

# Oriented Metrics for Bottom-Up Enumerative Synthesis

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## Goal:

**Unify and generalize**

the existing approaches to bottom-up enumerative synthesis

# Syntax-Guided Synthesis (SyGuS)

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Specification: Examples (*In, Out*)

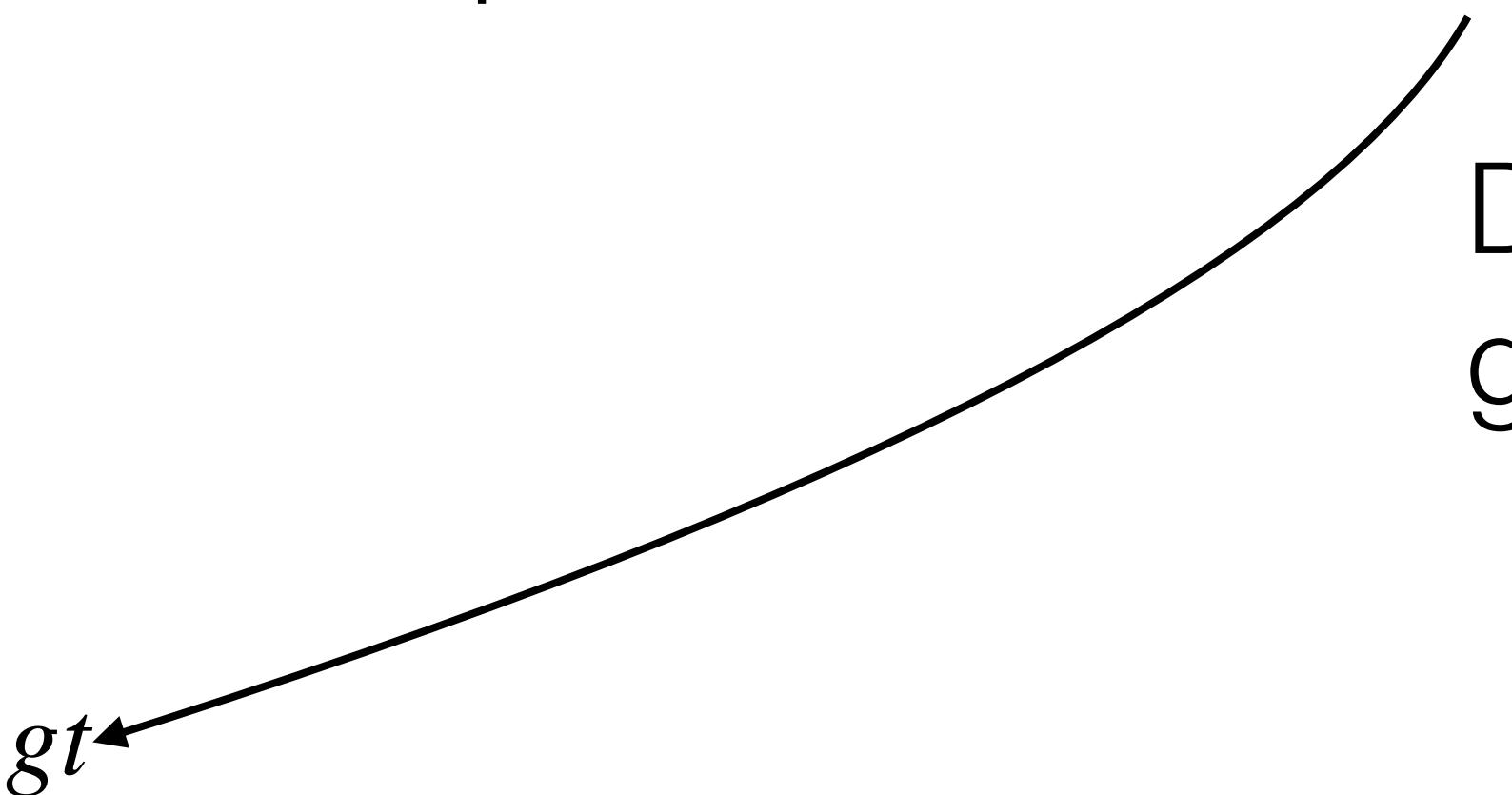
# Syntax-Guided Synthesis (SyGuS)

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Specification: Examples (*In, Out*)

Defines the  
ground truth  $gt$

$gt$



# Syntax-Guided Synthesis (SyGuS)

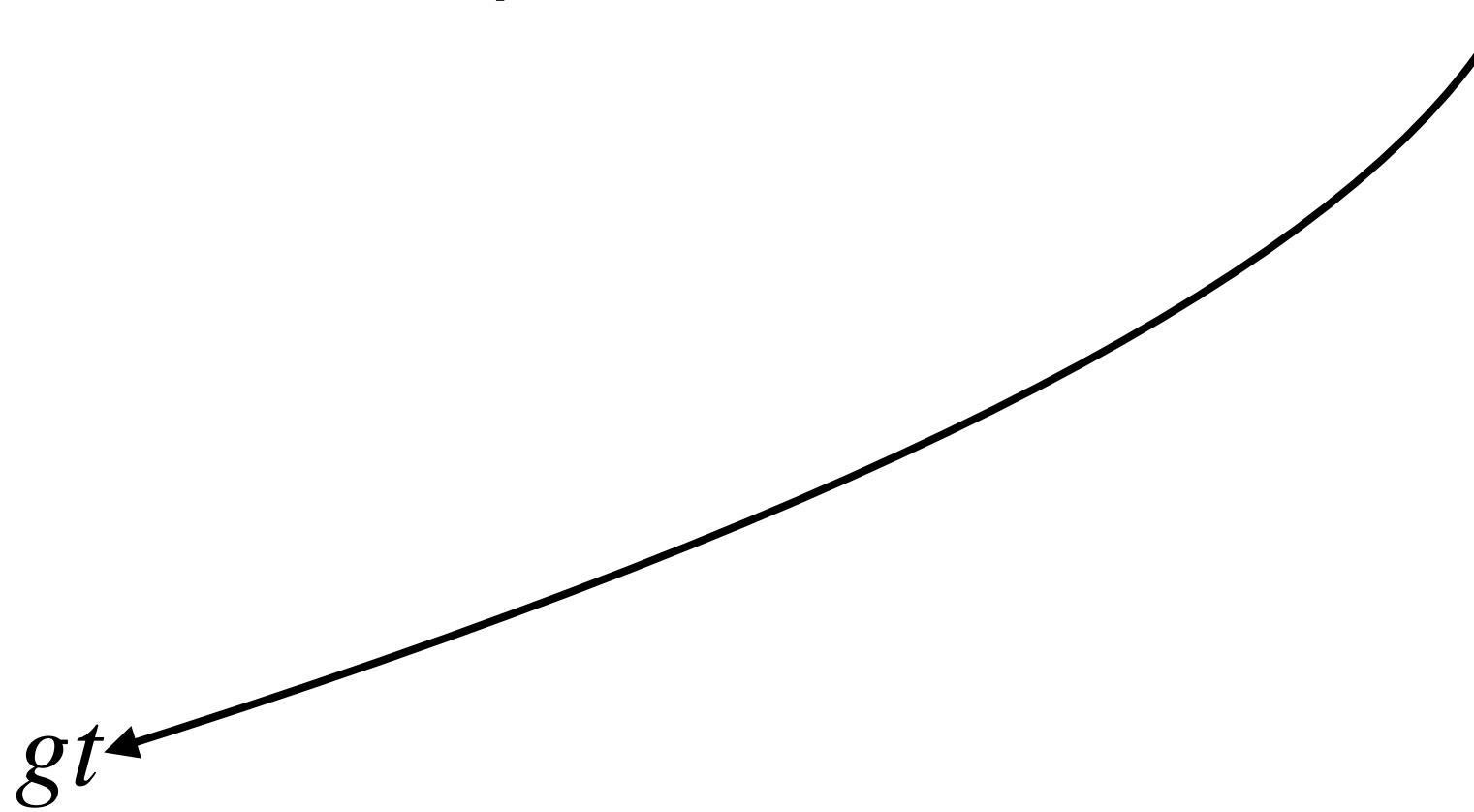
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Grammar:  $\mathcal{G}$

Specification: Examples (*In*, *Out*)

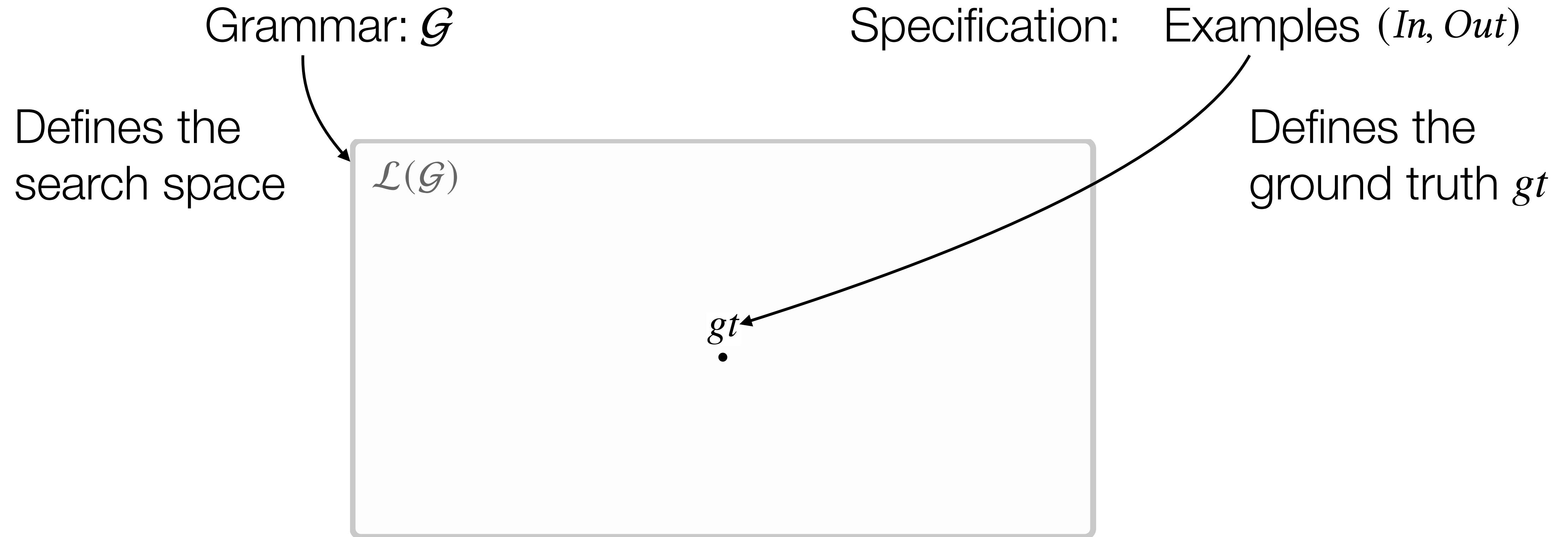
Defines the  
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$gt$

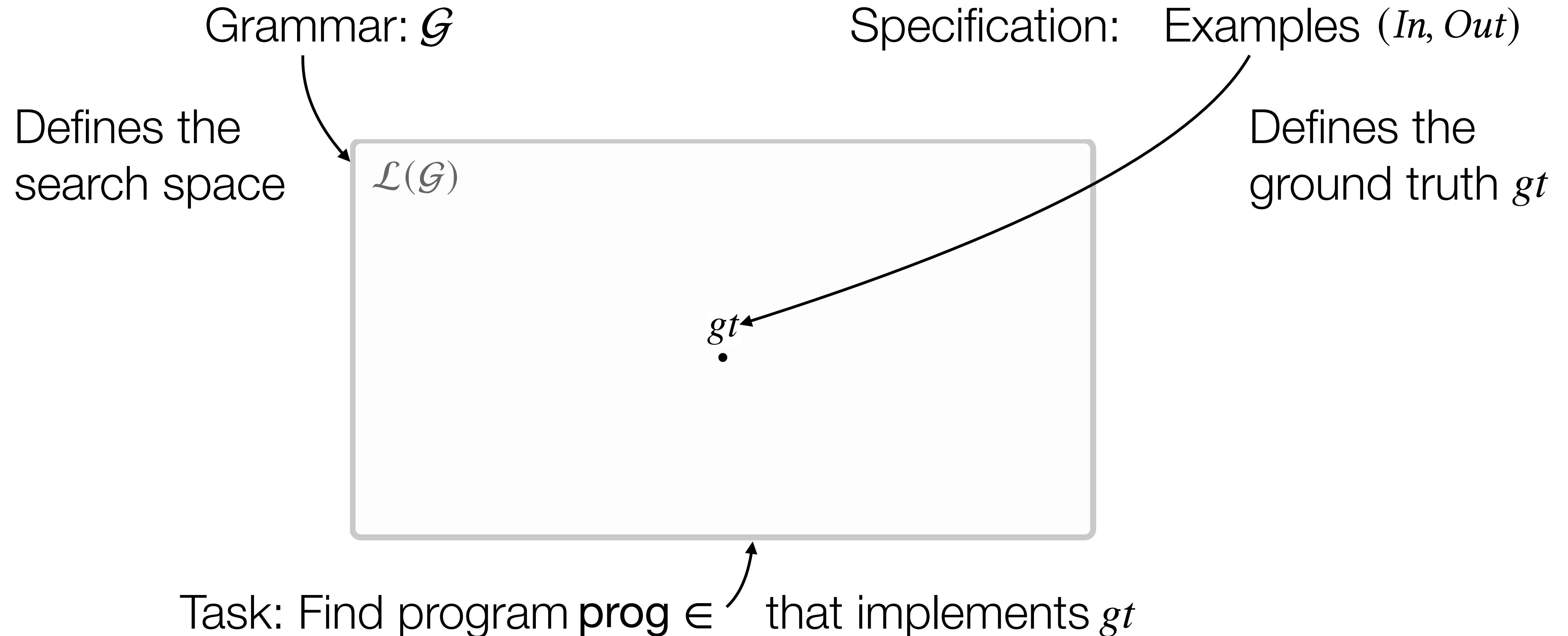


# Syntax-Guided Synthesis (SyGuS)

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# Syntax-Guided Synthesis (SyGuS)



# Syntax-Guided Synthesis (SyGuS)

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<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
"Rennes_City"	"Rennes"
"PLDI_Conf"	"PLDI"
"Seoul_City"	"Seoul"

# Syntax-Guided Synthesis (SyGuS)

$S ::= V \mid \text{replace}(S, S, S) \mid \text{concat}(S, S)$

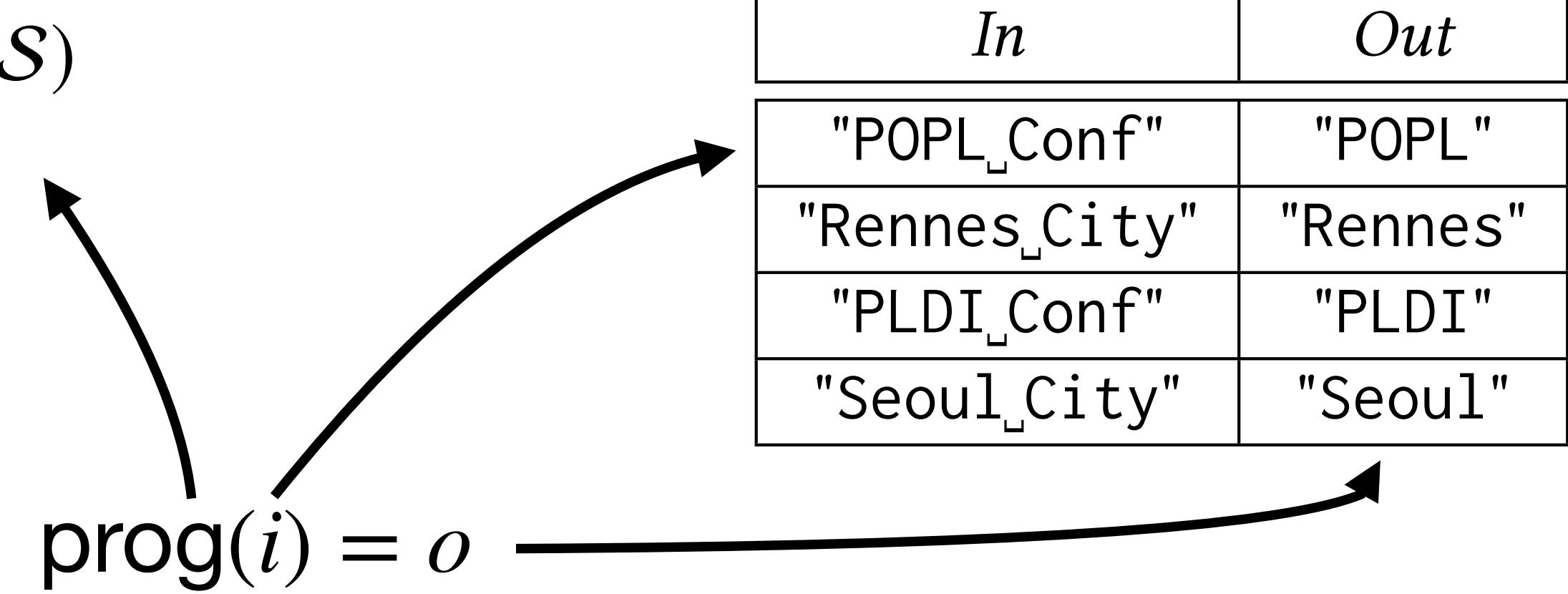
$V ::= x \mid \epsilon \mid \text{"\_Conf"} \mid \text{"\_City"}$

<i>In</i>	<i>Out</i>
"POPL\_Conf"	"POPL"
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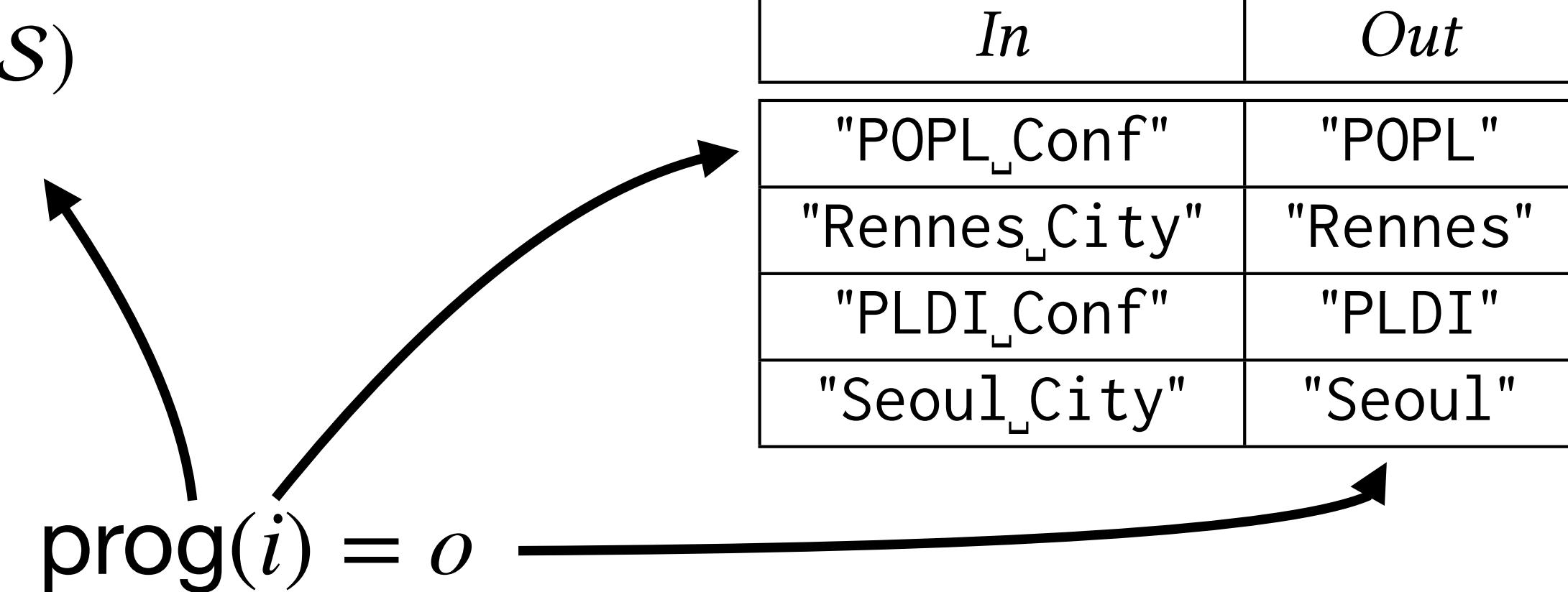
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<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
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$\text{prog}(i) = o$

