Family Name : ....

Exam number : ....

First name : ....

Reg. number : ....

Instructions:

Write your name and registration number in the corresponding fields. Do **not** use pencils, green or red pens (used in marking). Write your name and reg. number on **each sheet**, number your sheets **consecutively** and write only on **one side** of the sheets! Memorize or write down the **exam number**.

You are allowed to use a non-programmable pocket calculator and an A4 sheet of paper for formulas.

Task	1	2	3	4	5	Σ (60)
Mark						

Task 1 (11 Points)

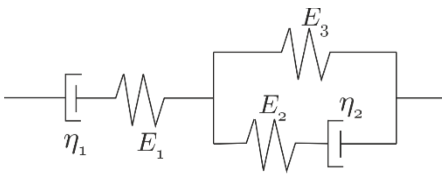
Mark the following statements as true (T) or false (F) (1 point/question).

1. Visco-elastic models always show creep and relaxation.
2. Visco-elastic models show a rate-dependent behaviour.
3. When passing from tension to compression the creep response of the Kelvin-Voigt model approaches zero asymptotically.
4. In linear elasto-plastic materials we have an additive decomposition of the strains in small deformation.
5. An elasto-plastic material model requires at least a yield criterion, a hardening law and a flow rule.
6. The Kuhn-Tucker conditions are valid only in case of linear elasticity.
7. Kinematic hardening increases the elastic stiffness when the history variable becomes negative.
8. The amount of damage depends only on the current state of the material.
9. Damage does not degrade the shear modulus G.
10. The deformation gradient includes the effect of rigid-body rotations.
11. Almansi strain and first Piola-Kirchhoff stresses are work conjugate of the deformed configuration.

Task 2 (16 Points)

Viscoelasticity

For the given rheological model,



a) derive the constitutive equation (assuming that the constitutive law for the Maxwell-element is known).

b) determine if it is fluid or solid-like. Why?

c) suppose $E_1 = E_2 = \infty$; is the model now fluid or solid-like? What is such a model called?

d) suppose $\eta_1 = \eta_2 = \infty$; is the model now fluid or solid-like? What is the name of such a model?

Task 3 (13 Points)

Hyperelasticity

Given strain energy density ψ and deformation gradient $\mathbf{F}$.

a) Compute the right and left Cauchy-Green deformation tensors,

b) their invariants I_1, I_2, I_3 and J.

c) Calculate the constant β such that the given strain energy density ψ is zero in the underformed configuration ($\alpha \neq 0$).

d) With the result in part c), compute the second Piola-Kirchhoff stress tensor.

Given: $\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda_z \end{bmatrix}$, $\psi = \alpha(I_1 - 3) + 10\alpha(J^2 - 1) - \beta \ln(J + I_1)$.

Task 4 (13 Points)

Plasticity

An elasto-plastic material is subjected to a state of pure shear, characterized by the strain tensor $\boldsymbol{\varepsilon}_s$. The material is linear elasto-plastic, with elastic modulus E, Poisson's ratio ν and initial yield limit σ_Y.

a) Determine the stress tensor.

b) Compute the strain $\varepsilon = \varepsilon_y$ at which the material starts to yield following the Tresca and Von Mises criteria.

c) Consider now an elasto-plastic material with isotropic hardening, characterized by hardening modulus H, and subjected to a uniaxial strain.
Sketch the stress-strain relation for the following load path:
$0 \to \varepsilon_{max} \to 0 \to -\varepsilon_{max} \to 0$.

Given: $E = 70$ GPa, $\nu = 0.5$, $\sigma_Y = 210$ MPa, $\boldsymbol{\varepsilon}_s = \begin{bmatrix} -\varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & 0 \end{bmatrix}$,
$\varepsilon_{max} = 0.007$, $H = 10$ GPa.

Task 5 (7 points)

Damage

An initially linear-elastic material with elastic modulus E is subjected to monotonic tension. For monotonic loading, the damage is supposed to evolve following the damage law

$$D(k) = \begin{cases} 0 & \text{if} \quad \omega(k) \leq 0 \\ 1 - \omega(k) & \text{if} \quad 0 < \omega(k) < 1 \\ 1 & \text{if} \quad \omega(k) \geq 1, \end{cases}$$

where k is the history variable taken as the maximum strain that occurred during the loading history ($k = max\{\varepsilon(t)\}$) and $\omega(k) = \frac{5}{4}(1 - \frac{\varepsilon_0}{k})$.

a) Calculate the stress at ε_0, $2\varepsilon_0$, $5\varepsilon_0$ and $6\varepsilon_0$.

b) Sketch the monotonic stress strain response between 0 and $5\varepsilon_0$ and add the unloading response at $4\varepsilon_0$.