Family Name : .... **Exam number :** ....

First name : .... **Reg. number :** ....

Instructions:

Write your name and registration number in the corresponding fields. Do **not** use pencils, green or red pens (used in marking). Write your name and reg. number on **each sheet**, number your sheets **consecutively** and write only on **one side** of the sheets! Memorize or write down the **exam number**.

You are allowed to use a non-programmable pocket calculator and an A4 sheet of paper for formulas.

Task	1	2	3	4	5	6	Σ (60)
Mark							

Task 1 (15 Points)

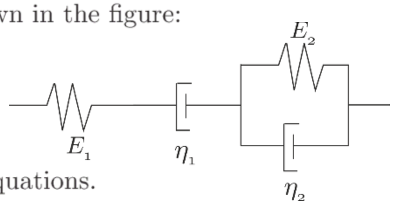
1. Sketch the graph of the excitation and the expected experimental response in time for the creep and relaxation tests.
2. When is it convenient to adopt the frequency domain in viscoelasticity? Define the loss and storage moduli for a relaxation test and clarify their meaning from the energy standpoint. Define the loss factor and sketch the phase shift angle δ in case of a relaxation test for i) a spring, ii) a dashpot and iii) a combination of these two elements.
3. What is generally needed to uniquely define the behavior of an elasto-plastic material? (select the correct answer/s)

   ☐ Elastic response ☐ Unloading stiffness ☐ Flow rule
   ☐ Yielding condition ☐ Viscosity ☐ Hardening law
4. Write down the Tresca criterion for yielding and the related equivalent stress. Sketch the Tresca and Von Mises criteria in the $\sigma_1 - \sigma_2$ plane (i.e., assuming $\sigma_3 = 0$).
5. How can we safely distinguish between an elasto-damaging and an elasto-plastic material? How would a mixed elasto-damaging-plastic material behave in terms of $\sigma - \varepsilon$ graph?
6. Define the energy and the energy rate in an isotropic elasto-damaging material. For the latter define also the damage energy release rate, the stored elastic energy rate and the dissipated energy rate.
7. Define the 3 work conjugates and the configurations they belong to.
8. What is the relationship between the Cauchy stress tensor and the first Piola-Kirchoff stress tensor?
9. What is the definition of the energy potential in hyperelasticity? How can $\mathbf{P}$ and $\mathbf{S}$ be obtained from it?

Task 2 (10 Points)

Viscoelasticity

Given the rheological model shown in the figure:



a) Determine the governing equations.

b) Assuming the dashpot η_1 is removed, compute and sketch the creep function for a step-like loading applied at $t = 0$ and removed at $t = t^*$.

Task 3 (6 Points)

Plasticity

Given the strain tensor $\boldsymbol{\varepsilon} = \begin{bmatrix} \bar{\varepsilon} & 2\bar{\varepsilon} & 0 \\ 2\bar{\varepsilon} & \frac{1}{2}\bar{\varepsilon} & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

For an isotropic linearly elastic material, with elastic constants E and ν,

a) calculate the Cauchy stress tensor,
b) compute the hydrostatic and deviatoric stress tensors,
c) define for which strain $\bar{\varepsilon}$ the material starts to yield following the Von Mises criterion.

Given: $E = 210$ GPa, $\nu = 0.3$, $\sigma_y = 214$ MPa.

Task 4 (9 Points)

Plasticity

Consider an elasto-plastic material with linear isotropic hardening (hardening modulus H) subjected to uniaxial loading conditions.

Starting from the initial state, $\varepsilon_0 = 0$, $\varepsilon_0^p = 0$, $\alpha_0 = 0$, write down the incremental return mapping for an applied strain increment of $\Delta\varepsilon = 2\varepsilon_y$ and sketch the $\sigma - \varepsilon$ graph of the return mapping including the trial and actual states.

Given: $E = 210$ GPa, $\nu = 0.3$, $\sigma_y = 214$ MPa, $\varepsilon_y = \frac{\sigma_y}{E}$, $H = 4$ GPa.

Task 5 (8 Points)

Damage

Some experimental data on a damaging material can be fitted to the following monotonic relation:

$$\sigma(\varepsilon) = \frac{\sigma_0}{\varepsilon_0}\varepsilon e^{-\varepsilon/\varepsilon_0}.$$

Determine

a) the initial stiffness,

b) the expression for $D(k)$, using $k = \max(\varepsilon)$,

c) the residual stiffness after a monotonic loading up to $\varepsilon = 3\varepsilon_0$ and sketch the related $\sigma - \varepsilon$ and $D - \varepsilon$ diagrams.

Task 6 (12 points)

Large deformations

For the given motion

$$\mathbf{x}(\mathbf{X}) = \begin{Bmatrix} 3X_1 \\ 1.5X_2 \\ \lambda_3 X_3 \end{Bmatrix},$$

calculate

a) the deformation gradient and compute the value of λ_3 such that the change in volume is $V/V_0 = 1.5$.

b) the right and left Cauchy-Green deformation tensors.

c) the Green, Almansi and infinitesimal strain tensors.

d) the final configuration of the segment $\mathbf{L_1} = \{1\ 1\ 1\}^T$ and the initial configuration of the segment $\boldsymbol{\ell}_2 = \{2\ 1\ 0.5\}^T$.