Solution:  $r(r(x, \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon)$

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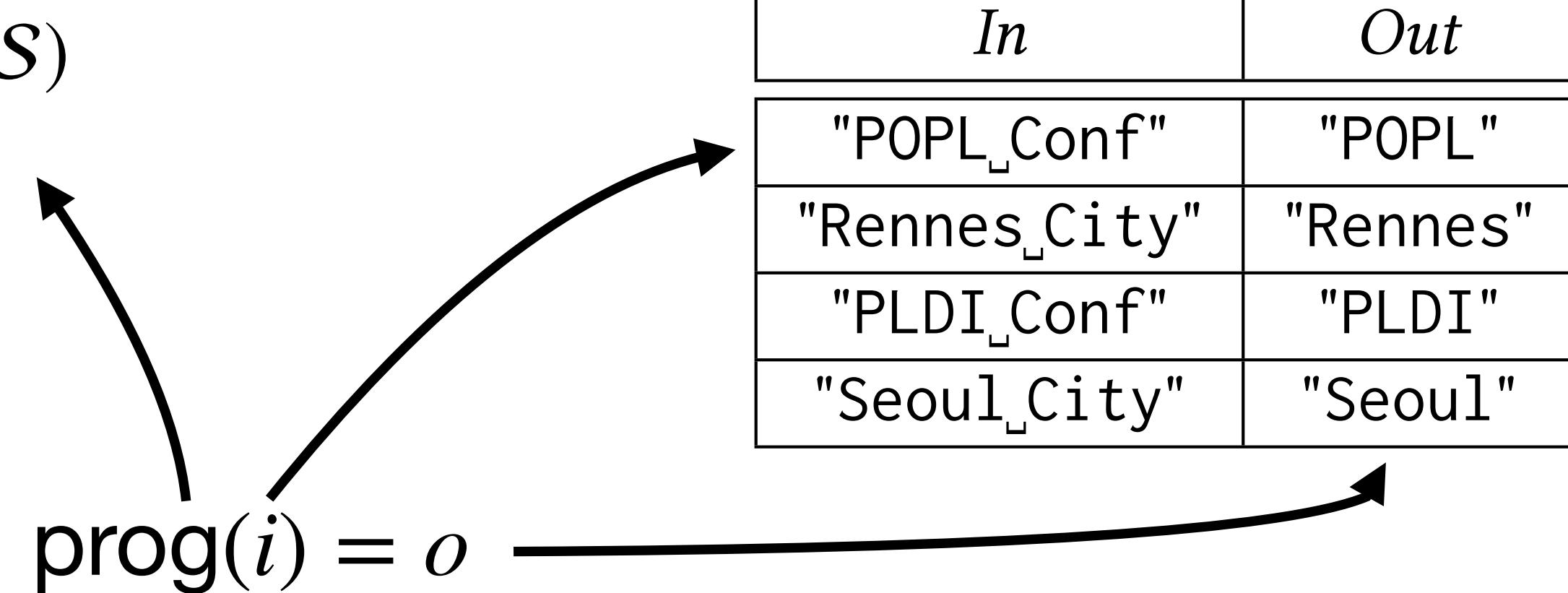
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Solution:  $r(r(x, \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon)$

$r(\underbrace{r("POPL\_Conf", \text{"\_Conf"}, \epsilon)}_{\text{prog}(i) = o}, \text{"\_City"}, \epsilon)$

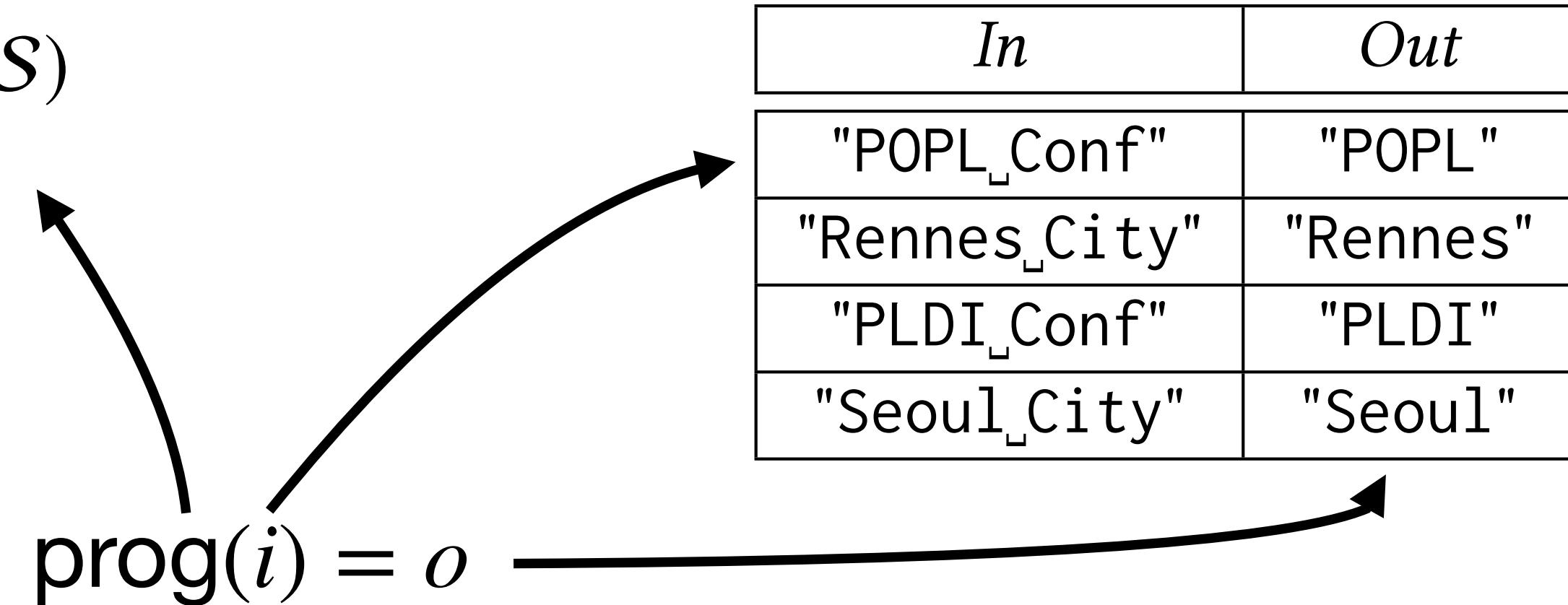
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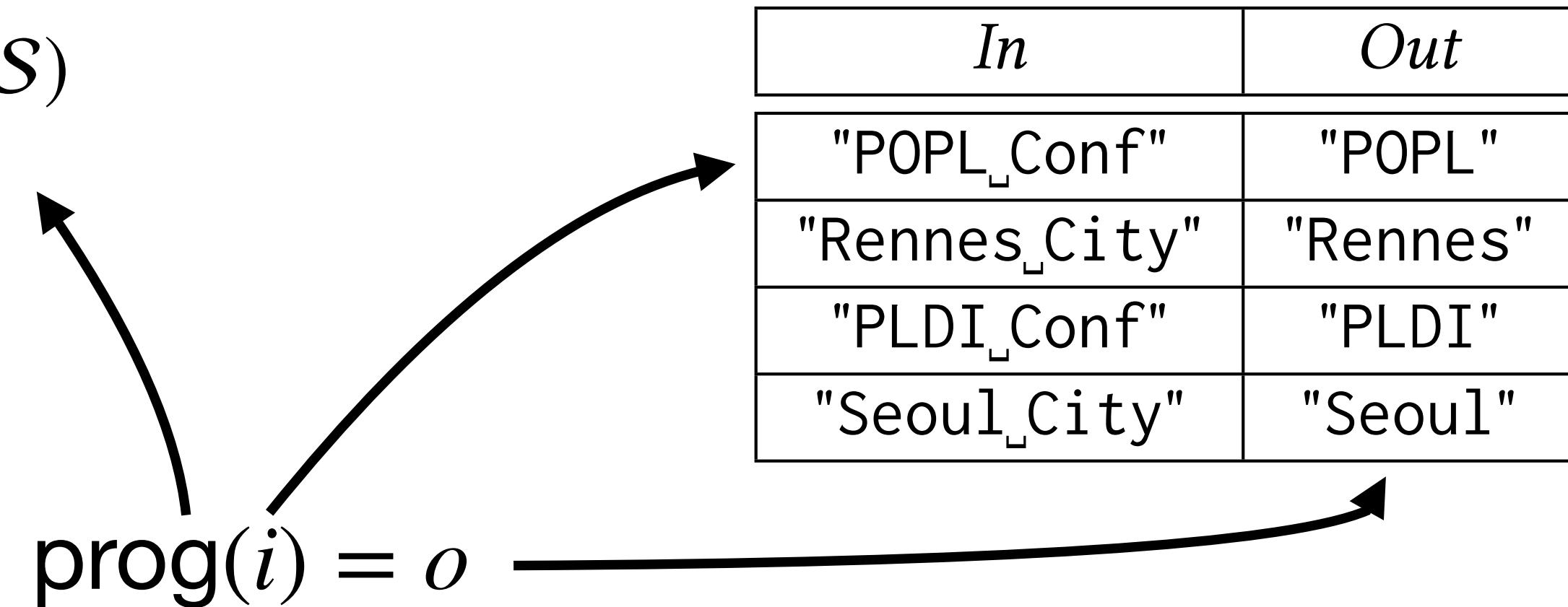
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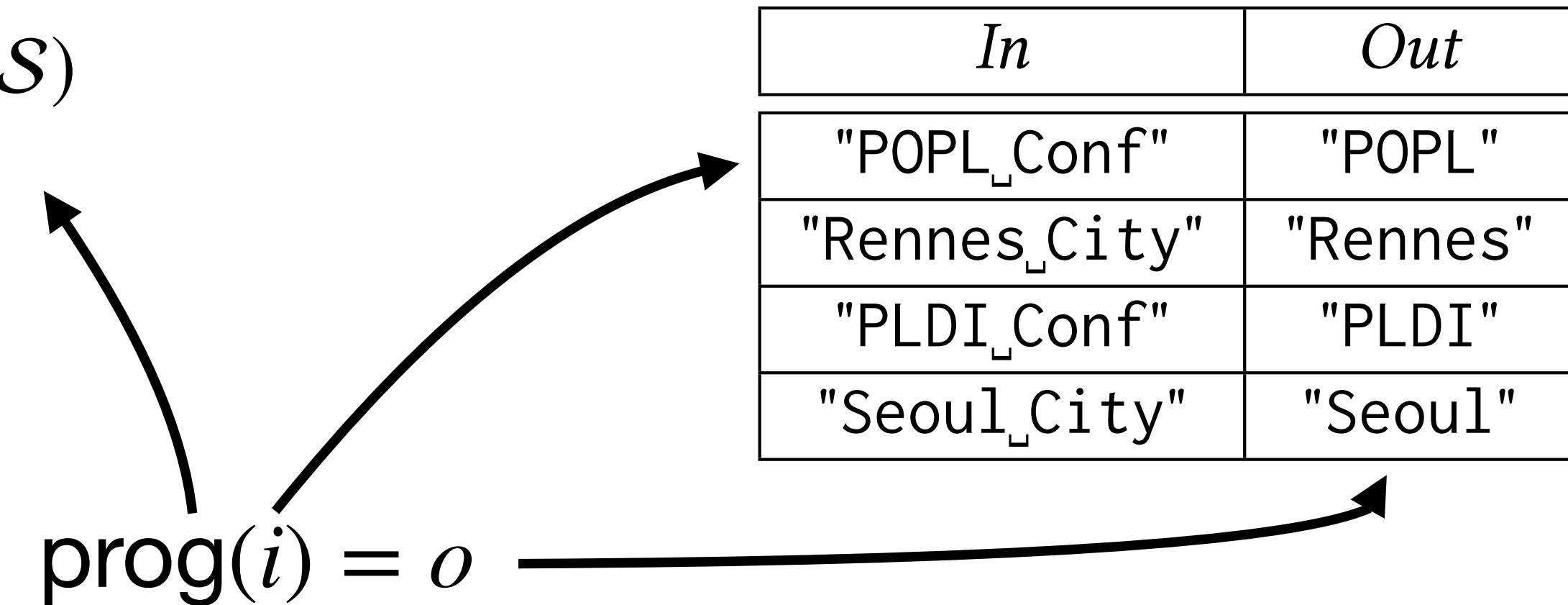
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Solution:  $r(r(x, \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon)$

$r(\underbrace{r("POPL\_Conf", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"POPL"}}, \underbrace{\text{"POPL"}_{\text{}}}_{\text{"POPL"}}$

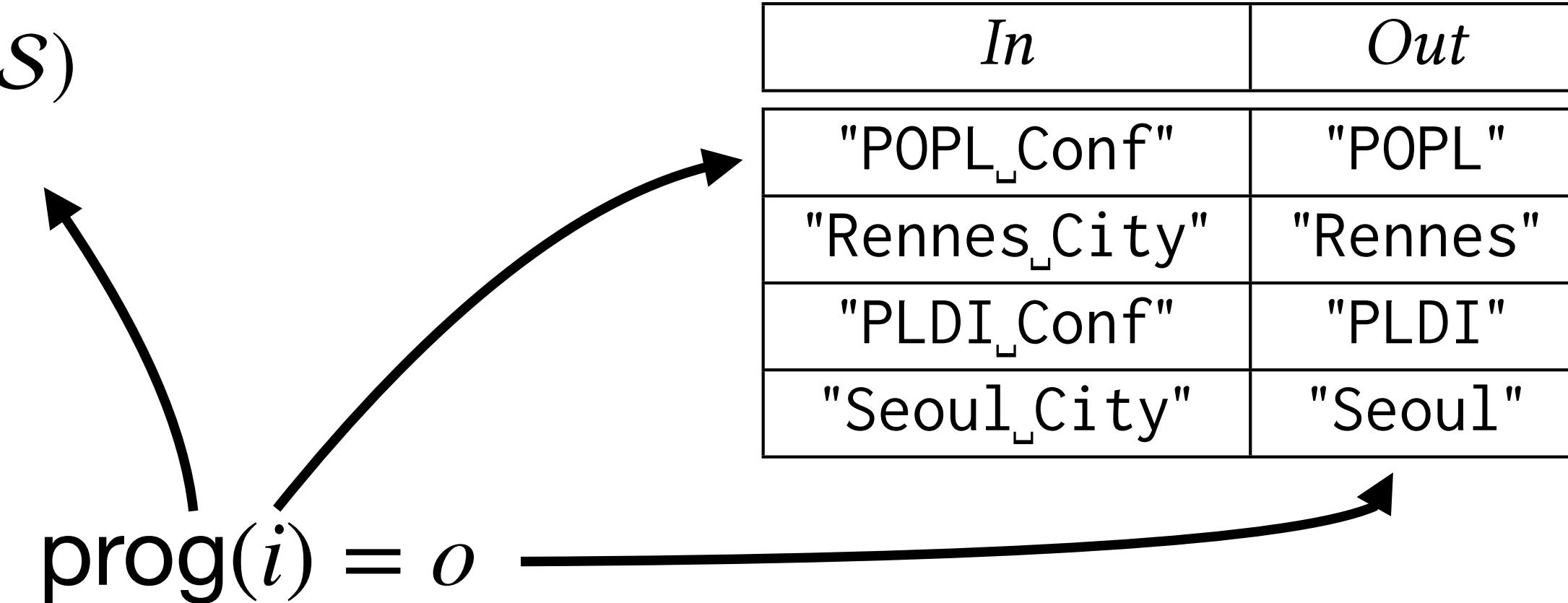
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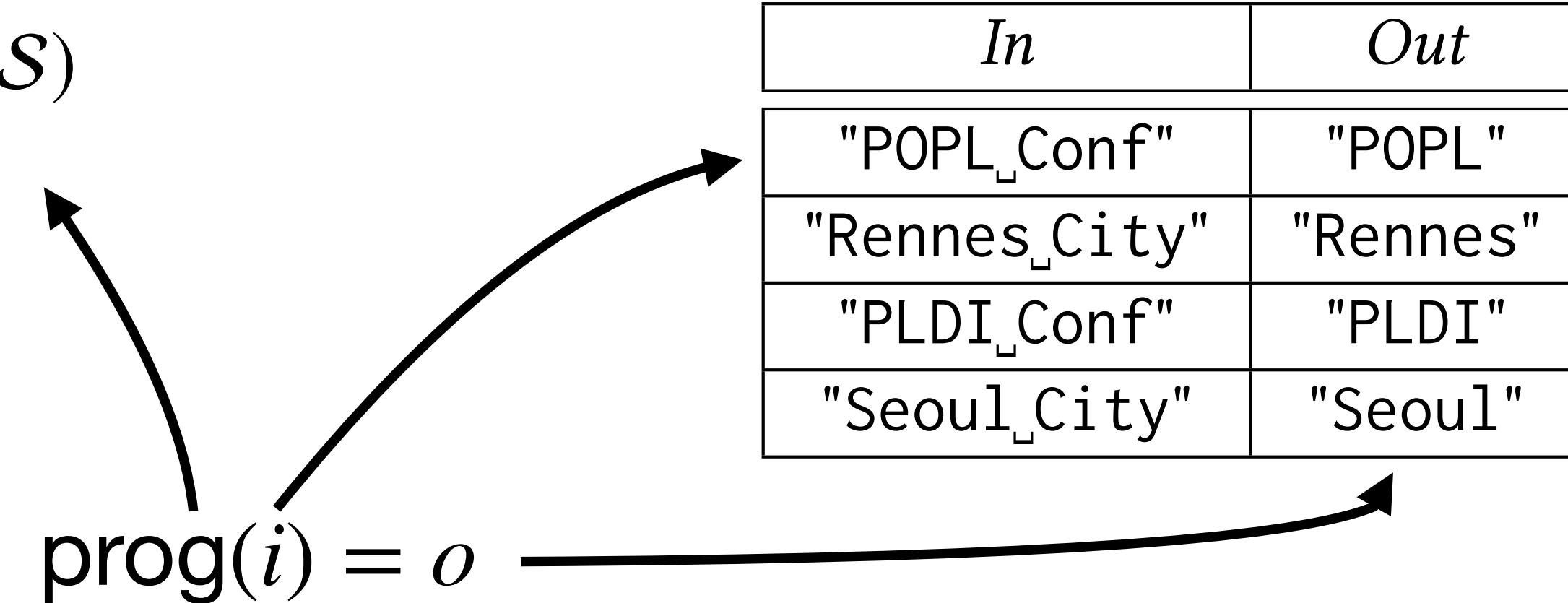
$r(\underbrace{r("POPL\_Conf", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"POPL" }})$

