Family Name : .... **Exam number :** ....

First name : .... **Matric. number :** ....

Notes on the exam:

Write name and matriculation number in the corresponding fields. Do **not** use pencils, green or red pens (used in marking). Place name and matric. number on **each sheet**, number sheets **consecutively** and write only on **one side** of the sheets! Memorize or write down the **exam number**.
You are allowed to use a non-programmable pocket calculator and two pages of equations.

Task	1	2	3	4	5	Σ (50)
Mark						

1. Task

Please answer briefly the following questions:

General

1) Refering to the four types of nonlinear behavior considered in the lecture (visco-elasticity, plasticity, damage, hyper-elasticity), name one example each where these phenomena will be relevant.

Visco-elasticity

2) Under which conditions can the Boltzmann superposition principle be used?
3) Given a description of a material in the time domain (e.g. constitutive relation, relaxation function), how can the corresponding behavior in the frequency domain (complex dynamic modulus) be found?

Damage

4) How can one experimentally distinguish between damage and other types of nonlinear behavior?

Plasticity

5) Why is a procedure like return mapping required at all in the numerical treatment of plasticity?
6) What does the principle of maximum plastic dissipation state, and what follows for the direction of the plastic flow?

Large deformation kinematics

7) What is the reason for the large strain measures $(\mathbf{E}, \mathbf{e})$ not being affected by rigid body rotations?
8) When is it appropriate to use a strain energy density formulation based on invariants or principal stretches (e.g. $\Psi = \Psi(I_1, I_2, J)$)?

2. Task

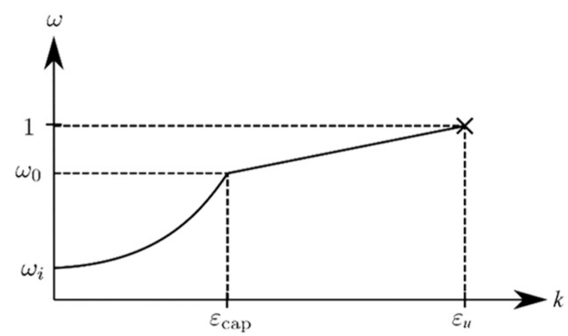
Damage

For concrete in tension, it is known that the damage parameter ω as function of maximum strain k is, as depicted in the figure:

$$\omega(k) = \begin{cases} \omega_i + C_1 k^\beta & \text{for} \quad 0 \leq k \leq \varepsilon_{\text{cap}} \\ \omega_0 + C_2(k - \varepsilon_{\text{cap}}) & \text{for} \quad \varepsilon_{\text{cap}} \leq k \leq \varepsilon_u \end{cases}$$

Assuming no initial damage ($\omega_i = 0$), an exponent $\beta = 2$ and an inital Young's modulus E, determine

a) the constants C_1 and C_2,
b) the stress $\sigma(k)$ for $0 \leq k \leq \varepsilon_u$.



3. Task

Plasticity

A thin sheet of material is subjected to equi-biaxial tension, characterized by the strain tensor $\boldsymbol{\varepsilon}_{\text{eb}}$. The material is linear elastic-plastic, with Young's modulus E, Poisson's ratio ν and yield limit σ_{Y}. A state of plane stress shall be assumed.

a) Calculate the Cauchy stress.

b) At what strain ε_0 will the material start to yield, based on the von-Mises and the Treca yield criterion, respectively?

In another test, the material is subjected to uniaxial tension. It shows nonlinear isotropic hardening, specified by the hardening modulus $H(\alpha)$, for $0 \leq \alpha \leq 1$.

c) Calculate the stress and plastic strain at twice the yield strain (assuming monotonical increasing strain).

Given:

$$E = 100\text{GPa},\ \nu = 0.25,\ \sigma_{\text{Y}} = 200\text{MPa},$$

$$\boldsymbol{\varepsilon}_{\text{eb}} = \begin{pmatrix} \varepsilon_0 & 0 \\ 0 & \varepsilon_0 \end{pmatrix},$$

$$H(\alpha) = \frac{E}{2}(1-\alpha)$$

Remark:

The stress-strain relation for isotropic, linear elastic material in plane stress is:

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1+\nu \end{bmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{pmatrix}$$

To simplify the result in c) you may use the linearization

$$\sqrt{x+dx} \simeq \sqrt{x}\left(1 + \frac{1}{2}\frac{dx}{x}\right)$$

4. Task

Large deformations

A hyperelastic material of neo-Hookean type, with strain energy density $\Psi(\mathbf{C})$, is subjected to a motion $\mathbf{x}(\mathbf{X})$.
Compute

a) the deformation gradient $\mathbf{F}$, the Right Cauchy-Green deformation tensor $\mathbf{C}$ and its invariants $I_1, J = \sqrt{I_3}$,

b) 2^{nd}-Piola-Kirchhoff stress $\mathbf{S}$ and Cauchy stress $\boldsymbol{\sigma}$ at point $\mathbf{P} = (1,4,5)^{\text{T}}$.

Given:

$$\mathbf{x}(\mathbf{X}) = \begin{pmatrix} X_1 + \frac{1}{4}X_1X_2 + 5 \\ \frac{5}{4}X_2 + 4 \\ X_3 \end{pmatrix},\ \Psi(\mathbf{C}) = C_1(I_1 - 3) + C_2(J-1)^2$$

5. Task

Visco-elasticity

A Maxwell element has a constitutive relation $\dot{\varepsilon}(\sigma, \dot{\sigma})$ and relaxation function $G(t)$ as given below.
Determine

a) the stress $\sigma(t)$ for a strain $\varepsilon(t)$ as shown in the figure,

b) the corresponding viscous strain $\varepsilon_d(t)$.

Given:

$$\dot{\varepsilon} = \frac{1}{\eta}(\sigma + \tau\dot{\sigma}),\ G(t) = Ee^{-\frac{t}{\tau}}$$