Family Name : …………………………… **Exam number :** ……………………………

First name : …………………………… **Reg. number :** ……………………………

Notes on the exam:

Write name and registration number in the corresponding fields. Do **not** use pencils, green or red pens (used in marking). Place name and reg. number on **each sheet**, number sheets **consecutively** and write only on **one side** of the sheets! Memorize or write down the **exam number**.
You are allowed to use a non-programmable pocket calculator and two pages of equations.

Task	1	2	3	4	Σ (40)
Mark					

1. Task (16 Points)

Answer briefly the following questions:

General

1) Sketch a diagram of fracture toughness versus ultimate strength. Mark the regions where plastics, ductile metals, brittle metals and ceramics are expected. Add lines that mark critical crack lengths of approx. 1μm, 1mm, and 1cm.
2) Why is there such a huge difference between the fracture toughess of ductile compared to otherwise similar brittle materials?

Linear elastic fracture mechanics (LEFM)

3) Under which conditions and for which materials should LEFM be used?
4) In mixed-mode fracture, what tendency exists regarding the final direction of crack propagation? As an example, sketch the crack path in a center-cracked large plate under uniaxial tension, with inital crack orientation of 45° relative to the load axis.

Elastic-plastic fracture mechanics (EPFM)

5) What are cohesive zone models? Provide an example of a traction-separation law. How are these models used in a finite element setting?

Creep

6) Which criterion is used to calculate the crack propagation velocity in the visco-elastic Dugdale model?
7) What is the relation between J-integral and energy release rate?

Fatigue

8) Create a sketch of the crack growth per cycle versus the stress intensity amplitude. Mark the Paris' line and the lower threshold of fatigue crack growth. Why does the latter exist in many materials?

2. Task (10 Points)

Fatigue

A thin-walled cylinder with a longitudinal crack, as depicted, is subjected to cyclic loading by an internal pressure of amplitude Δp.
Given initial crack lengths of $a_1 = 2$mm and $a_2 = 4$mm, determine the number of cycles required to make the crack grow by 1mm, assuming

a) a Paris' law of type $\frac{da}{dN} = C(\Delta K_\mathrm{I})^n$,

b) a modified Paris' law of type $\frac{da}{dN} = C[(\Delta K_\mathrm{I})^n - (\Delta K_\mathrm{th})^n]$.

c) If the load consisted of 70% cycles with Δp, 20% cycles with $2\Delta p$ and 10% cycles with $3\Delta p$, what would be the equivalent load amplitude?

given:

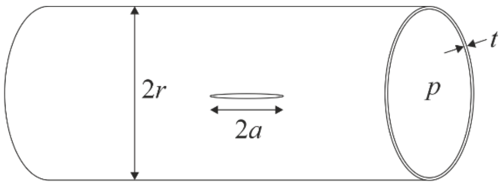
$\Delta p = 1\mathrm{MPa}$, $t = 1\mathrm{mm}$, $r = 100\mathrm{mm}$, $C = 2.5 \cdot 10^{-18} \frac{\mathrm{m}}{(\mathrm{N/m^{3/2}})^n}$, $n = 2$,

$\Delta K_\mathrm{th} = 5 \cdot 10^6 \mathrm{N/m^{3/2}}$

Remark:

The stress intensity factor for the given geometry is

$K_\mathrm{I} = 1.1\sigma_0\sqrt{\pi a}$ with $\sigma_0 = p\frac{r}{t}$



3. Task (8 Points)

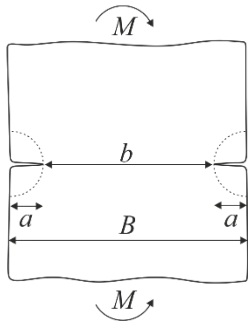
Stress intensity factor

A beam of width B and thickness t is subjected to bending by a bending moment M. It has sharp cracks of depth a on both edges.

a) At what bending moment will the crack propagate assuming LEFM?

b) Check whether Irwin's correction should and can be used and calculate the crack length correction.

c) To remove the effect of the cracks, round notches of radius $r = a$ are machined (dotted lines in figure). Check when the beam will fail due to yielding, keeping in mind the stress concentration factor K_t.



given:

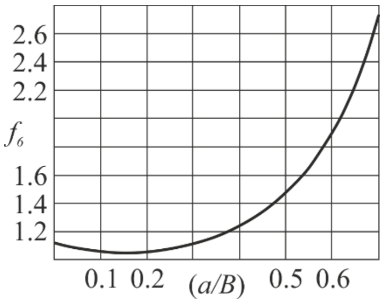
$B = 30\text{mm}$, $a = 2.5\text{mm}$, $t = 3\text{mm}$,

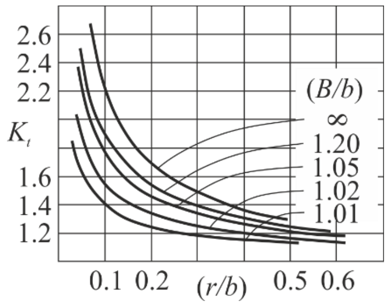
$K_{\text{IC}} = 70\text{MPa}\sqrt{\text{mm}}$, $\sigma_{\text{Y}} = 100\text{MPa}$,

$K_{\text{I}} = \sigma_0 \sqrt{\pi a} f_6$ with $\sigma_0 = 6\frac{M}{tB^2}$

Remark:

The stress concentration factor K_t increases the stress relative to the stress in an unnotched beam of width b.





4. Task (6 Points)

Energy release rate

To assess the fracture toughness of an adhesive joint, a circular bar (diameter d) has been glued into a solid block, as shown in the figure. The adhesive has thickness t and Poisson's ratio ν. Bar and block can be assumed to be infinitely stiff. In pulling out the bar from the block, a linear relation between displacement and force is observed, see the diagram.

a) Calculate the critical energy release rate G_{c}.

b) Calculate the critical stress intensity factor K_{II}.

given:

$d = 10\text{mm}$, $l = 100\text{mm}$, $t = 0.5\text{mm}$, $\nu = 0.5$