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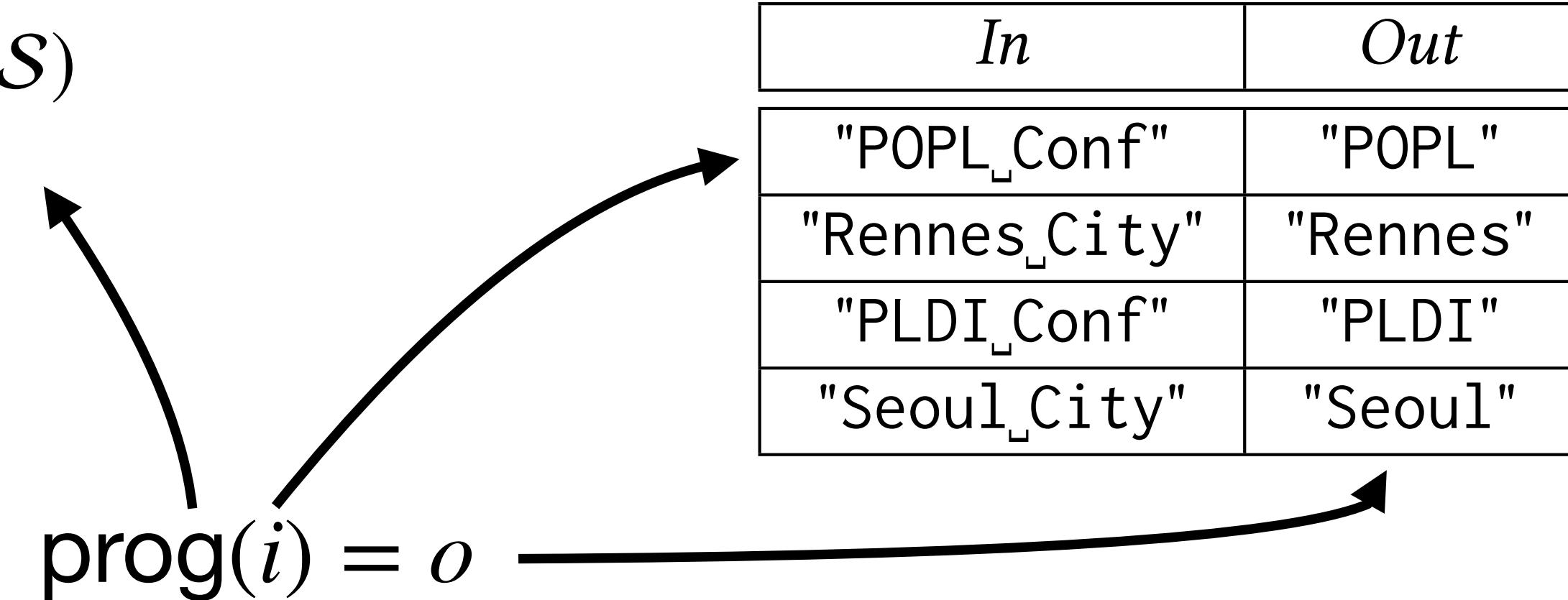
$r(\underbrace{r("POPL\_Conf", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"POPL" }})$

$r(\underbrace{r("Rennes\_City", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"Rennes" }})$

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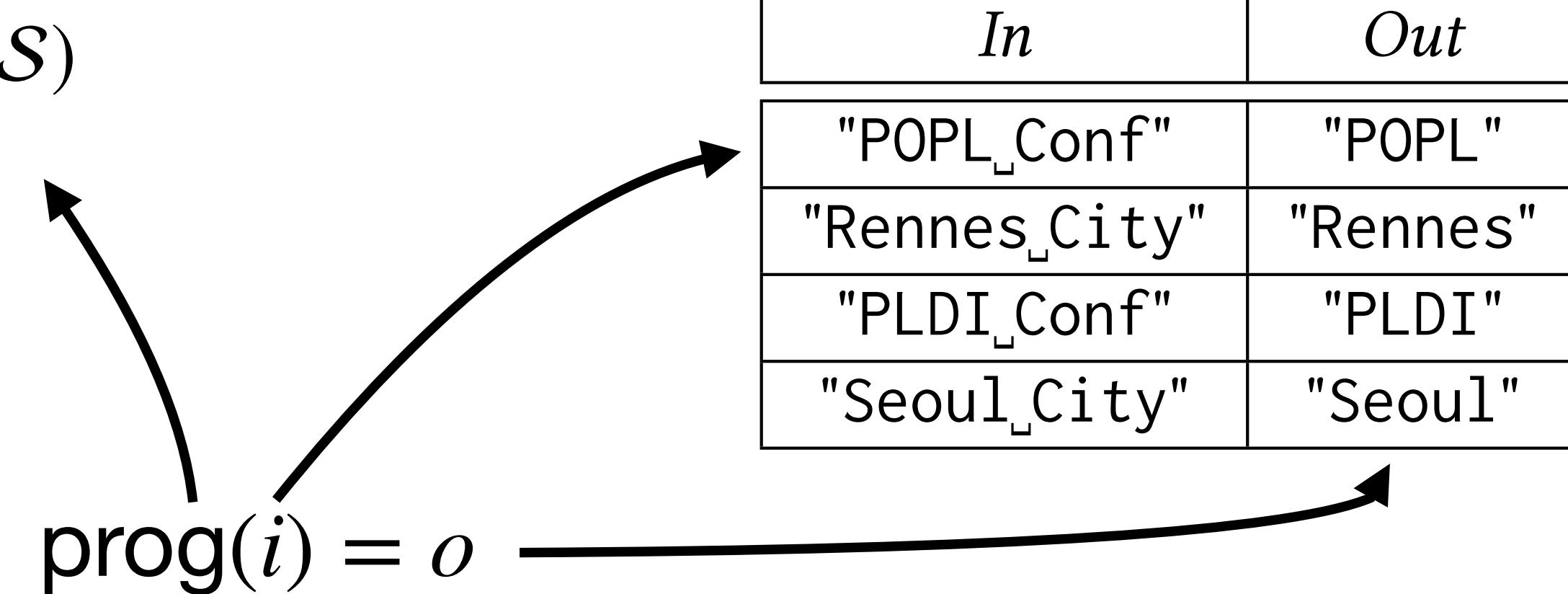
$r(\underbrace{r("POPL\_Conf", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"POPL" }})$

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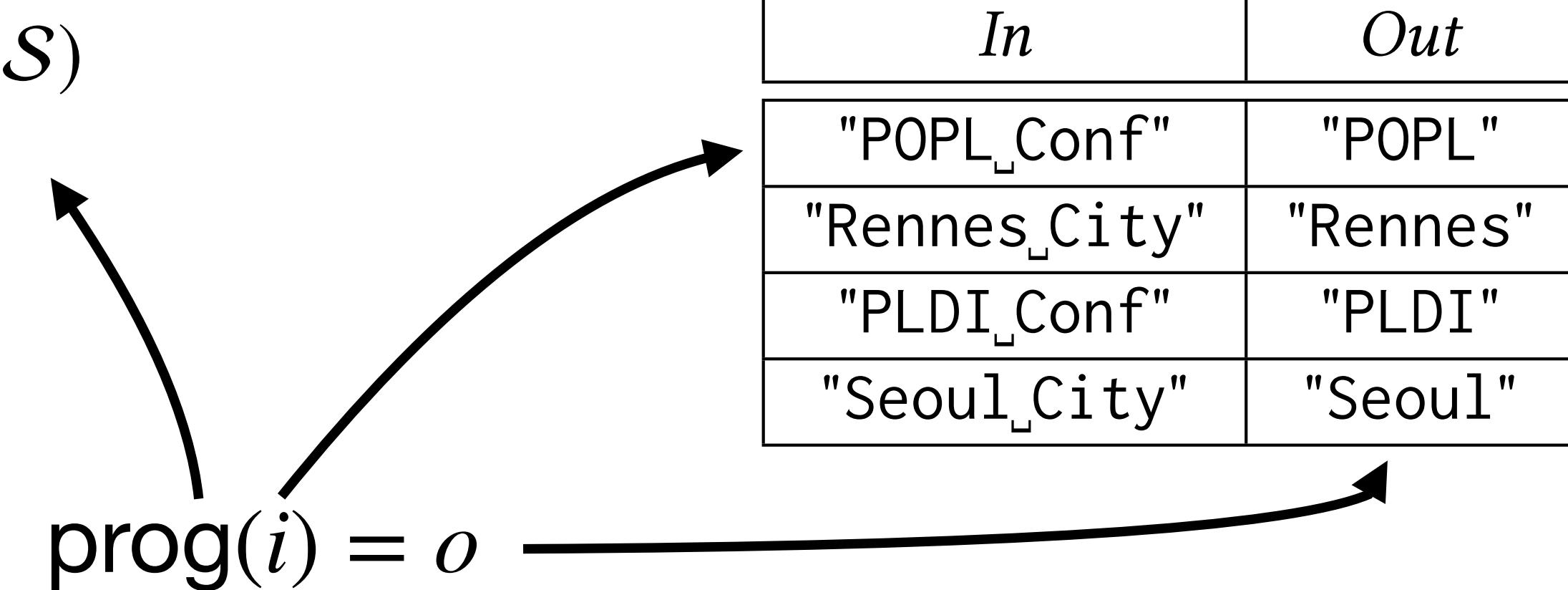
$r(\underbrace{r("POPL\_Conf", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"POPL" }}, \text{"POPL"})$

$r(\underbrace{r("Rennes\_City", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"Rennes\_City" }}, \text{"Rennes\_City"})$

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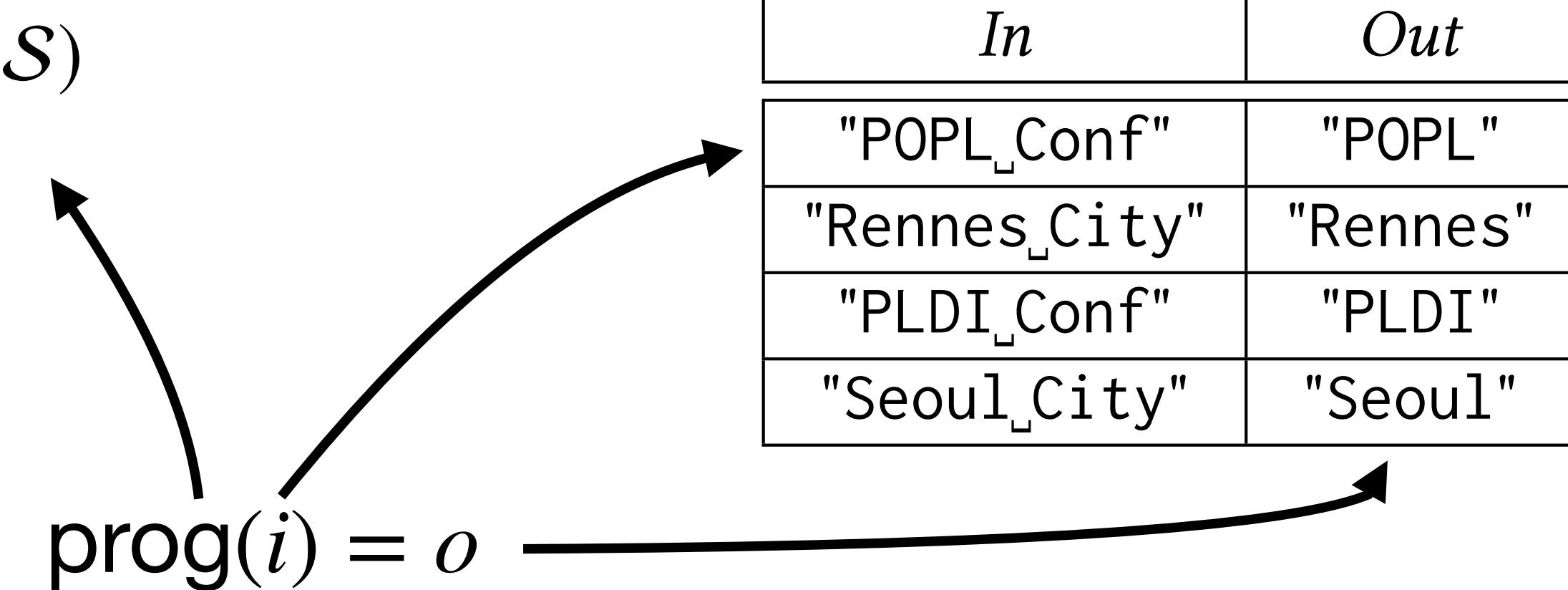
$r(\underbrace{r("POPL\_Conf", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"POPL" }}, \epsilon)$

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Solution:  $r(r(x, \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon)$

$r(\underbrace{r("POPL\_Conf", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"POPL" }}, \epsilon)$

$r(\underbrace{r("Rennes\_City", \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon}_{\text{"Rennes\_City" }}, \epsilon)$

## Understanding 1: Existing approaches

have a way to **enumerate**

$$\begin{array}{lcl}
 \mathcal{S} & ::= & V \mid \text{replace}(\mathcal{S}, \mathcal{S}, \mathcal{S}) \mid \text{concat}(\mathcal{S}, \mathcal{S}) \\
 V & ::= & x \mid \epsilon \mid \text{"Conf"} \mid \text{"City"}
 \end{array}$$

# Bottom-Up Enumeration

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"POPL_Conf"	"POPL"
"Rennes_City"	"Rennes"
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 \mathcal{S} & ::= & V \mid \text{replace}(\mathcal{S}, \mathcal{S}, \mathcal{S}) \mid \text{concat}(\mathcal{S}, \mathcal{S}) \\
 V & ::= & x \mid \epsilon \mid \text{"\_Conf"} \mid \text{"\_City"}
 \end{array}$$

# Bottom-Up Enumeration

$$P_1 = \{x, \epsilon, \text{"\_Conf"}, \text{"\_City"}\}$$

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# Bottom-Up Enumeration

$$P_1 = \{x, \epsilon, \text{"\_Conf"}, \text{"\_City"}\}$$

$$P_2 = \emptyset$$

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 \mathcal{S} & ::= & V \mid \text{replace}(\mathcal{S}, \mathcal{S}, \mathcal{S}) \mid \text{concat}(\mathcal{S}, \mathcal{S}) \\
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## Bottom-Up Enumeration

$$P_1 = \{x, \epsilon, \text{"\_Conf"}, \text{"\_City"}\}$$

$$P_2 = \emptyset$$

$$\begin{aligned}
 P_3 = \{ & x.x, x.\epsilon, x.\text{"\_Conf"}, x.\text{"\_City"}, \epsilon.x, \epsilon.\epsilon, \epsilon.\text{"\_Conf"}, \epsilon.\text{"\_City"}, \\
 & \text{"\_Conf".}x, \text{"\_Conf".}\epsilon, \text{"\_Conf".}\text{"\_Conf"}, \text{"\_City".}\text{"\_City"}, \\
 & \text{"\_Conf".}\text{"\_City"}, \text{"\_City".}x, \text{"\_City".}\epsilon, \text{"\_City".}\text{"\_Conf"} \}
 \end{aligned}$$

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 & \text{"\_Conf".}\text{"\_City"}, \text{"\_City".}x, \text{"\_City".}\epsilon, \text{"\_City".}\text{"\_Conf"} \}
 \end{aligned}$$

$$P_4 = \{r(x, x, \text{"\_City"}), r(x, \text{"\_City"}, \text{"\_Conf"}), r(x, \text{"\_Conf"}, \epsilon), \dots\}$$

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 \end{aligned}$$

$$P_4 = \{r(x, x, \text{"\_City"}), r(x, \text{"\_City"}, \text{"\_Conf"}), r(x, \text{"\_Conf"}, \epsilon), \dots\}$$

$$P_5 = \{\dots\}$$

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 \end{aligned}$$

$$P_4 = \{r(x, x, \text{"\_City"}), r(x, \text{"\_City"}, \text{"\_Conf"}), r(x, \text{"\_Conf"}, \epsilon), \dots\}$$

$$P_5 = \{\dots\} \quad P_6 = \{\dots\}$$

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$$P_4 = \{r(x, x, \text{"\_City"}), r(x, \text{"\_City"}, \text{"\_Conf"}), r(x, \text{"\_Conf"}, \epsilon), \dots\}$$

$$P_5 = \{\dots\} \quad P_6 = \{\dots\} \quad P_7 = \{\dots, r(r(x, \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon), \dots\}$$

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 \end{array}$$

## Bottom-Up Enumeration

$$P_1 = \{x, \epsilon, \text{"\_Conf"}, \text{"\_City"}\}$$

$$P_2 = \emptyset$$

$$\begin{aligned}
 P_3 = \{ & x.x, x.\epsilon, x.\text{"\_Conf"}, x.\text{"\_City"}, \epsilon.x, \epsilon.\epsilon, \epsilon.\text{"\_Conf"}, \epsilon.\text{"\_City"}, \\
 & \text{"\_Conf".}x, \text{"\_Conf".}\epsilon, \text{"\_Conf".}\text{"\_Conf"}, \text{"\_City".}\text{"\_City"}, \\
 & \text{"\_Conf".}\text{"\_City"}, \text{"\_City".}x, \text{"\_City".}\epsilon, \text{"\_City".}\text{"\_Conf"} \}
 \end{aligned}$$

$$P_4 = \{r(x, x, \text{"\_City"}), r(x, \text{"\_City"}, \text{"\_Conf"}), r(x, \text{"\_Conf"}, \epsilon), \dots\}$$

$$P_5 = \{\dots\} \quad P_6 = \{\dots\} \quad P_7 = \{\dots, r(r(x, \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon), \dots\}$$

<i>In</i>	<i>Out</i>
"POPL\_Conf"	"POPL"
"Rennes\_City"	"Rennes"
"PLDI\_Conf"	"PLDI"
"Seoul\_City"	"Seoul"

Exponential Blowup!

$$\begin{array}{lcl}
 \mathcal{S} & ::= & V \mid \text{replace}(\mathcal{S}, \mathcal{S}, \mathcal{S}) \mid \text{concat}(\mathcal{S}, \mathcal{S}) \\
 V & ::= & x \mid \epsilon \mid \text{"\_Conf"} \mid \text{"\_City"}
 \end{array}$$

## Bottom-Up Enumeration

$$P_1 = \{x, \epsilon, \text{"\_Conf"}, \text{"\_City"}\}$$

$$P_2 = \emptyset$$

$$\begin{aligned}
 P_3 = \{ & x.x, x.\epsilon, x.\text{"\_Conf"}, x.\text{"\_City"}, \epsilon.x, \epsilon.\epsilon, \epsilon.\text{"\_Conf"}, \epsilon.\text{"\_City"}, \\
 & \text{"\_Conf".}x, \text{"\_Conf".}\epsilon, \text{"\_Conf".}\text{"\_Conf"}, \text{"\_City".}\text{"\_City"}, \\
 & \text{"\_Conf".}\text{"\_City"}, \text{"\_City".}x, \text{"\_City".}\epsilon, \text{"\_City".}\text{"\_Conf"} \}
 \end{aligned}$$

$$P_4 = \{r(x, x, \text{"\_City"}), r(x, \text{"\_City"}, \text{"\_Conf"}), r(x, \text{"\_Conf"}, \epsilon), \dots\}$$

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<i>In</i>	<i>Out</i>
"POPL\_Conf"	"POPL"
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Exponential Blowup!

