Family Name : …………………………… **Exam number : ……………………………**

First name : …………………………… **Reg. number : ……………………………**

Notes on the exam:

Write name and registration number in the corresponding fields. Do **not** use pencils, green or red pens (used in marking). Place name and reg. number on **each sheet**, number sheets **consecutively** and write only on **one side** of the sheets! Memorize or write down the **exam number**.
You are allowed to use a non-programmable pocket calculator and two pages of equations.

Task	1	2	3	4	5	6	Σ (36)
Mark							

1. Task (9 Points)

Answer briefly the following questions:

1) Compile a list of criteria used in fracture mechanics to decide whether a crack will propagate, or not.
2) Sketch the relation $\sigma(r)$ between stress and distance from crack tip in linear-elastic fracture mechanics with Irwin's correction, for a sharp crack tip.
3) Make a sketch of the Dugdale model and label it properly. What is the value of the J integral for this model?
4) What is a cohesive zone model and how is it used in finite element analysis?
5) Make a sketch of fatigue crack growth per cycle versus stress intensity amplitude for a material without permanent endurance limit. Mark the region in which the Paris law is considered to be valid.

2. Task (7 Points)

Failure theories

An uncracked, linear elastic material with Lame's constants μ, λ is subject to a strain state $\boldsymbol{\varepsilon}$. Data from uniaxial tests indicates that the material will fail at $\sigma_\mathrm{T} = 20\mathrm{MPa}$ in tension and $\sigma_\mathrm{C} = 100\mathrm{MPa}$ in compression.

Determine at which strain ε the material will fail based on

a) The maximum stress criterion.
b) The Drucker-Prager criterion (failure surface F as given below).

given:

$\lambda = \mu = 5\mathrm{GPa}$; $F(I_\sigma, II_s) = \alpha I_\sigma + \sqrt{II_s} - k = 0$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} 2\varepsilon & 0 & 0 \\ 0 & -\varepsilon & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Remark: $\boldsymbol{\sigma} = \lambda \mathrm{tr}(\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon}$

3. Task (6 Points)

Fatigue: Cycles-Till-Failure

A structure with a disc-shaped crack of initial radius a is subjected to cyclic tensile loading between $\sigma = 0$ and $\sigma = \sigma_\mathrm{o}$. The critical (static) stress intensity K_IC for the material be known. The given Paris equation for crack growth be valid.
Determine

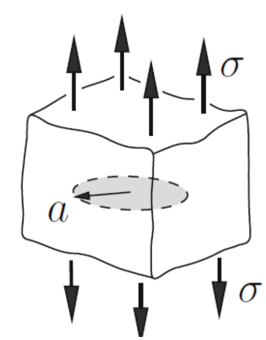
a) The stress σ^stat the part will withstand for one cycle.
b) The number of cycles the part will withstand if the load amplitudes alternated (one cycle with $\sigma_\mathrm{o}^{(1)}$, one with $\sigma_\mathrm{o}^{(2)}$, ...).

given:

$a = 1\,\mathrm{mm}$, $\sigma_\mathrm{o}^{(1)} = 0.5\,\mathrm{MPa}$, $\sigma_\mathrm{o}^{(2)} = 1\,\mathrm{MPa}$

$K_\mathrm{I} = \frac{2}{\pi}\sigma\sqrt{\pi a}$, $K_\mathrm{IC} = 10\,\mathrm{MPa}\sqrt{\mathrm{mm}}$

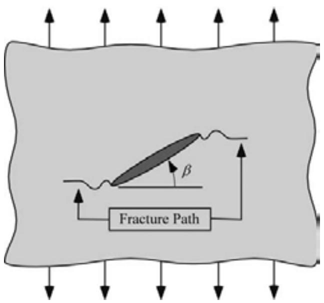
$\frac{\mathrm{d}a}{\mathrm{d}N} = 0.01\,\mathrm{mm}\left(\frac{\Delta K}{\mathrm{MPa}\sqrt{\mathrm{mm}}}\right)^4$



4. Task (4 Points)

Mixed-mode fracture

An angled crack, as shown in the picture, is under mixed-mode loading (Why? Which modes?). Describe **qualitatively** how the direction of crack propagation (relative to the initial direction β) can be found. What criterion is used? What general rule could be given concerning the final propagation direction?



5. Task (5 Points)

Irwin's correction

A hollow, thin-walled (thickness t) sphere with an initial crack of length $2a$ is subjected to an internal pressure p. The material has fracture thoughness K_{IC} and yield limit σ_Y.

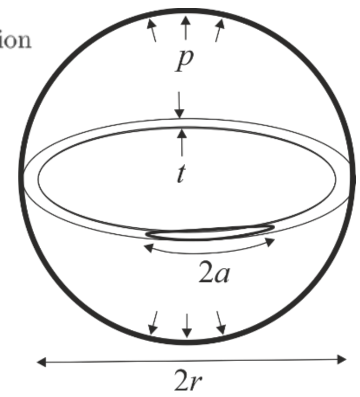
a) Why should Irwin's correction be used in this situation?
Calculate the crack length correction!

b) At what pressure p would crack propagation start?

c) Verify again that LEFM+Irwin's correction is appropriate.

given:

$t = 1\mathrm{mm}$, $r = 100\mathrm{mm}$, $a = 10\mathrm{mm}$,
$K_{\mathrm{IC}} = 25\mathrm{MPa}\sqrt{\mathrm{m}}$, $\sigma_Y = 500\mathrm{MPa}$,
and $K_{\mathrm{I}} = 1.6\sigma_0\sqrt{\pi a}$ for this geometry,
with $\sigma_0 = \frac{pr}{2t}$.



6. Task (5 Points)

Energy release rate

To determine the fracture toughness G_C of an adhesive a set-up as shown below is used: a ring ($R >> w$) is glued onto a base with an adhesive layer of thickness t. Torque is applied to the ring until the adhesive layer fails. The measured torque-angle relation $M(\varphi)$ is shown below. Ring and base can be considered rigid.

a) Determine the critical energy release rate G_{C}.

b) Assuming the material was linear-elastic (Poisson's ratio ν), calculate the critical stress intensity factor K_{IIC}.

c) By what factor whould the result from a) change if, due to imperfect application, the adhesive only covered 95% of the surface between ring and base?

given:

$t = 1\mathrm{mm}$, $R = 100\mathrm{mm}$, $w = 10\mathrm{mm}$, $\nu = 0.33$

Remark:

The relation between Young's and shear modulus is $E = 2G(1+\nu)$.