. **1**, 173–179 (1986).
[28]It can be downloaded from www.physik.uni-muenchen.de/didaktik/Computer/Doppelspalt/dslit.html.
[29]W. Theis, ''Begründung diskreter Energiewerte für gebundene Zustände ohne Lösung der Eigenwertgleichung,'' Phys. Didaktik **22**, 198–204 (1994).
[30]M. Lichtfeldt, *Schülervorstellungen in der Quantenphysik und ihre Möglichen Veränderungen durch Unterricht* (Westarp, Essen, 1992).
[31]The milq pages can be found at www.cip.physik.uni-muenchen.de/~milq.