Method	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$
No Pruning or Factorization	4	-	16	64	128	1280	4352

# Enumeration, Factorization, Pruning

---

$\mathcal{L}(\mathcal{G})$

$gt$   
•

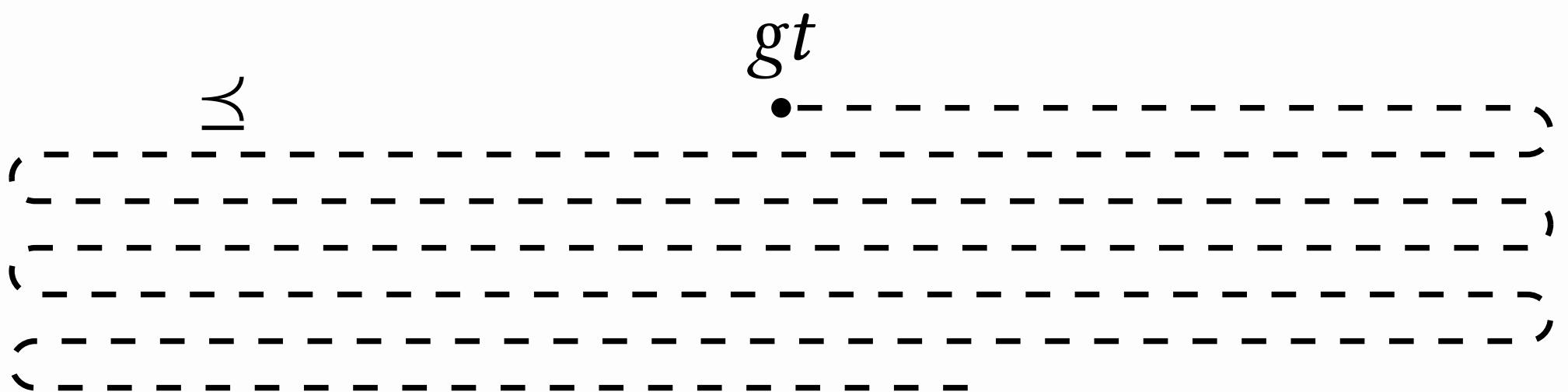
# Enumeration, Factorization, Pruning

---

## Enumeration Order:

Size-based

$$\mathcal{L}(\mathcal{G})$$



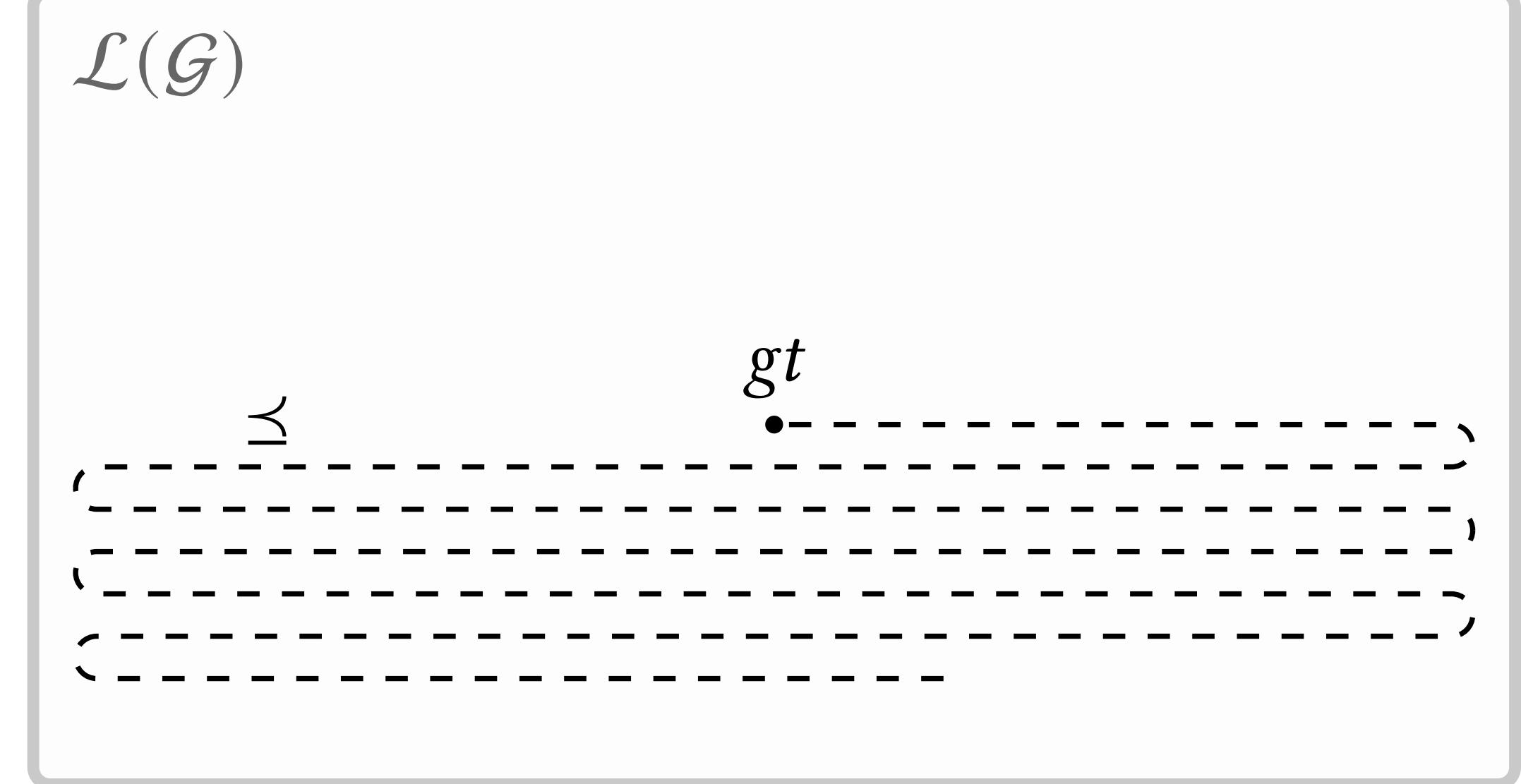
# Enumeration, Factorization, Pruning

---

## Enumeration Order:

Size-based

Deduction [Alur et al. 2017, Lee 2021, Yoon et al. 2023, Ding and Qiu 2024, Ding and Qiu 2025]



# Deduction

---

# Deduction

---

$\epsilon$

# Deduction

---

$\epsilon \in \text{City}$

# Deduction

---

$\epsilon \in \text{"City"}$   $\wedge \dots$

$\dots$

# Deduction

---

$\epsilon \vdash \text{"City"} \wedge \dots$

$\dots \vdash r(x, \text{"Conf"}, \epsilon)$

# Deduction

---

$\epsilon \ \text{Iy} \ \text{"City"} \ \text{Iy} \ \dots$

$\dots \ \text{Iy} \ r(x, \text{"Conf"}, \epsilon) \ \text{Iy} \ \dots$

# Deduction

---

$\epsilon \ \text{I}\!\text{Y} \ \text{"\_City"} \ \text{I}\!\text{Y} \ \dots$

$\dots \ \text{I}\!\text{Y} \ r(x, \text{"\_Conf"}, \epsilon) \ \text{I}\!\text{Y} \ \dots \ \text{I}\!\text{Y} \ r(r(x, \text{"\_Conf"}, \epsilon), \text{"\_City"}, \epsilon)$

# Deduction

---

$\epsilon \vdash "City" \wedge \dots$   
 $\dots \vdash \underline{r(x, "Conf", \epsilon)} \wedge \dots \vdash r(r(x, "Conf", \epsilon), "City", \epsilon)$

# Deduction

€ γ "City" γ ...

$$\dots \preceq r(x, "Conf", \epsilon) \preceq \dots \preceq r(r(x, "Conf", \epsilon), "City", \epsilon)$$

# Deduction

€ γ "City" γ ...

...  $\preceq$   $r(x, "Conf", \epsilon)$   $r(r(x, "Conf", \epsilon), "City", \epsilon)$

# Deduction

€ ↳ "City" ↳ ...

$$\dots \preceq r(x, \text{``Conf''}, \epsilon) \preceq r(r(x, \text{``Conf''}, \epsilon), \text{``City''}, \epsilon)$$

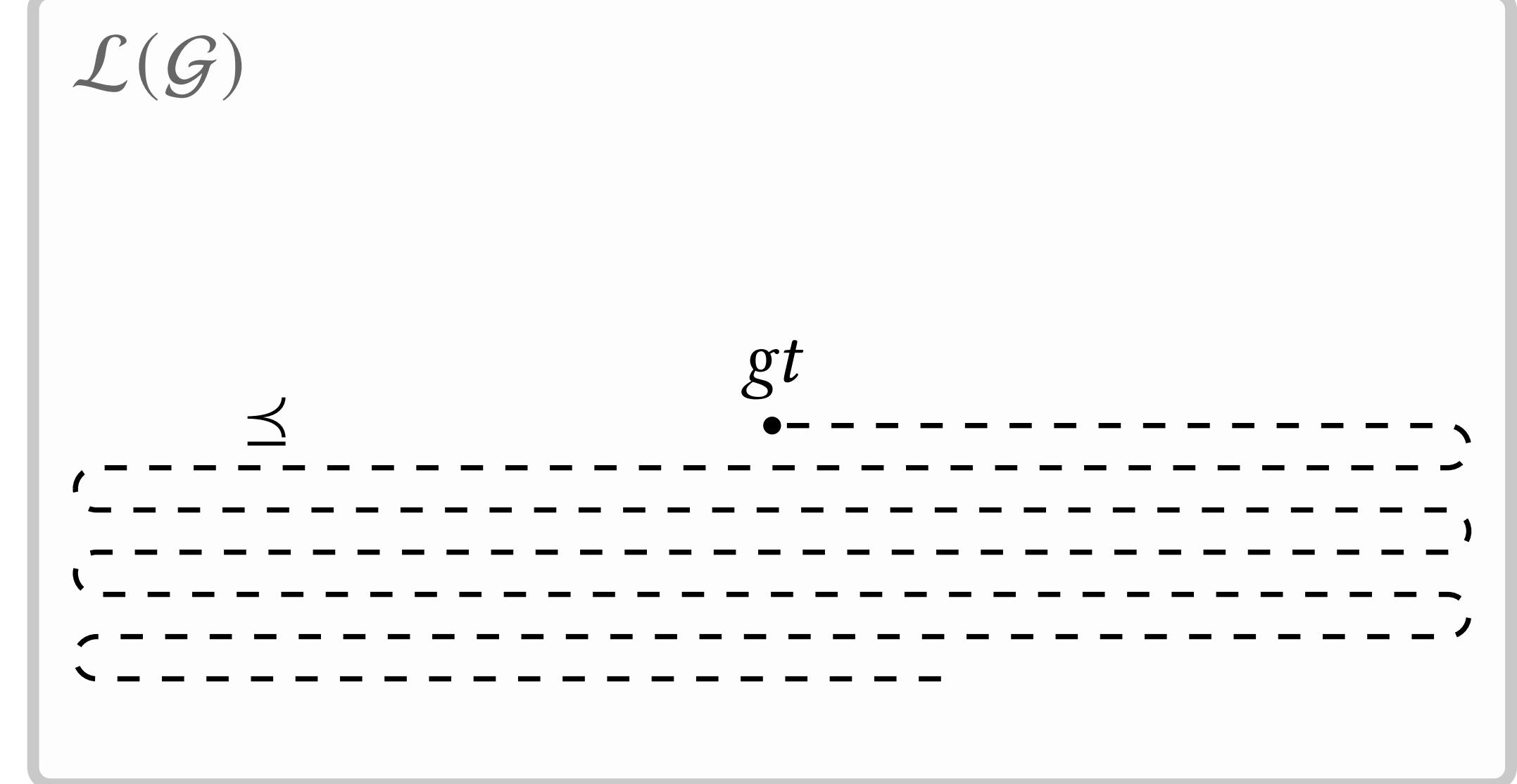
# Enumeration, Factorization, Pruning

---

## Enumeration Order:

Size-based

Deduction [Alur et al. 2017, Lee 2021, Yoon et al. 2023, Ding and Qiu 2024, Ding and Qiu 2025]



## Understanding 2: Existing approaches

have a way to **factorize** the search space

# Enumeration, Factorization, Pruning

# Enumeration Order:

## Size-based

Deduction [Alur et al. 2017, Lee 2021, Yoon et al. 2023, Ding and Qiu 2024, Ding and Qiu 2025]

# Factorizations:

# Observational Equivalence Factorization [Udapa et al. 2013, Albarghouthi et al. 2013]

$$\mathcal{L}(G)$$

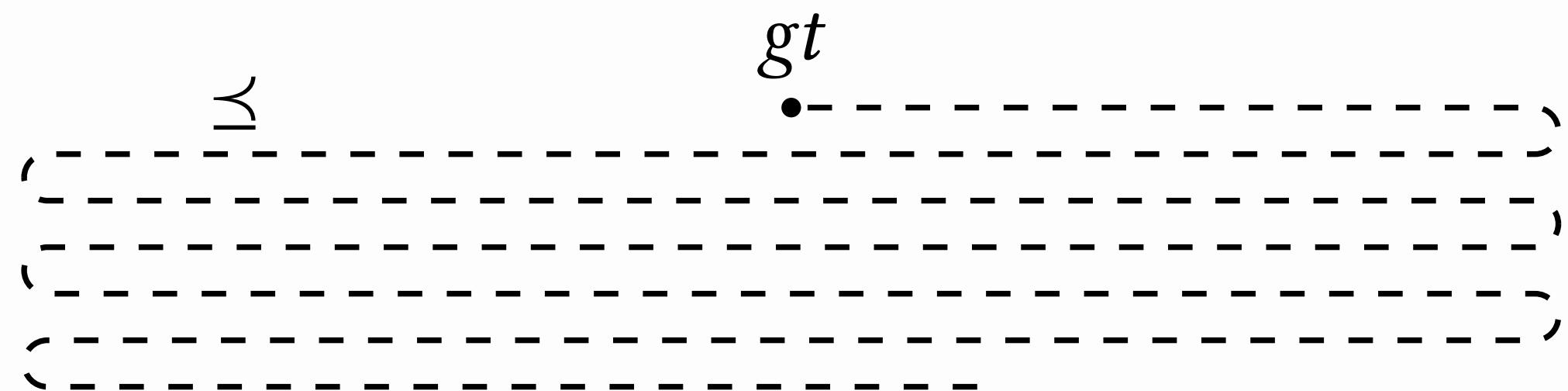
# Observational Equivalence Factorization

Observational Equivalence:

$$\forall i \in In : \text{prog}_1(i) = \text{prog}_2(i)$$

$$x \approx x . \epsilon$$

$$\mathcal{L}(\mathcal{G})$$



# Observational Equivalence Factorization

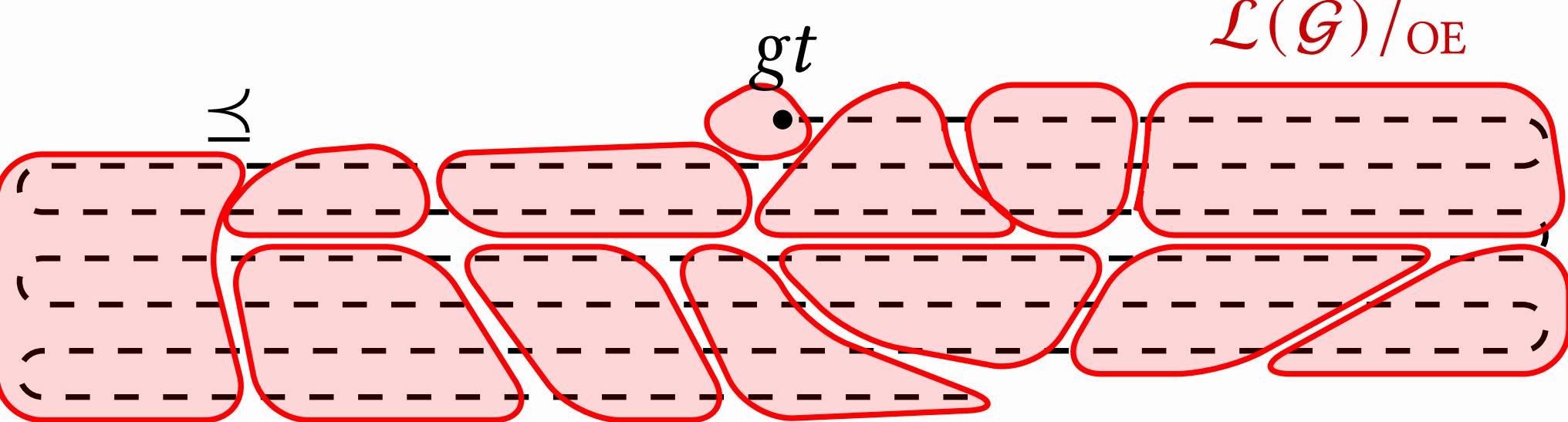
Observational Equivalence:

$$\forall i \in In : \text{prog}_1(i) = \text{prog}_2(i)$$

$$\mathbf{x} \approx \mathbf{x} . \epsilon$$

Factorizes search space

$$\mathcal{L}(\mathcal{G})$$



# Observational Equivalence Factorization

Observational Equivalence:

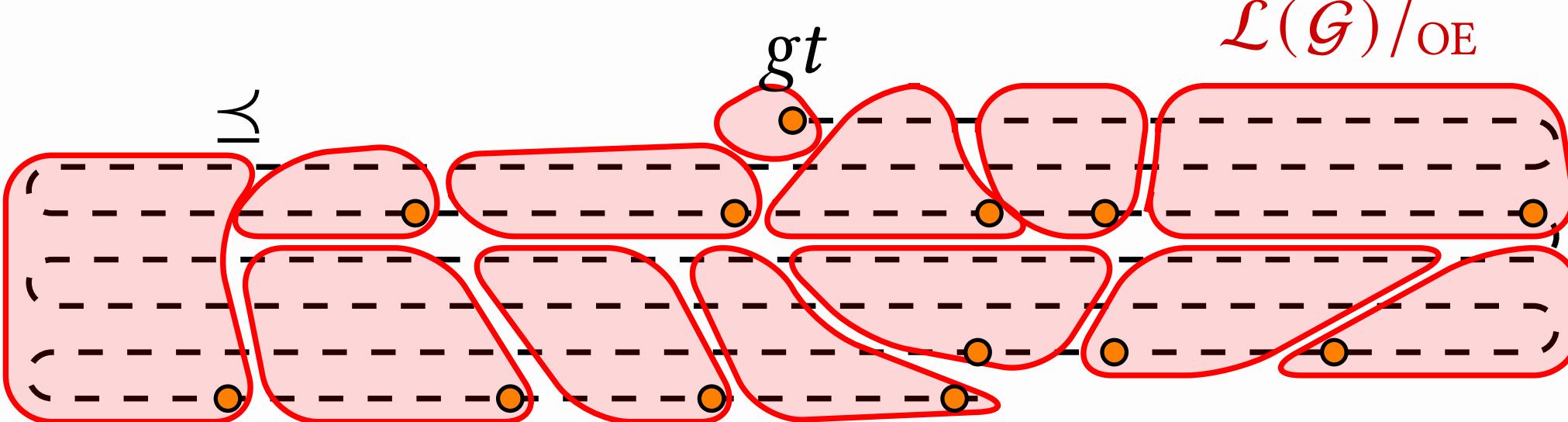
$$\forall i \in In : \text{prog}_1(i) = \text{prog}_2(i)$$

$$\mathbf{x} \approx \mathbf{x} \cdot \epsilon$$

Factorizes search space

Only keep one representative per class

$$\mathcal{L}(\mathcal{G})$$



# Observational Equivalence Factorization

Observational Equivalence:

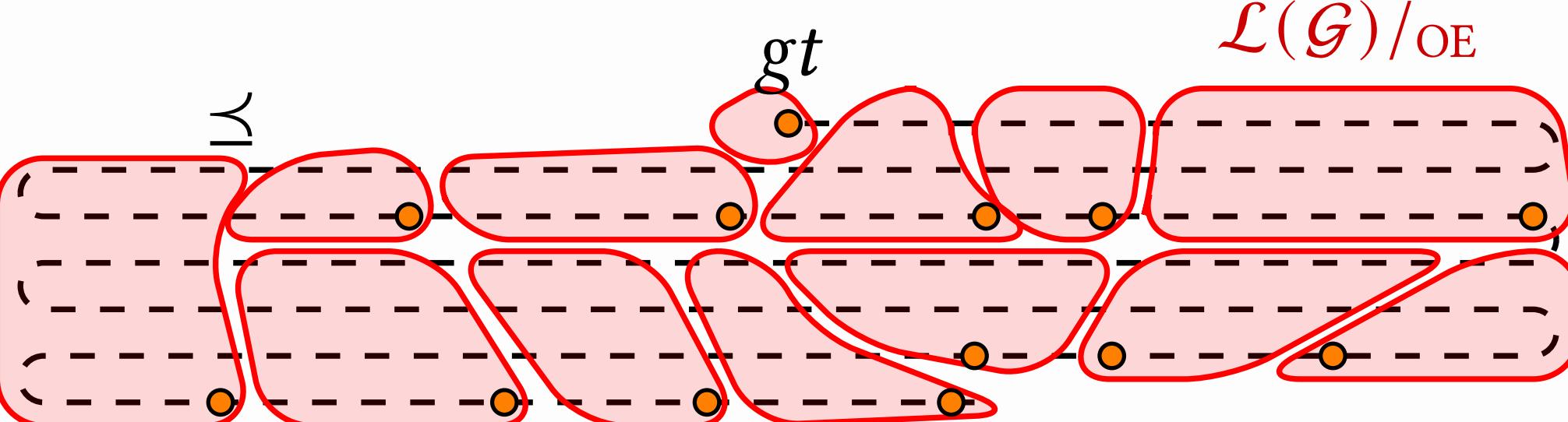
$$\forall i \in In : \text{prog}_1(i) = \text{prog}_2(i)$$

