Last Name : ....

Exam Number : ....

First Name : ....

Matriculation No. : ....

General examination rules:

Write your name and matriculation number on **all** sheets.

Please remember or note your **exam number**.

Please do **NOT** write with a lead pencil or red or green colors.

New task = **new sheet**. Write only on one side of each sheet.

All calculations must be **comprehensible**.

Allowed tools: a scientific calculator and a double sided hand written A4 sheet.

Task	1	2	3	4	5	Σ60
Points						

Problem 1 (15 Points)

Please mark the correct statement(s) or write down the answer.

1. The infinitesimal strain tensor $\boldsymbol{\varepsilon}$ is a linear measure of strain. Give its definition and explain under which conditions it should be used.
2. Write down strain tensors for uniaxial tension, dilation, and simple shear.
3. How is the volumetric-deviatoric split of the strain tensor done, and why is it used?
4. Why is the Cauchy stress tensor $\boldsymbol{\sigma}$ symmetric? Make a sketch to explain.
5. Balance of linear momentum in continuum mechanics is true
   - ☐ always
   - ☐ only in small deformations
   - ☐ only in linear elasticity
   - ☐ only in isotropic material
   - ☐ only in absence of tractions
   - ☐ only in absence of thermal expansion

6. What are the relations between strain energy density (elasticity potential), stress and elasticity tensor?
7. How many independent components does the elasticity tensor have in linear elastic, isotropic material, and how would you measure them?
8. What are the balance and boundary conditions in thermal problems?
9. The Airy stress function
   - ☐ can be used only in linear elasticity
   - ☐ can be used only for 2D problems
   - ☐ can be used only in absence of body forces
   - ☐ must fulfill the Neumann boundary conditions
   - ☐ must fulfill the Dirichlet boundary conditions
   - ☐ must be biharmonic
   - ☐ depends on the material properties

Problem 2 (19 Points)

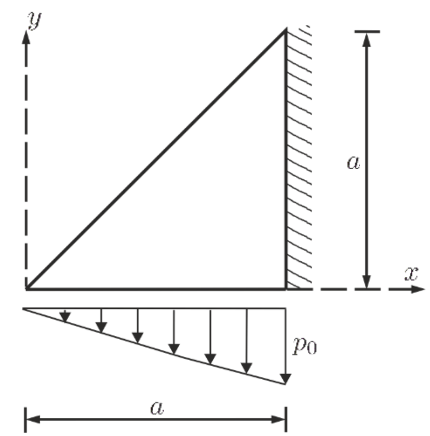
A triangular plate is exposed to a linearly increasing surface normal traction $p(x)$, as shown in the figure. The traction is p_0 at $x = a$. A proper choice of an Airy's stress function $\phi(x, y)$ is given below. The thickness of the plate is t.

$$\phi(x, y) = Ax^3 + Bx^2y + Cxy^2 + Dy^3$$

a) Compute $p(x)$ and the stresses in the whole plate.

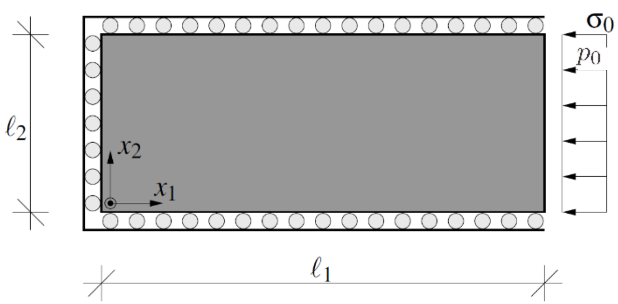
b) Apply the boundary conditions to determine the constants A to D.

c) Calculate the stresses along the fixed boundary.



Problem 4 (9 Points)

A block, as shown in the figure, is subjected to confined compression p_0 (no Strain in x_3-direction). The block is homogeneous and isotropic and has initial dimensions of $\ell_1 \times \ell_2 \times \ell_3$.



Compute

a) the stresses,

b) the strains,

c) and the displacement.

Problem 4 (6 Points)

The determinant of a matrix [A] is defined by

$$\det[A] = \frac{1}{6}\epsilon_{lmn}\epsilon_{xyz}A_{lx}A_{my}A_{nz}$$

If $[A]$ is symmetric and $tr[A] = 0$,

prove that the determinant reduces to

$$\det[A] = \frac{1}{3}A_{lm}A_{mn}A_{nl}$$

Given:

$$\epsilon_{ijk}\epsilon_{pqr} = \delta_{ip}\left(\delta_{jq}\delta_{kr} - \delta_{jr}\delta_{kq}\right) + \delta_{iq}\left(\delta_{jr}\delta_{kp} - \delta_{jp}\delta_{kr}\right) + \delta_{ir}\left(\delta_{jp}\delta_{kq} - \delta_{jq}\delta_{kp}\right)$$

Problem 5 (12 Points)

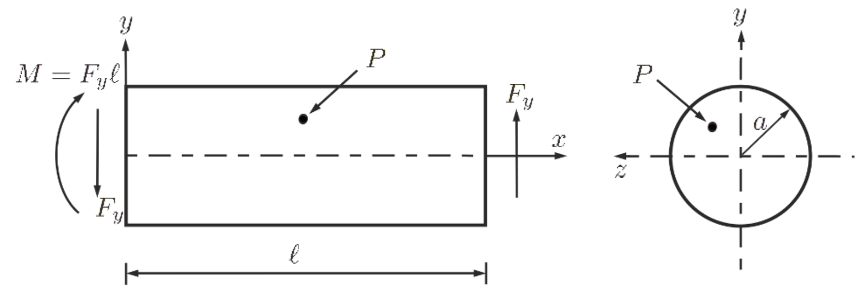
For a beam of circular cross-section, components of the displacement vector $\mathbf{u}$ at any point (x, y, z), with respect to the given coordinate system, are given as:

$$u_x = A\left[\frac{1}{2}a^2y + \left(\frac{1}{2}x^2 - \ell x\right)y - \frac{1}{4}\left(y^3 + yz^2\right)\right],$$

$$u_y = A\left[\frac{1}{2}a^2x + \frac{1}{2}\ell x^2 - \frac{1}{6}x^3 + \frac{1}{4}\left(\ell - x\right)\left(y^2 - z^2\right)\right],$$

$$u_z = A\left[\frac{1}{2}\left(\ell - x\right)yz\right],$$

where $A = \frac{4F_y}{\pi E a^4}$, E is Young's modulus and a, ℓ, F_y are as shown in the figure.



a) Find the components of the linear strain tensor ε and the infinitesimal rotation tensor ω.

b) Evaluate the strain components at point $P(\ell/2, a/2, a/2)$ and further determine the stretch along the direction

$$\mathbf{n} = \frac{1}{\sqrt{3}}[1; 1; 1]^T.$$