Last Name : ....

Exam Number : ....

First Name : ....

Matriculation No. : ....

General examination rules:

Write your name and matriculation number on **all** sheets.

Please remember or note your **exam number**.

Please do **NOT** write with a lead pencil or red or green colors.

New task = **new sheet**. Write only on **one side** of each sheet.

All calculations must be **comprehensible**.

Allowed tools: a scientific calculator and a double-sided hand written A4 sheet for formulae etc.

Task	1	2	3	4	5	Σ60
Points						

Problem 1 (11 Points)

a) Prove that the expression $a_i b_j \delta_{ij}$ represents the dot product of two vectors $\mathbf{a}$ and $\mathbf{b}$, while

b) $\epsilon_{ijk} a_j b_k \mathbf{e}_i$ represents the cross product of two vectors $\mathbf{a}$ and $\mathbf{b}$.

c) Given $\epsilon_{jki}\epsilon_{nmi} = (\delta_{jn}\delta_{jn} - \delta_{jm}\delta_{kn})$, prove that

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c},$$

where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors in 3D space.

Problem 2 (15 Points)

a) With reference to a rectangular Cartesian coordinate system, the state of strain at a point is given by the tensor,

$$\boldsymbol{\varepsilon} = \begin{bmatrix} 5 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times 10^{-4}.$$

i) Compute the unit elongation in the direction of $2\mathbf{e}_1 + 2\mathbf{e}_2 + \mathbf{e}_3$.

ii) Calculate the change in angle between two perpendicular lines (in the undeformed state) emanating from the point and in the directions $2\mathbf{e}_1 + 2\mathbf{e}_2 + \mathbf{e}_3$ and $3\mathbf{e}_1 - 6\mathbf{e}_3$.

b) For the displacement field:

$$u_1 = kX_1^2,\ u_2 = kX_2X_3,\ u_3 = k(2X_1X_3 + X_1^2),\ k = 10^{-6},$$

find the maximum unit elongation for an element that is initially at $(1, 0, 0)$.

Problem 3 (12 Points)

A material is called *incompressible* if there is no change of volume under any and all states of stresses.
for an incompressible isotropic linearly elastic solid with finite Young's modulus E, Prove that

a) Poisson's ratio $\nu = \frac{1}{2}$,

b) the shear modulus $G = \frac{1}{3}E$,

c) Bulk's modulus $k \to \infty$,

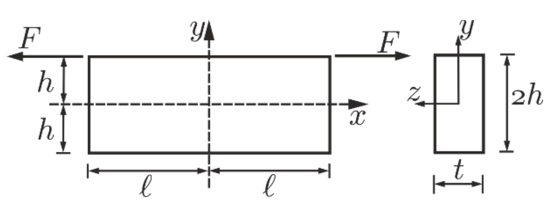
d) Lamme's constant $\lambda \to \infty$, and

e) $k - \lambda = \frac{2}{3}G$

Problem 4 (9 Points)

A rectangular plate of length 2ℓ, height $2h$ and thickness t is loaded in accordance to the Figure below.
For the approximation of the bending stress an Airy's stress function is given as:

$$\Phi = \frac{1}{2}Cy^2 + \frac{1}{6}Dy^3,$$



a) Determine the in-plane stresses.

b) Apply the necessary boundary conditions to obtain the constants C and D of the given Airy's stress function

Problem 5 (13 Points)

Mark the correct statement(s) or write down the answer.

1. The kinematics of a body describes its
☐ rigid body motion ☐ shape change ☐ volume change
☐ surface tractions ☐ body forces ☐ elastic properties
☐ thermal properties ☐ velocity and acceleration
2. What is the definition of the infinitesimal strain tensor ε, and under which two conditions shall it not be used?
3. The invariants of a tensor are scalar measures which do not change under a coordinate system transformation. Prove that the determinant of a tensor is an invariant.
4. What does it mean that a material is isotropic? Name one isotropic and one anisotropic material.
5. If a material is elastic, then the stress
☐ is a function of current strain ☐ is a function of strain rate
☐ is a linear function of strain ☐ is equal in all directions
☐ is a bijective function of strain ☐ can be derived from a potential
6. What is the definition of the traction vector $\mathbf{t}$, and how can it be calculated?
7. What is the relation between principal normal and shear stresses, both in terms of magnitude and direction?
8. Write down the weak form of the mechanical boundary value problem. Why is this form called "weak"?
9. In a thermo-elasticity problem, what are the equilibrium and possible boundary conditions on the thermal field?

T2) i) $\varepsilon'_{11} = \underline{e}'^{T}_{1} \underline{\underline{\varepsilon}} \, \underline{e}'_{1} = \frac{58}{9} 10^{-4}$

ii) $2\varepsilon'_{12} = 2 \, \underline{e}^{T\prime}_{1} \underline{\underline{\varepsilon}} \, \underline{e}'_{2} = \frac{32}{\sqrt{2}} 10^{-4}$ rad.

b) The largest Eigenvalue of $\underline{\underline{\varepsilon}}$

$= 3k$

T3) a) incompressible material $\Rightarrow \varepsilon_{ii} = 0 \Rightarrow \nu = \frac{1}{2}$

b) $G = \frac{E}{2(1+\nu)}$

c) $\sigma^{hyd}_{ij} = \bar{\sigma} \, \delta_{ij}$

$\sigma^{hyd}_{ii} = 3\bar{\sigma}$; $e = \frac{1}{E}(1-2\nu) 3\bar{\sigma}$

$K = \frac{\bar{\sigma}}{e} \Rightarrow k \longrightarrow \infty$ (for $\nu = 1/2$)

T4) BCs

$$\int_{-h/2}^{h/2} \sigma_x(\ell, y) t \, dy = F$$

$$\int_{-h/2}^{h/2} \sigma_x(-\ell, y) t \, dy = F$$

$\Rightarrow C = F/2ht$

$$t \int_{-h/2}^{h/2} \sigma_x(\ell, y) \, y \, dy = F \cdot h \Rightarrow D = \frac{3}{2} \frac{F}{h^2 t}$$