$x \approx \cancel{x} \epsilon$

Factorizes search space

Only keep one representative per class

$$\mathcal{L}(\mathcal{G})$$



# Observational Equivalence Factorization

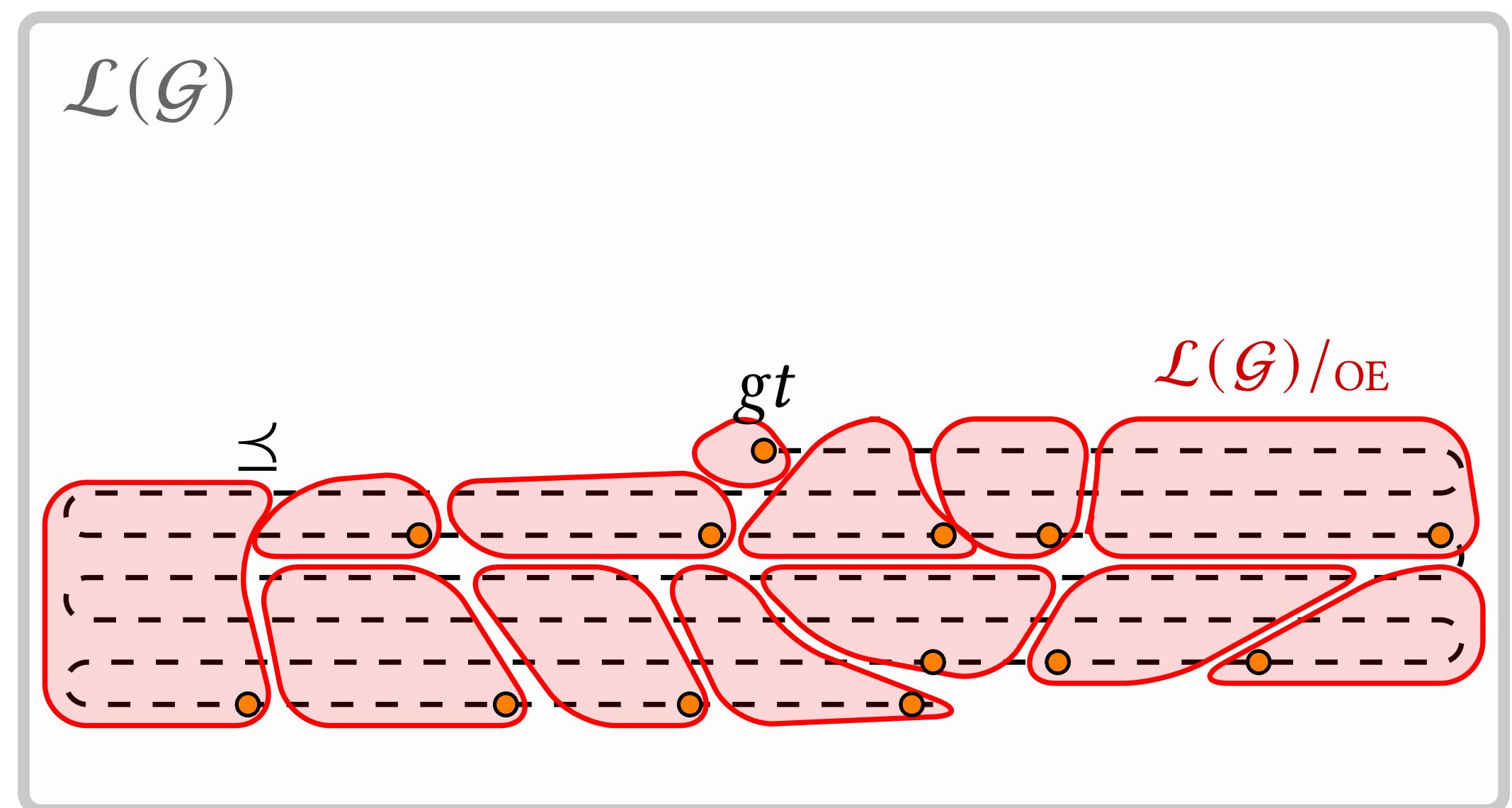
Observational Equivalence:

$$\forall i \in In : \text{prog}_1(i) = \text{prog}_2(i)$$

~~$x \approx x \epsilon$~~

Factorizes search space

Only keep one representative per class



Method	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$
No Pruning or Factorization	4	-	16	64	128	1280	4352
OE Factorization	4	-	9	6	27	56	119

# Enumeration, Factorization, Pruning

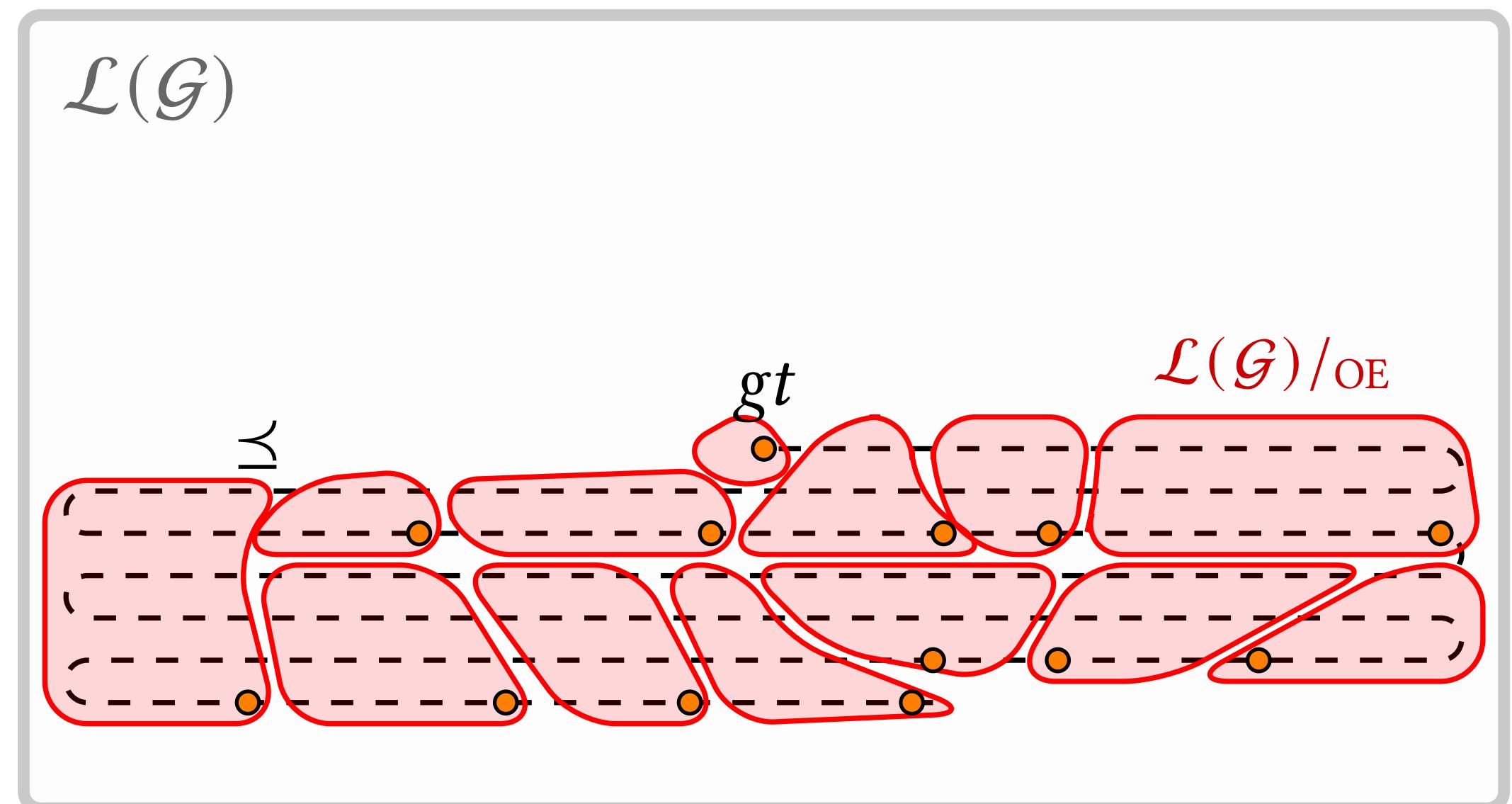
## Enumeration Order:

Size-based

Deduction [Alur et al. 2017, Lee 2021, Yoon et al. 2023, Ding and Qiu 2024, Ding and Qiu 2025]

## Factorizations:

Observational Equivalence Factorization [Udapa et al. 2013, Albarghouthi et al. 2013]



# Enumeration, Factorization, Pruning

## Enumeration Order:

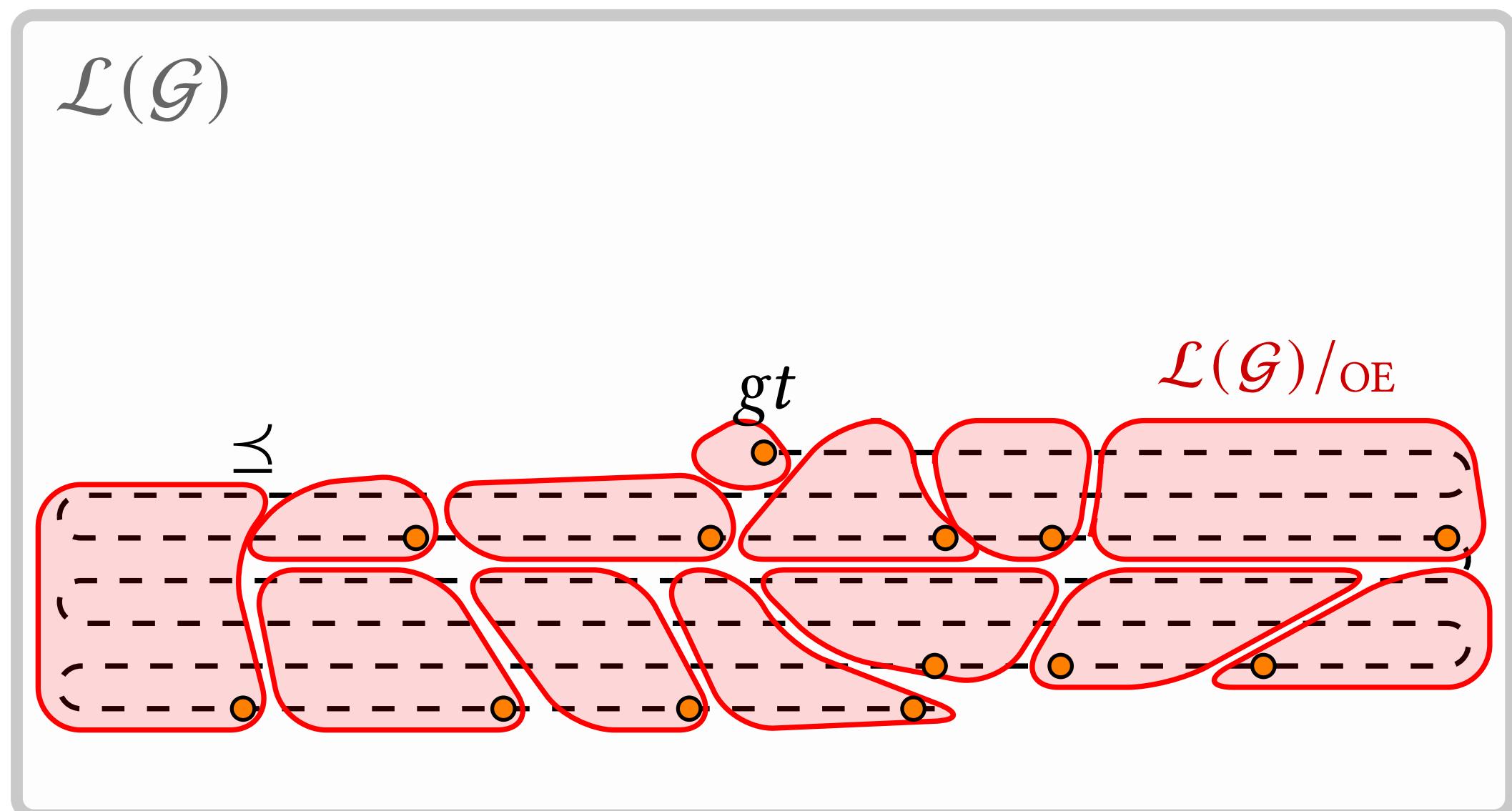
Size-based

Deduction [Alur et al. 2017, Lee 2021, Yoon et al. 2023, Ding and Qiu 2024, Ding and Qiu 2025]

## Factorizations:

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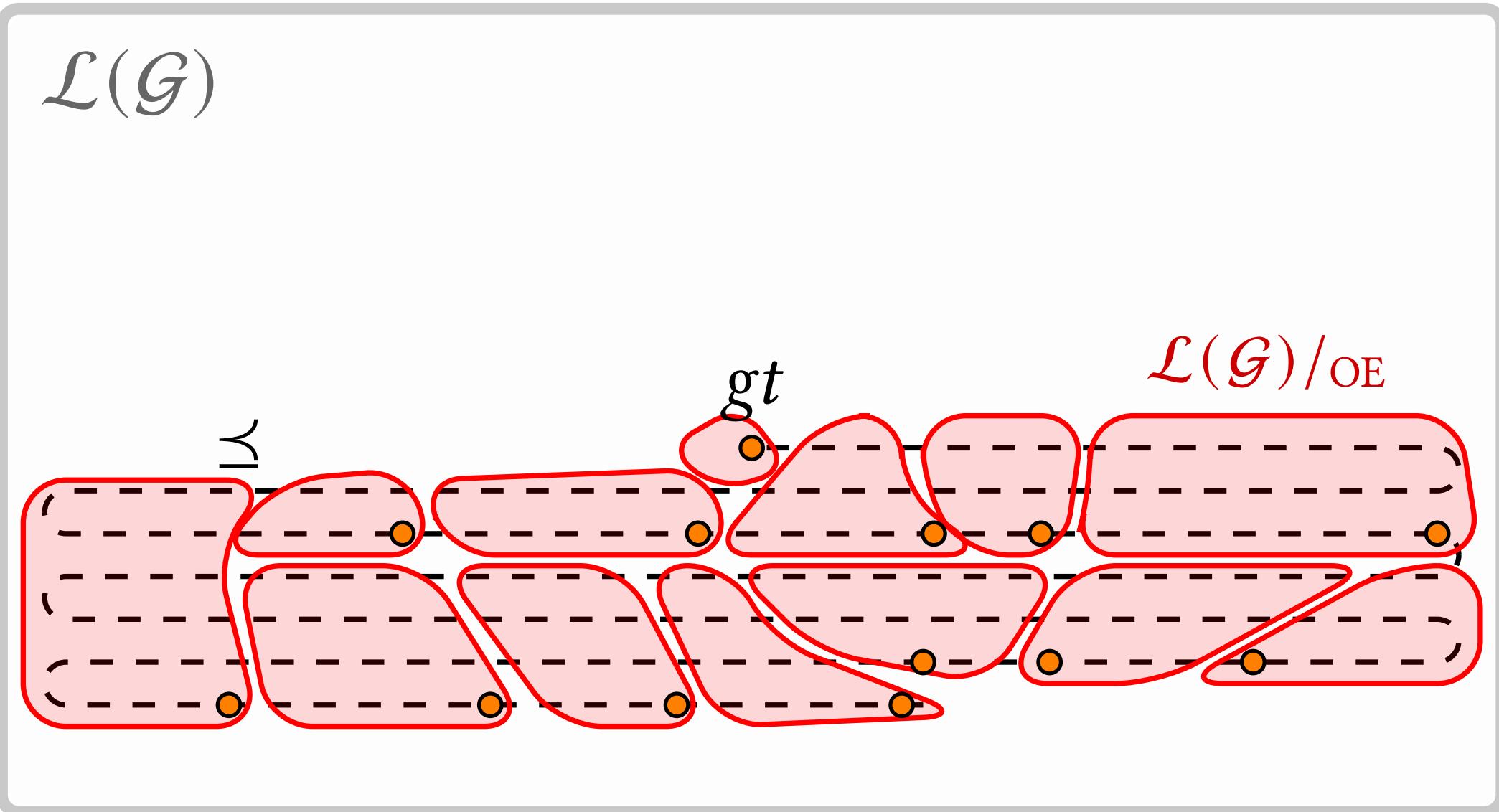
Abstraction [Wang et al. 2018]



# Abstraction

Perform OE on abstracted values

$$\forall i \in In : \alpha(\text{prog}_1(i)) = \alpha(\text{prog}_2(i))$$



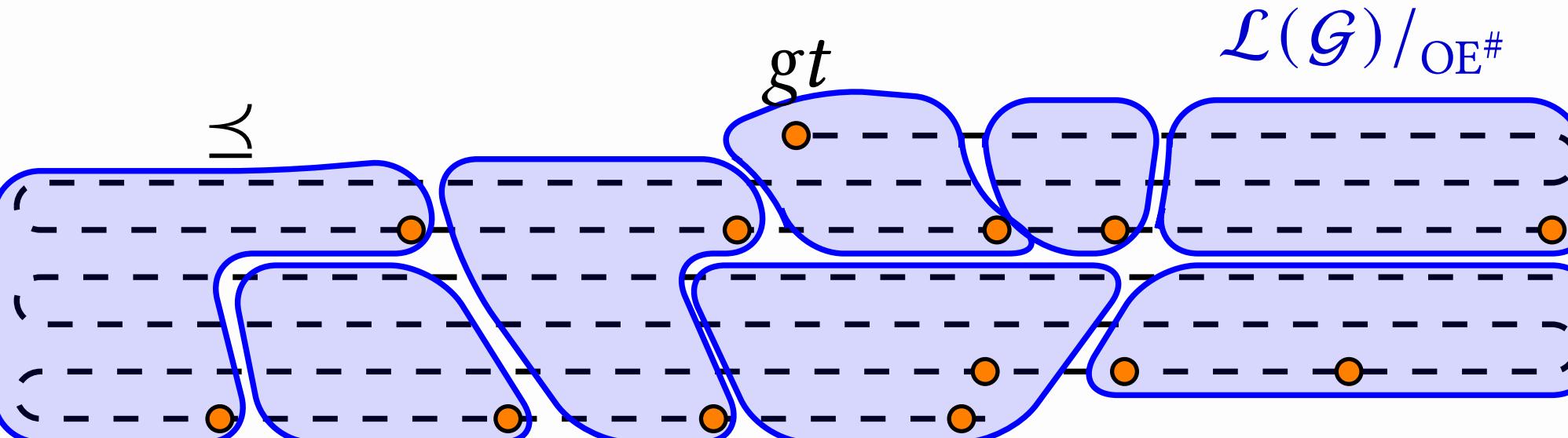
# Abstraction

Perform OE on abstracted values

$$\forall i \in In : \alpha(\text{prog}_1(i)) = \alpha(\text{prog}_2(i))$$

Coarser than OE

$$\mathcal{L}(\mathcal{G})$$



# Abstraction

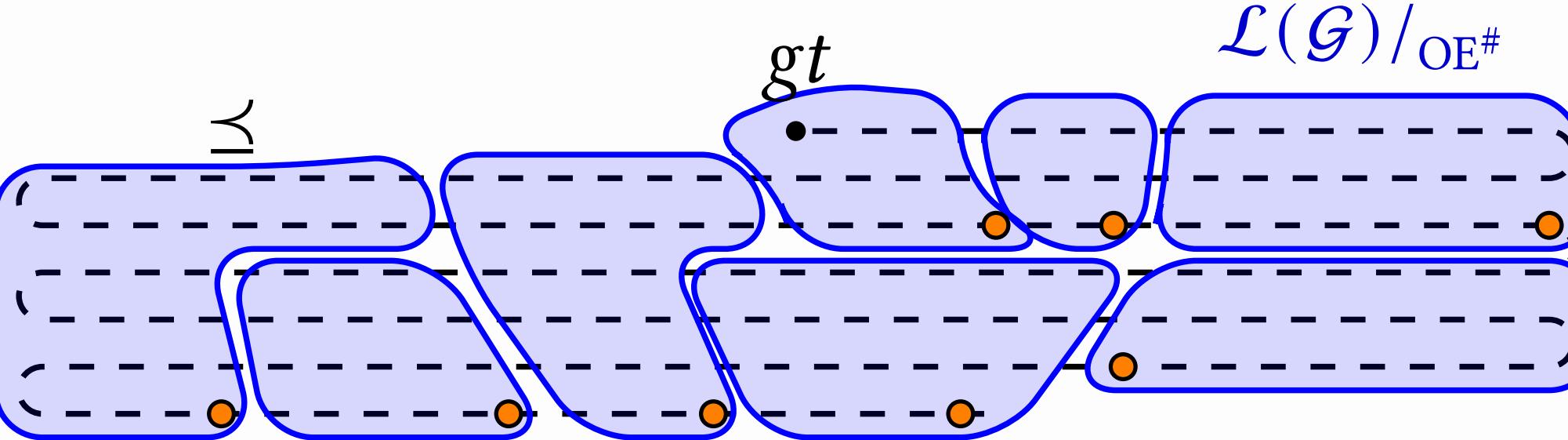
Perform OE on abstracted values

$$\forall i \in In : \alpha(\text{prog}_1(i)) = \alpha(\text{prog}_2(i))$$

Coarser than OE

Only keep one representative per class

$$\mathcal{L}(\mathcal{G})$$



# Enumeration, Factorization, Pruning

## Enumeration Order:

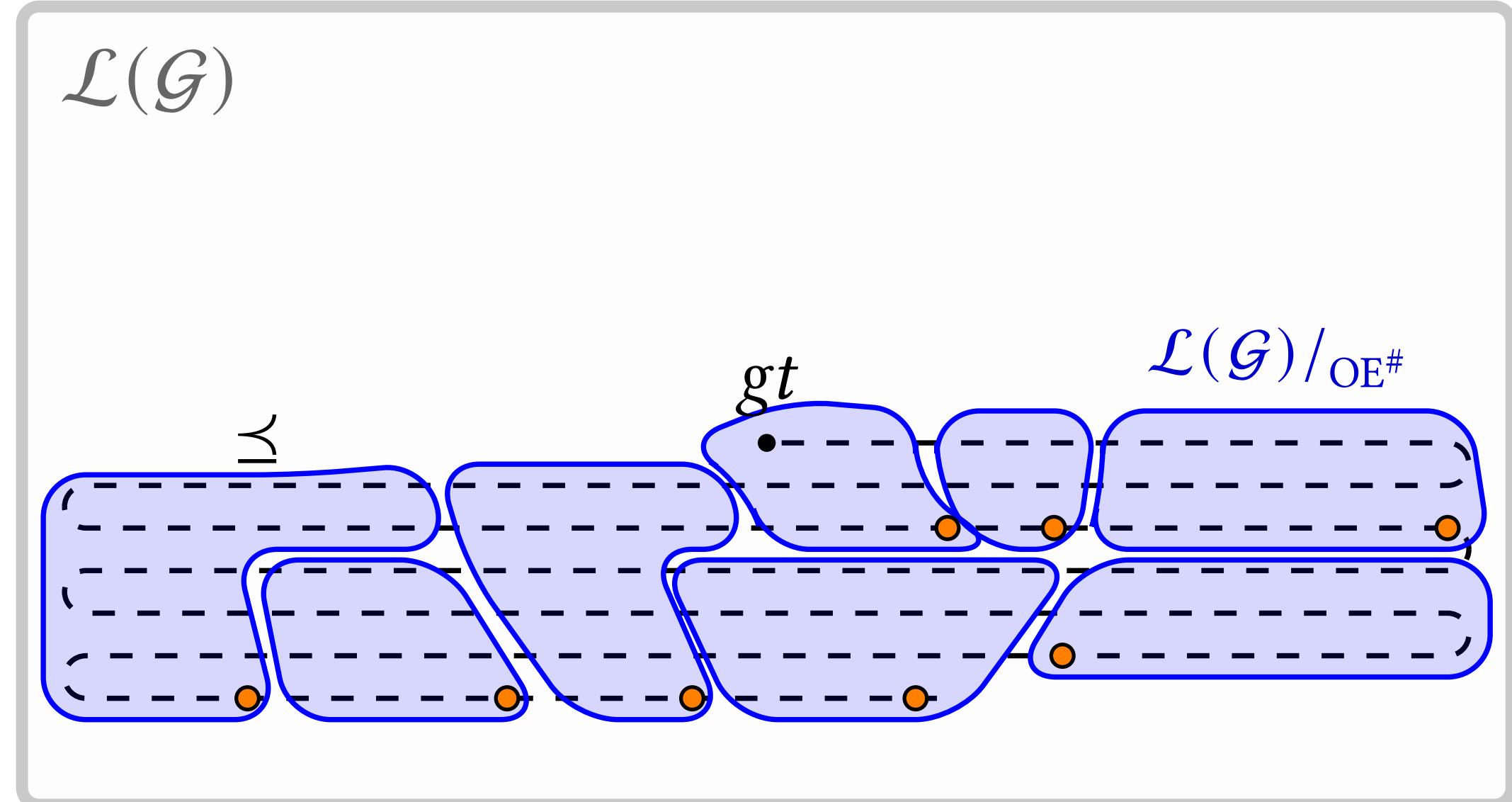
Size-based

Deduction [Alur et al. 2017, Lee 2021, Yoon et al. 2023, Ding and Qiu 2024, Ding and Qiu 2025]

## Factorizations:

Observational Equivalence Factorization [Udapa et al. 2013, Albarghouthi et al. 2013]

Abstraction [Wang et al. 2018]



## Understanding 3: Existing approaches

have a way to **prune** the search space

# Enumeration, Factorization, Pruning

## Enumeration Order:

Size-based

Deduction [Alur et al. 2017, Lee 2021, Yoon et al. 2023, Ding and Qiu 2024, Ding and Qiu 2025]

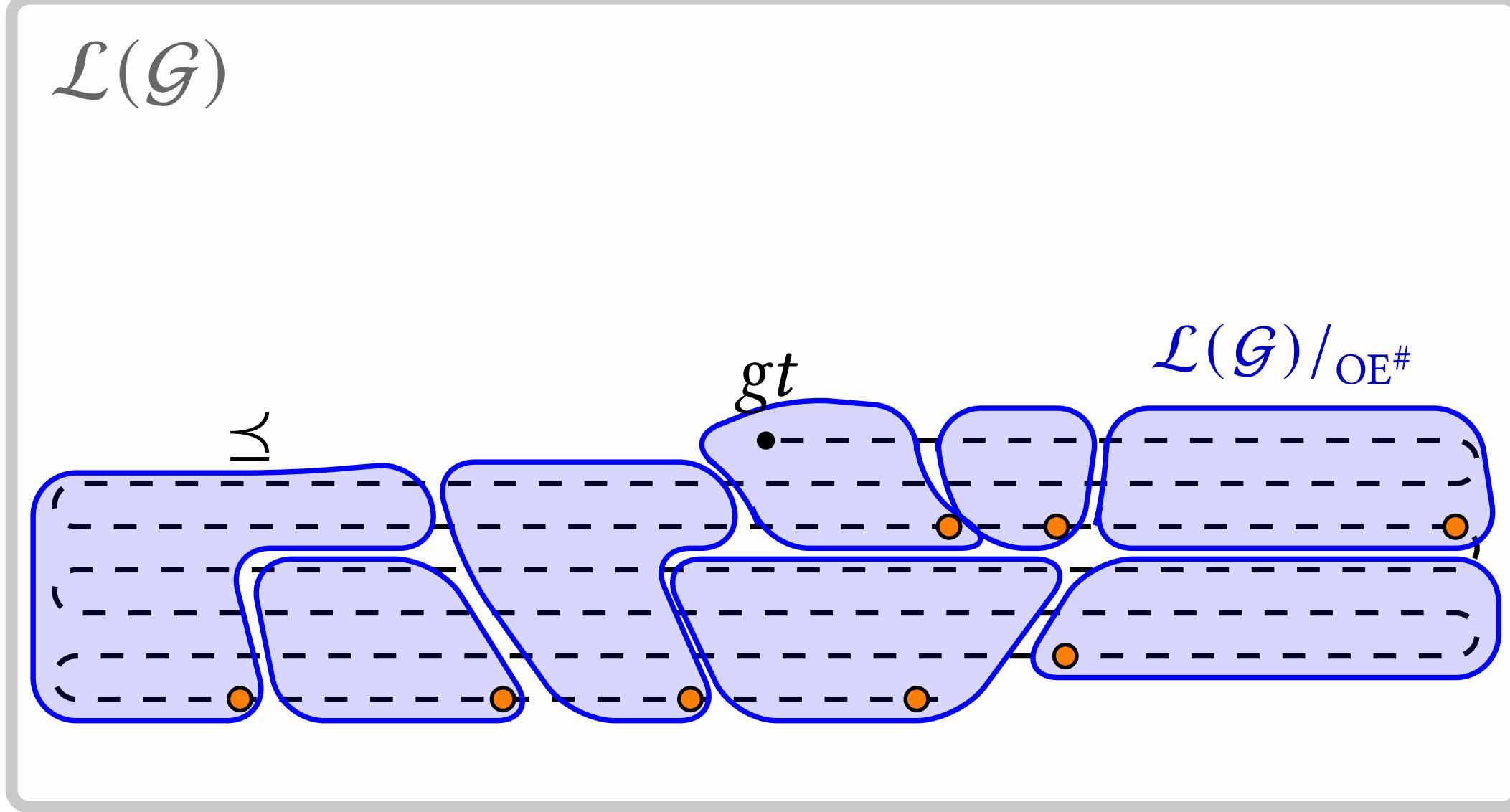
## Factorizations:

Observational Equivalence Factorization [Udapa et al. 2013, Albarghouthi et al. 2013]

Abstraction [Wang et al. 2018]

## Pruning:

Pruning with a ball [Feser et al. 2023]

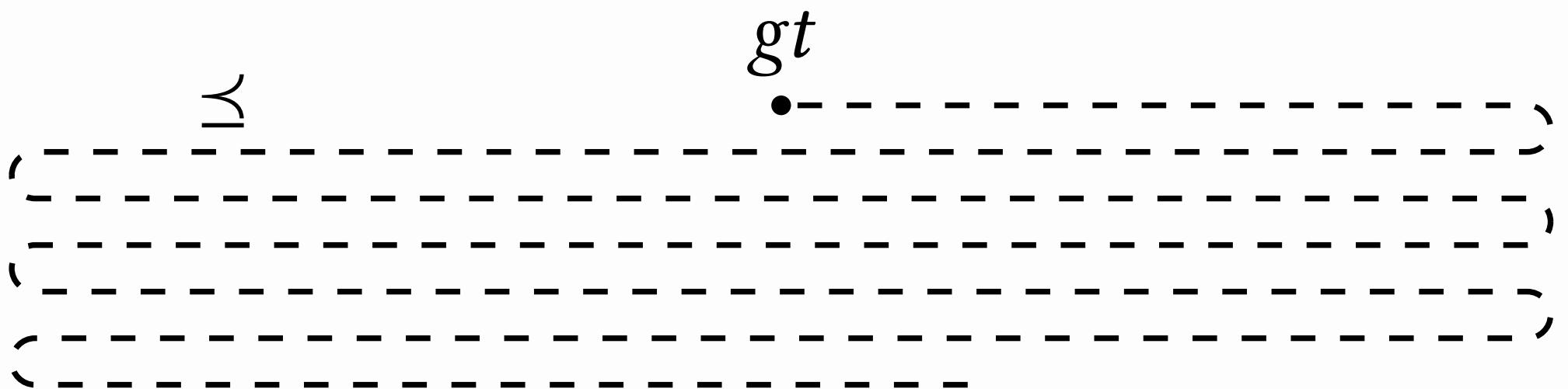


# Pruning with a Ball

---

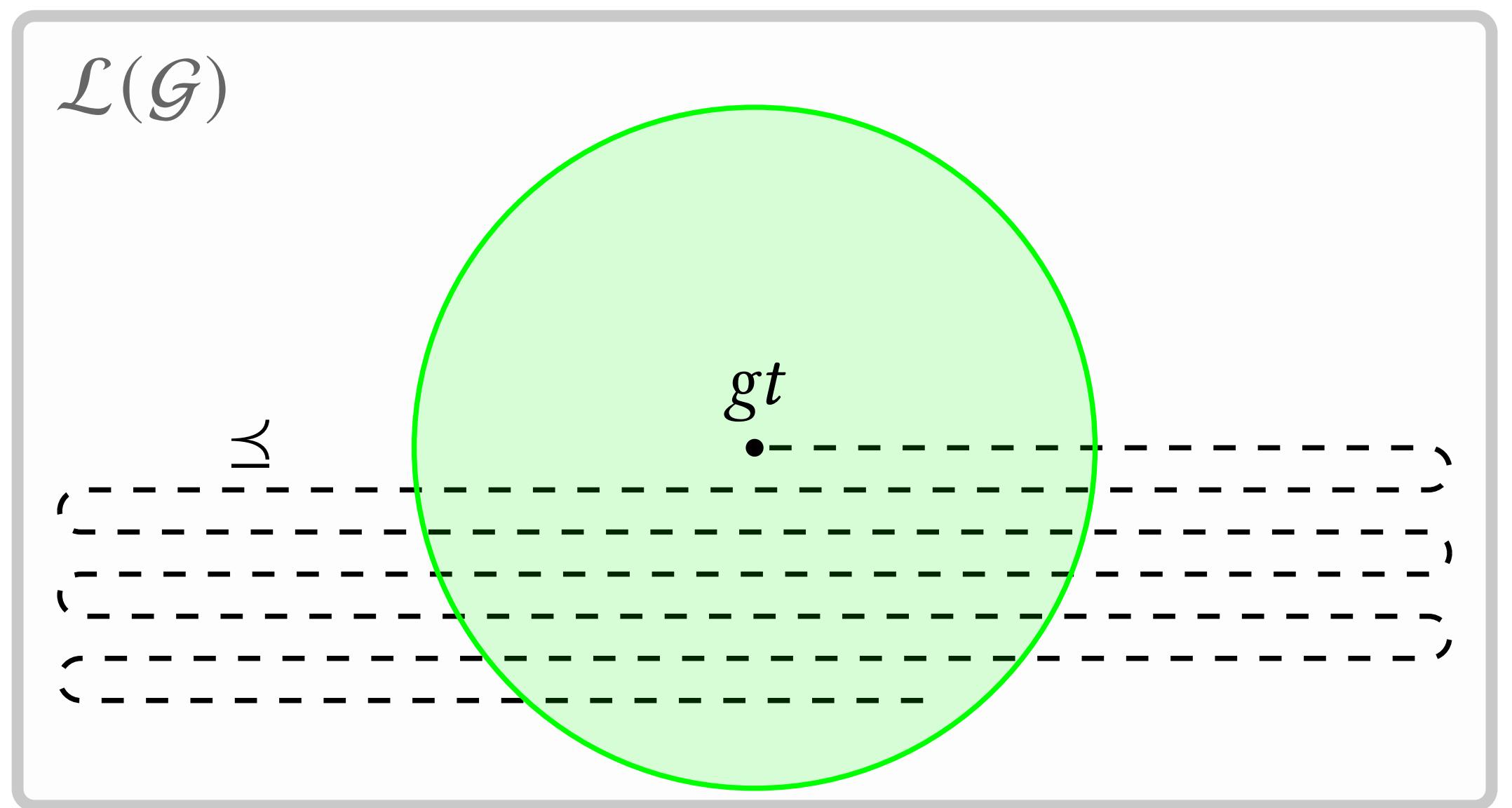
Use a metric to define a ball around  $gt$

$$\mathcal{L}(\mathcal{G})$$



# Pruning with a Ball

Use a metric to define a ball around  $gt$

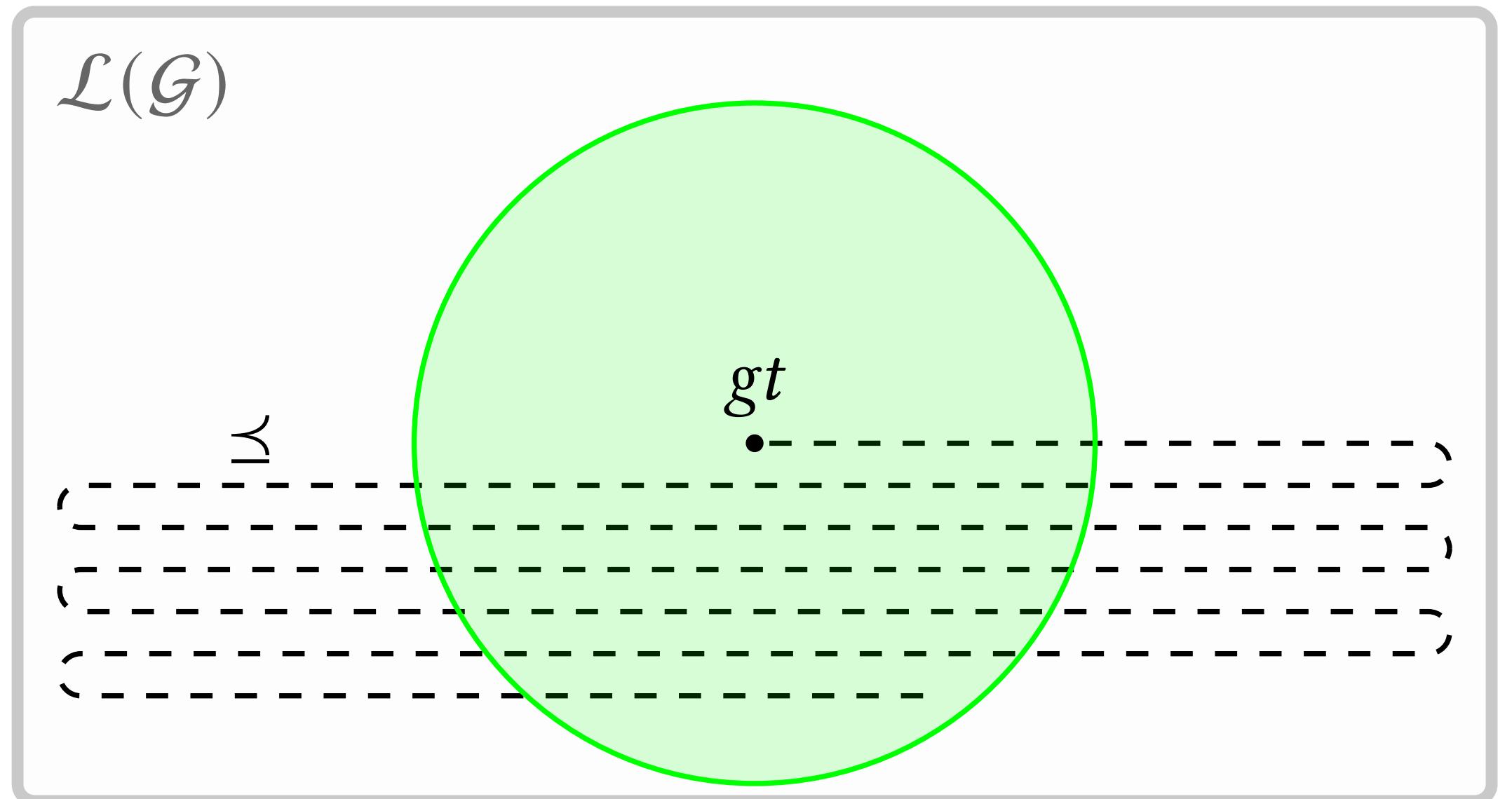


# Pruning with a Ball

---

Use a metric to define a ball around  $gt$

Only consider programs inside the ball



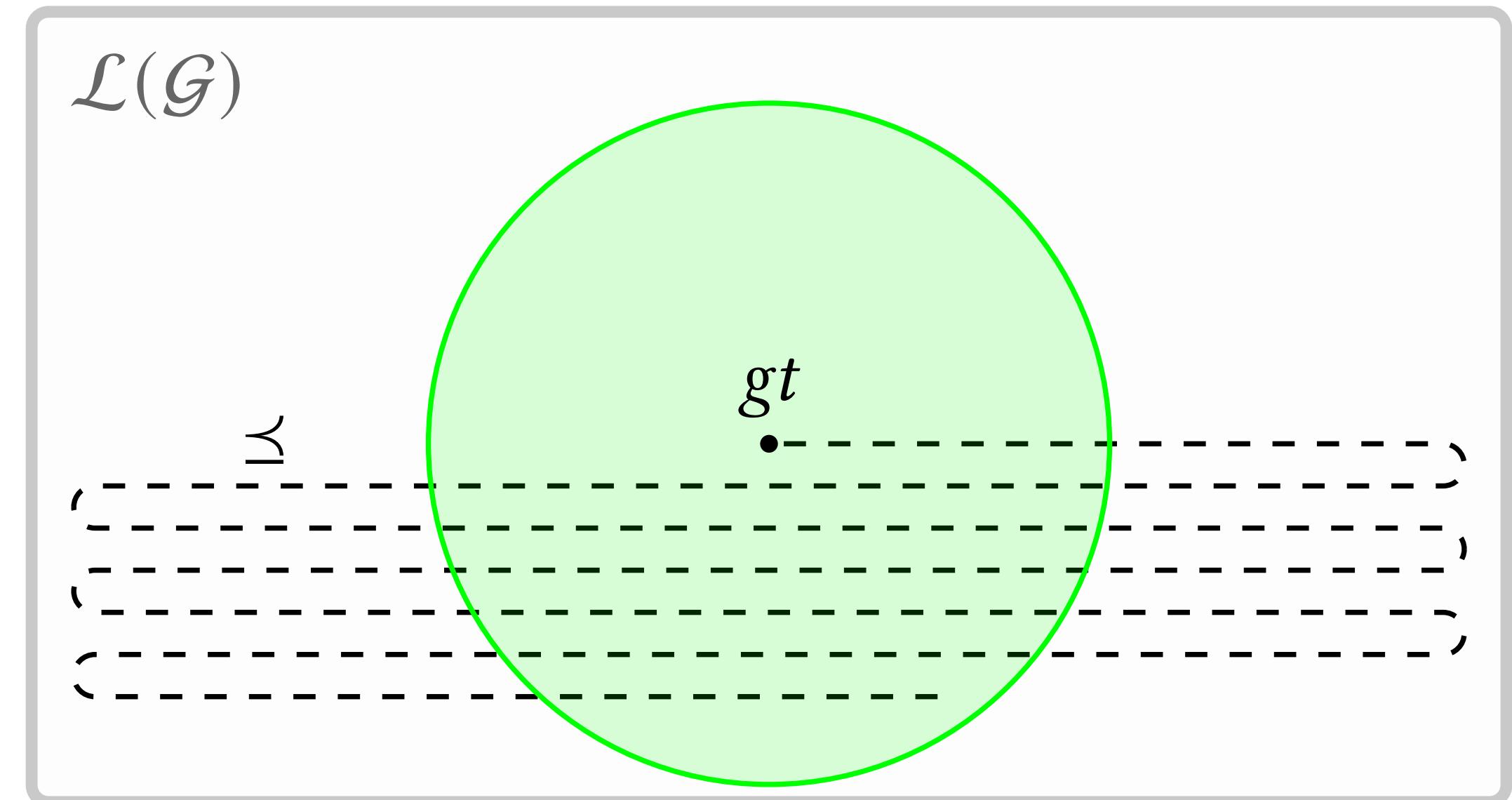
# Pruning with a Ball

---

Use a metric to define a ball around  $gt$

Only consider programs inside the ball

Deliberately incomplete



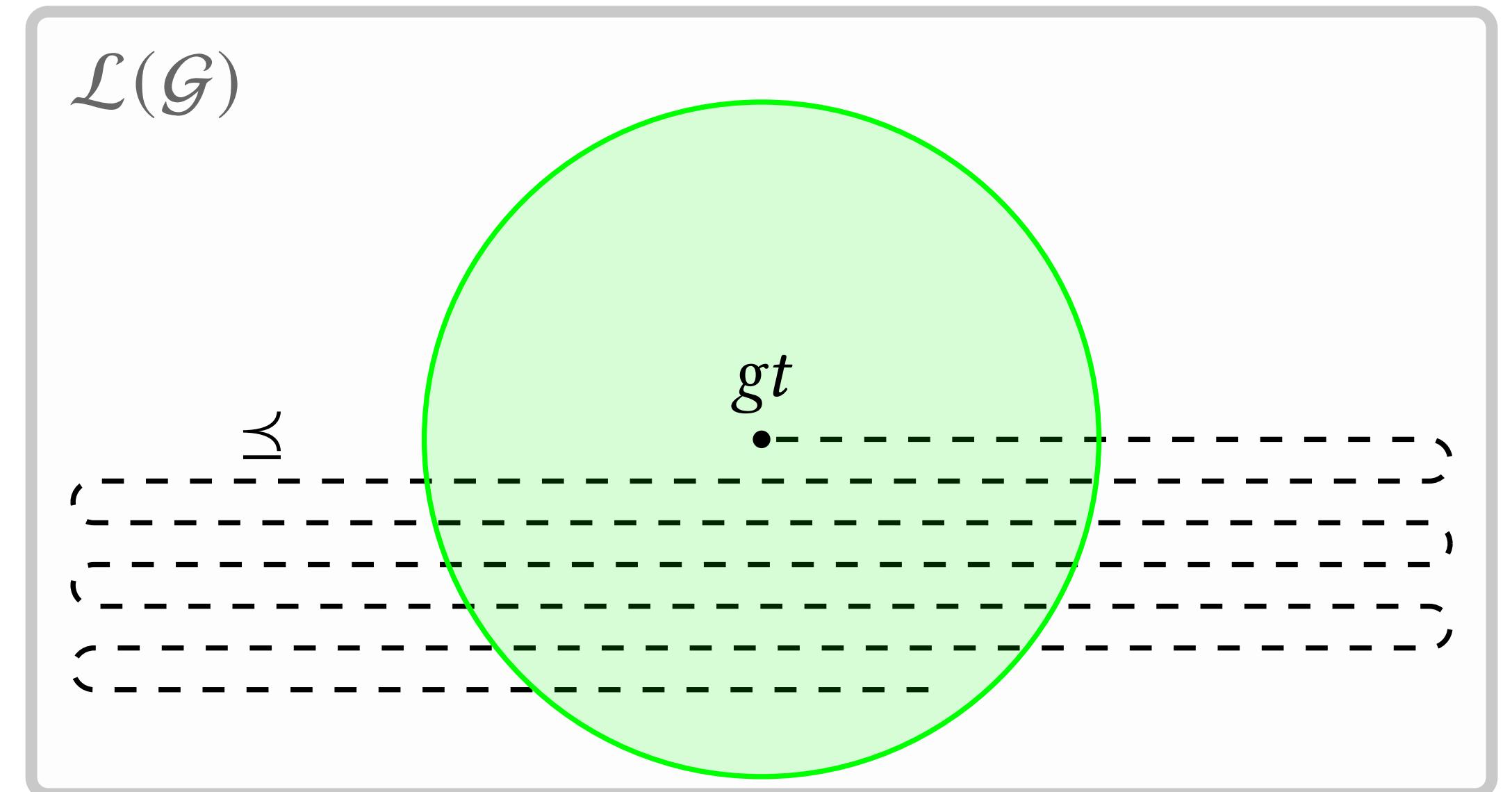
# Pruning with a Ball

Use a metric to define a ball around  $gt$

Only consider programs inside the ball

Deliberately incomplete

Control completeness / speed-up via radius



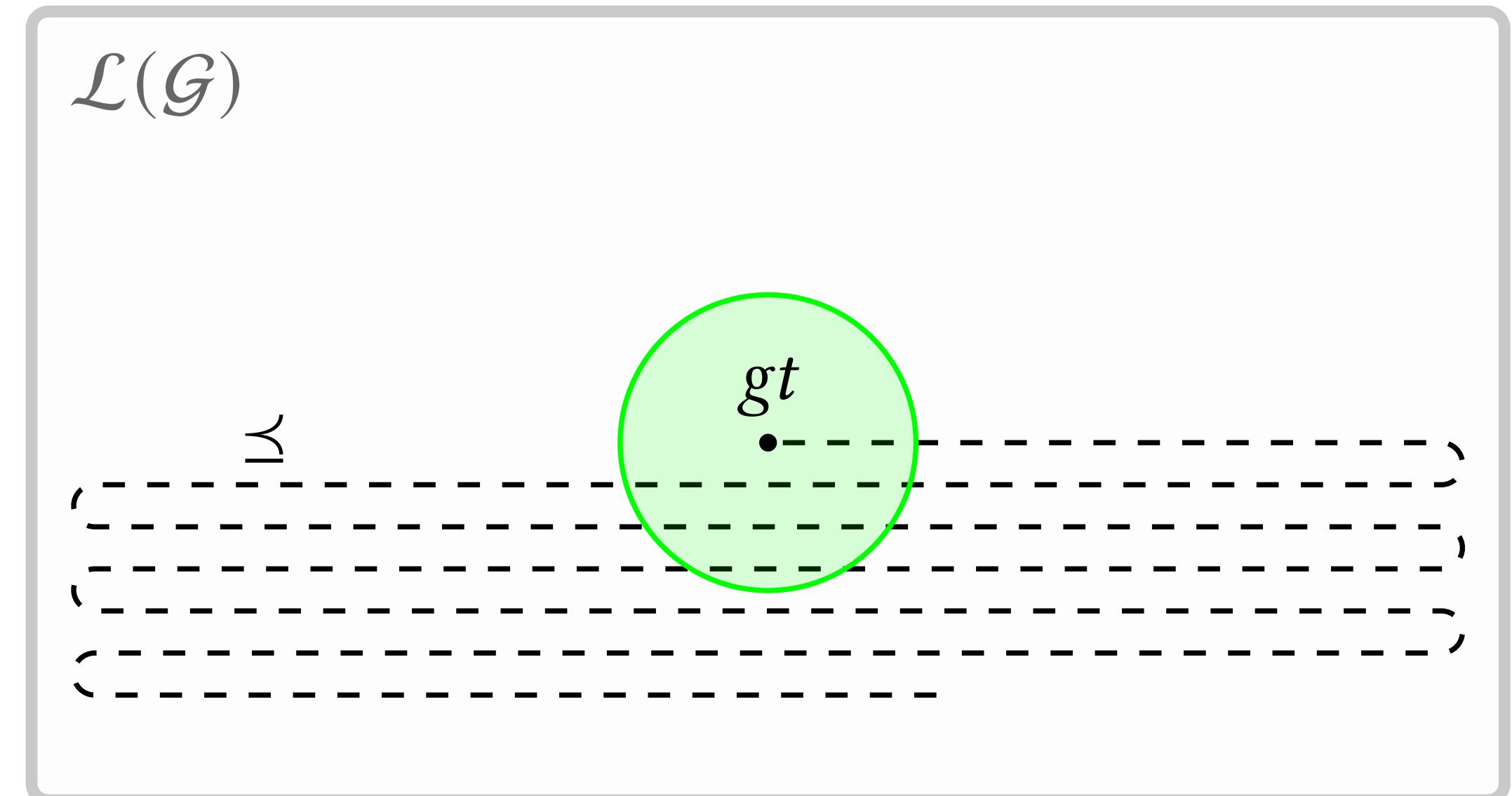
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Use a metric to define a ball around  $gt$

Only consider programs inside the ball

Deliberately incomplete

Control completeness / speed-up via radius



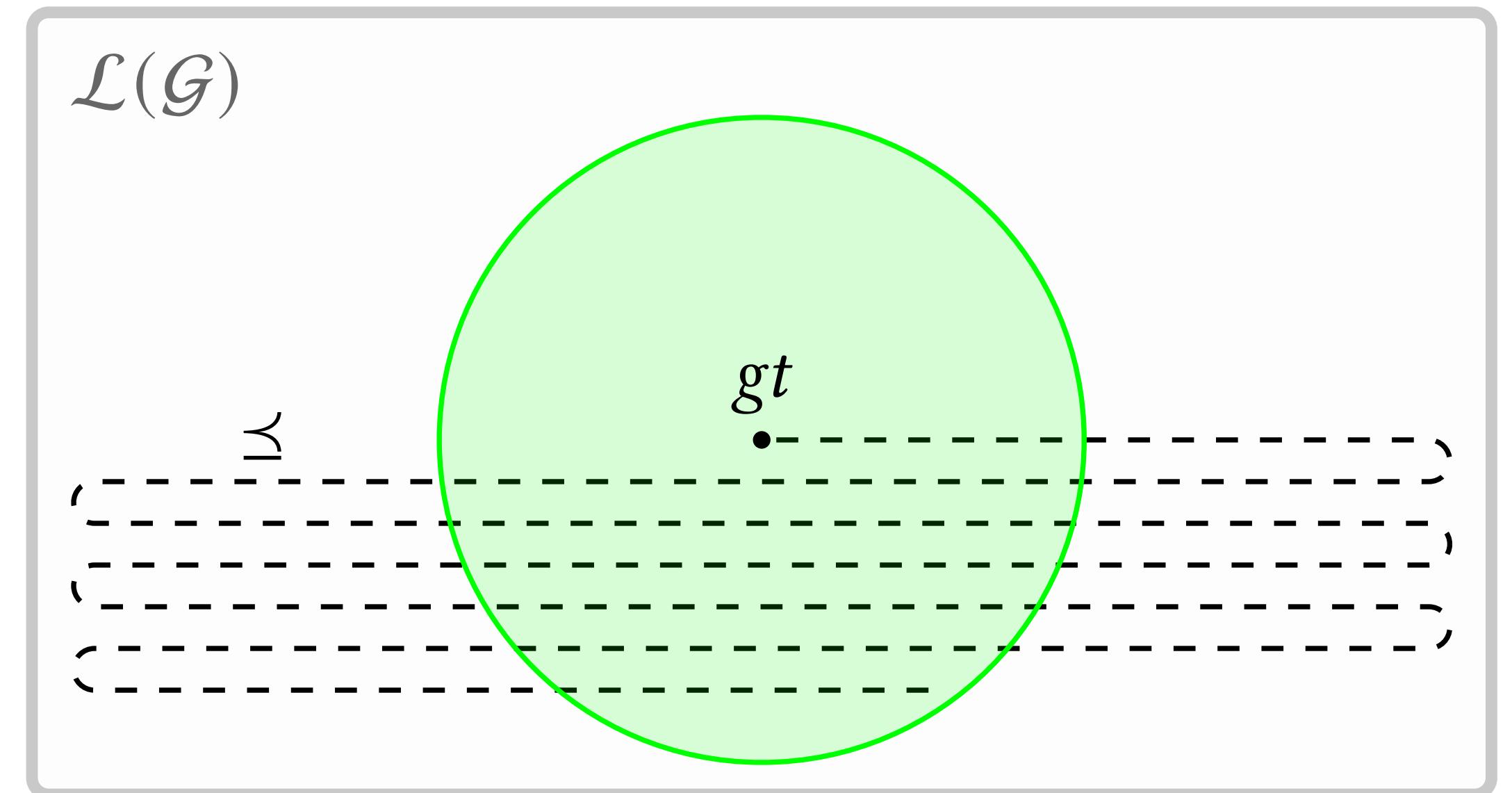
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Use a metric to define a ball around  $gt$

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# Pruning with a Ball

Use a metric to define a ball around  $gt$

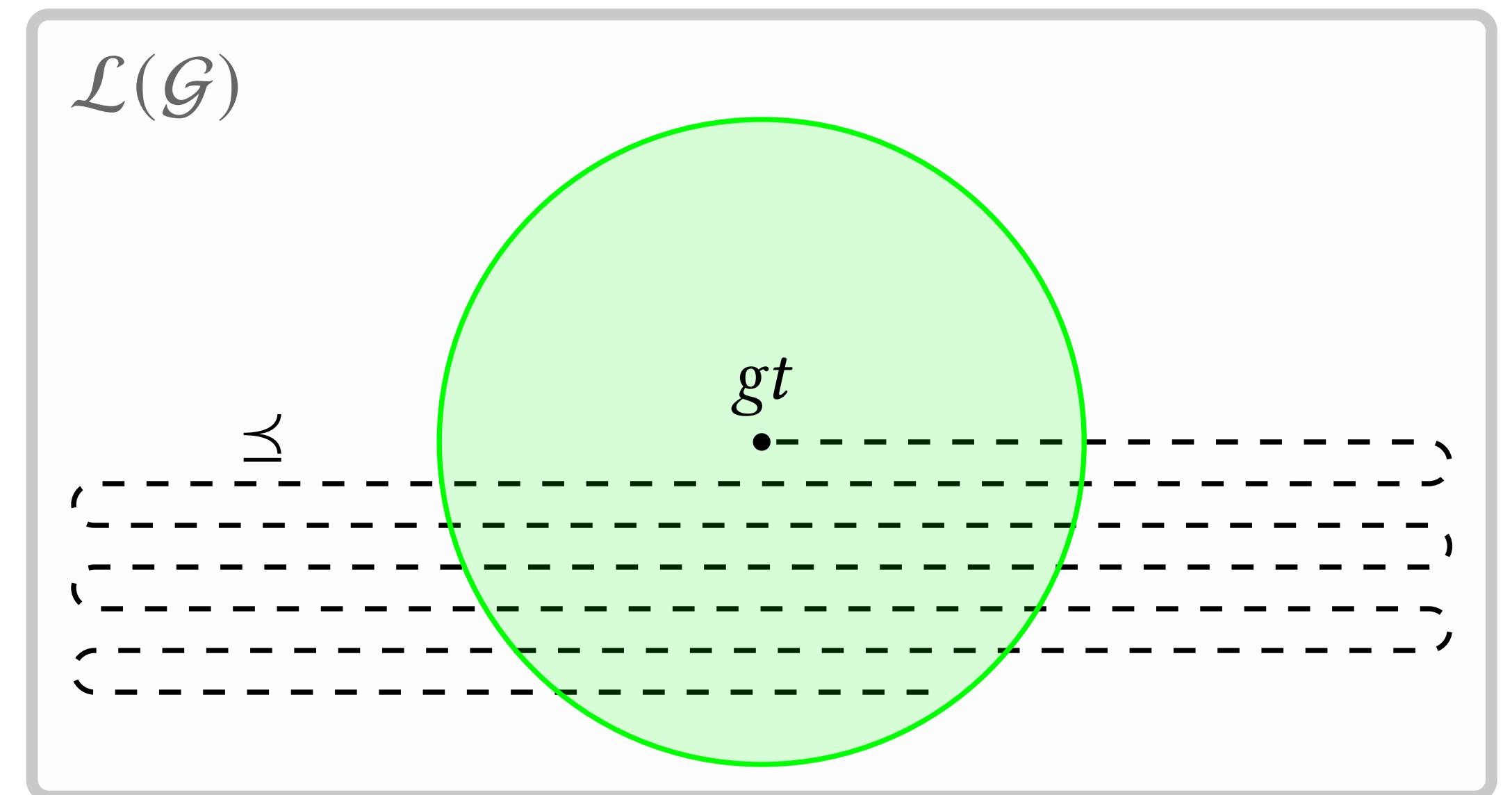
Only consider programs inside the ball

Deliberately incomplete

Control completeness / speed-up via radius

Downside:

Require symmetry



# Existing approaches

# Existing approaches

have a way to **enumerate**

# Existing approaches

have a way to **enumerate**

have a way to **factorize**

# Existing approaches

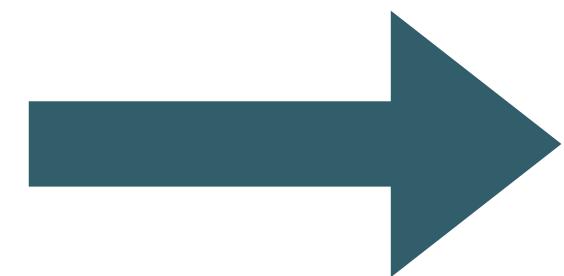
have a way to **enumerate**

have a way to **factorize**

have a way to **prune**  
symmetric (undesirable)

# Existing approaches

have a way to **enumerate**



Enumeration Order  $\preceq$

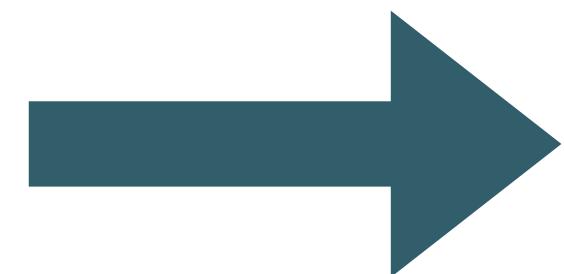
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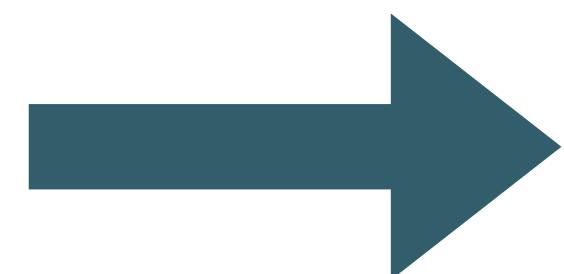
# Existing approaches

have a way to **enumerate**



Enumeration Order  $\preceq$

have a way to **factorize**

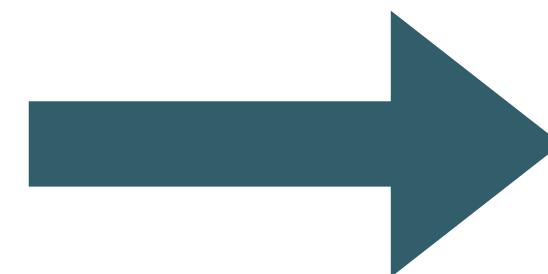


Equivalence  $\equiv$

have a way to **prune**  
symmetric (undesirable)

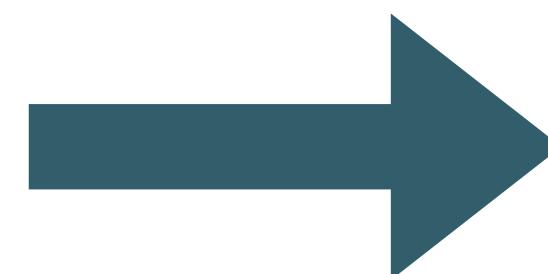
# Existing approaches

have a way to **enumerate**



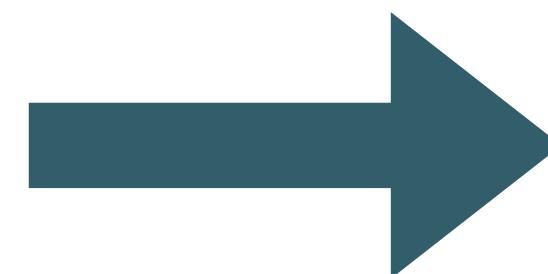
Enumeration Order  $\preceq$

have a way to **factorize**



Equivalence  $\equiv$

have a way to **prune**  
symmetric (undesirable)



Metric

## 3 Parameters of Bottom-Up Enumerative Synthesizers:

Enumeration Order  $\preceq$

Equivalence  $\equiv$

Metric

## 3 Parameters of Bottom-Up Enumerative Synthesizers:

Enumeration Order  $\preceq$

Equivalence  $\equiv$

Metric

**INSIGHT:**

Oriented Metric

## **2** Parameters of Bottom-Up Enumerative Synthesizers:

Enumeration Order  $\preceq$

~~Equivalence  $\equiv$  Metric~~

**INSIGHT:**

Oriented Metric

# Oriented Metrics (Orimetrics)

---

$$m : D \times D \rightarrow \mathbb{R}_{\geq 0}$$

$$m(a, a) = 0 \quad \text{(reflexivity)}$$

$$m(b, a) = 0 \Rightarrow m(a, b) = 0 \quad \text{(symmetry at zero)}$$

$$m(a, c) \leq m(a, b) + m(b, c) \quad \text{( $\Delta$ -inequality)}$$

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Allows for **asymmetry**

# Oriented Metrics (Orimetrics)

---

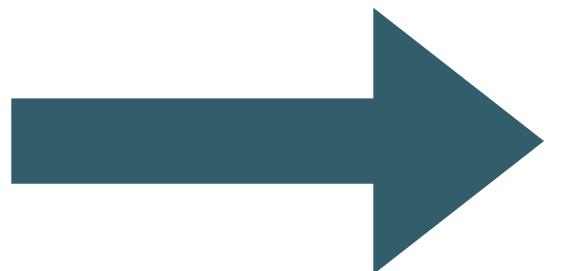
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Allows for **asymmetry**



Better pruning

# Oriented Metrics (Orimetrics)

---

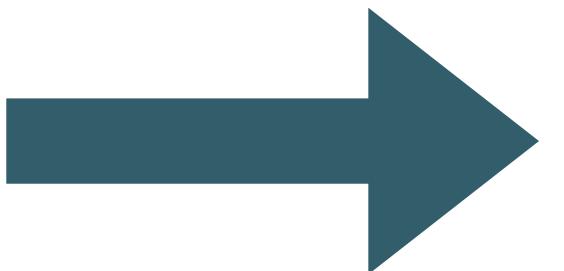
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Better pruning

Induces an equivalence

# Oriented Metrics (Orimetrics)

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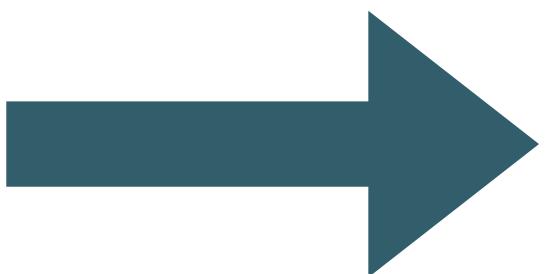
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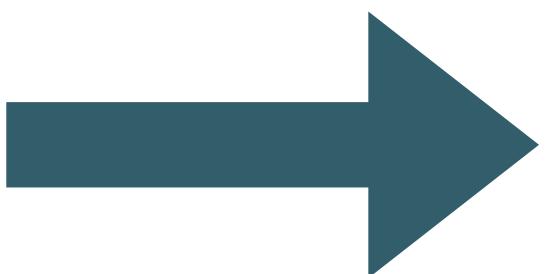
$$m(a, c) \leq m(a, b) + m(b, c) \quad (\Delta\text{-inequality})$$

Allows for **asymmetry**



Better pruning

Induces an equivalence



OE factorization, abstraction

## Why asymmetry?

SyGuS operators exhibit **asymmetric behavior**

# Asymmetric Behavior: $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

---

## Asymmetric Behavior: $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

---

Symmetry requires  $m(\text{"POPL"}, \text{"PO"}) = m(\text{"PO"}, \text{"POPL"})$

## Asymmetric Behavior: $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

---

Symmetry requires  $m("POPL", "PO") = m("PO", "POPL")$

$\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$  produces **super**strings of  $\mathcal{S}_1$  and  $\mathcal{S}_2$

## Asymmetric Behavior: $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

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"POPL" is a **super**string of "PO"

## Asymmetric Behavior: $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

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"POPL" is a **super**string of "PO"

"POPL" cannot produce "PO" with  $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

## Asymmetric Behavior: $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

---

Symmetry requires  $m("POPL", "PO") = m("PO", "POPL")$

$\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$  produces **super**strings of  $\mathcal{S}_1$  and  $\mathcal{S}_2$

"POPL" is a **super**string of "PO"

"POPL" cannot produce "PO" with  $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

"PO" is a **sub**string of "POPL"

## Asymmetric Behavior: $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

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"POPL" is a **super**string of "PO"

"POPL" cannot produce "PO" with  $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

"PO" is a **sub**string of "POPL"

"PO" might help produce "POPL" with  $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

## Asymmetric Behavior: $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

---

Symmetry requires  $m(\text{"POPL"}, \text{"PO"}) = m(\text{"PO"}, \text{"POPL"})$

big                                   small

$\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$  produces **super**strings of  $\mathcal{S}_1$  and  $\mathcal{S}_2$

"POPL" is a **super**string of "PO"

"POPL" cannot produce "PO" with  $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

"PO" is a **sub**string of "POPL"

"PO" might help produce "POPL" with  $\text{concat}(\mathcal{S}_1, \mathcal{S}_2)$

Asymmetric Behavior:  $\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$

---

Symmetry requires  $m(\text{"POPL"}, \text{"PO"}) = m(\text{"PO"}, \text{"POPL"})$

## Asymmetric Behavior: $\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$

---

Symmetry requires  $m("POPL", "PO") = m("PO", "POPL")$

$\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$  produces **sub**strings of  $\mathcal{S}_1$

## Asymmetric Behavior: $\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$

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small big

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# Asymmetric Behavior: $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

---

## Asymmetric Behavior: $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

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Symmetry requires  $m(110, 100) = m(100, 110)$

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Symmetry requires  $m(110, 100) = m(100, 110)$

$\text{and}(\mathcal{S}_1, \mathcal{S}_2)$  produces bitvectors bitwise **less** than  $\mathcal{S}_1$  and  $\mathcal{S}_2$

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---

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110 is bitwise **greater** than 100

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$\text{and}(\mathcal{S}_1, \mathcal{S}_2)$  produces bitvectors bitwise **less** than  $\mathcal{S}_1$  and  $\mathcal{S}_2$

110 is bitwise **greater** than 100

110 might help produce 100 with  $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

## Asymmetric Behavior: $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

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Symmetry requires  $m(110, 100) = m(100, 110)$

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110 is bitwise **greater** than 100

110 might help produce 100 with  $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

100 is bitwise **less** than 110

## Asymmetric Behavior: $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

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110 might help produce 100 with  $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

100 is bitwise **less** than 110

100 cannot produce 110 with  $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

## Asymmetric Behavior: $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

---

Symmetry requires  $m(110, 100) = m(100, 110)$

small                    big

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100 cannot produce 110 with  $\text{and}(\mathcal{S}_1, \mathcal{S}_2)$

We need **asymmetry**

# Oriented Metrics (Orimetrics)

---

$$m : D \times D \rightarrow \mathbb{R}_{\geq 0}$$

$$m(a, a) = 0 \quad (\text{reflexivity})$$

$$m(b, a) = 0 \Rightarrow m(a, b) = 0 \quad (\text{symmetry at zero})$$

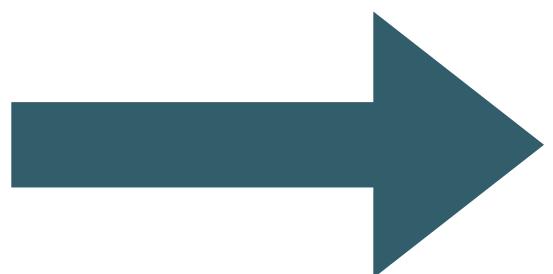
$$m(a, c) \leq m(a, b) + m(b, c) \quad (\Delta\text{-inequality})$$

Allows for **asymmetry**



Better pruning

Induces an equivalence



OE factorization, abstraction

# Oriented Metrics (Orimetrics)

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Equivalence at distance 0:  $a \equiv_m b$  if  $m(a, b) = 0$

# Oriented Metrics (Orimetrics)

---

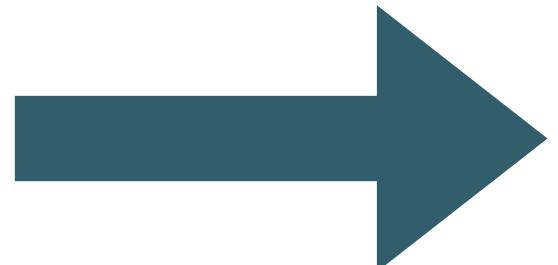
$$m : D \times D \rightarrow \mathbb{R}_{\geq 0}$$

$$m(a, a) = 0 \quad \text{(reflexivity)}$$

$$m(b, a) = 0 \Rightarrow m(a, b) = 0 \quad \text{(symmetry at zero)}$$

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OE factorization, abstraction

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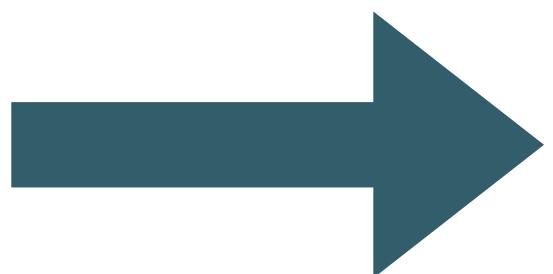
$$m(a, c) \leq m(a, b) + m(b, c) \quad (\Delta\text{-inequality})$$

Allows for **asymmetry**



Better pruning

Induces an equivalence



OE factorization, abstraction

## How to design an orimetric?

1. Construct an orimetric  $m$  on the data domain
2. Lift  $m$  to programs

Oriented Metric for  $\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$

---

$$m : D \times D \rightarrow \mathbb{R}_{\geq 0}$$

Reward superstrings

Oriented Metric for  $\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$

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$$m : D \times D \rightarrow \mathbb{R}_{\geq 0}$$

Reward superstrings

1. For strings  $i, o$ :

## Reward superstrings

1. For strings  $i, o$ :  $m(i, o) = \begin{cases} \text{len}(i) - \text{len}(o) & \text{if } i \text{ is a superstring of } o \\ 100 + |\text{len}(i) - \text{len}(o)| & \text{otherwise} \end{cases}$

Oriented Metric for  $\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$

$m : D \times D \rightarrow \mathbb{R}_{\geq 0}$

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$$m(\text{"PO"}, \text{"POPL"}) = 102$$

Oriented Metric for  $\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$

$m : D \times D \rightarrow \mathbb{R}_{\geq 0}$

---

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$$m(\text{"PO"}, \text{"POPL"}) = 102$$

$$m(\text{"POPL"}, \text{"PO"}) = 2$$

Oriented Metric for  $\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$

$m : D \times D \rightarrow \mathbb{R}_{\geq 0}$

---

Reward superstrings

1. For strings  $i, o$ :  $m(i, o) = \begin{cases} \text{len}(i) - \text{len}(o) & \text{if } i \text{ is a superstring of } o \\ 100 + |\text{len}(i) - \text{len}(o)| & \text{otherwise} \end{cases}$

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$$m(\text{"POPL"}, \text{"PO"}) = 2$$

2. For programs p, q:

Oriented Metric for  $\text{replace}(\mathcal{S}_1, \mathcal{S}_2, \epsilon)$

$m : D \times D \rightarrow \mathbb{R}_{\geq 0}$

---

Reward superstrings

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$$m(\text{"PO"}, \text{"POPL"}) = 102$$

$$m(\text{"POPL"}, \text{"PO"}) = 2$$

2. For programs  $p, q$ :  $m_{In}(p, q) = \sum_{i \in In} m(p(i), q(i))$

# Pruning with an Orimetric

<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
"Rennes_City"	"Rennes"
"PLDI_Conf"	"PLDI"
"Seoul_City"	"Seoul"

$$m(i, o) = \begin{cases} \text{len}(i) - \text{len}(o) & \text{if } i \text{ is a superstring of } o \\ 100 + |\text{len}(i) - \text{len}(o)| & \text{otherwise} \end{cases}$$

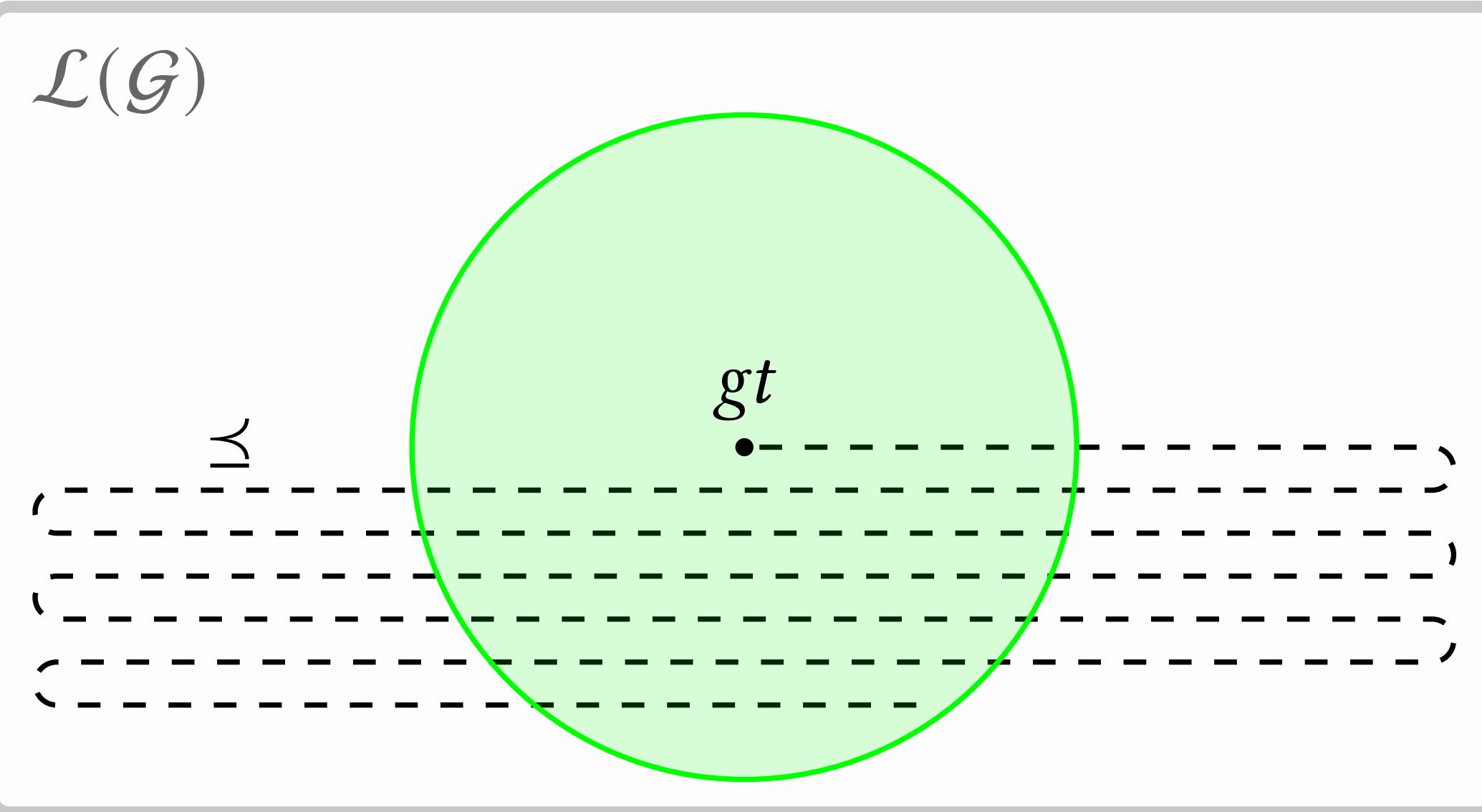
$$m_{In}(p, q) = \sum_{i \in In} m(p(i), q(i))$$

# Pruning with an Orimetric

<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
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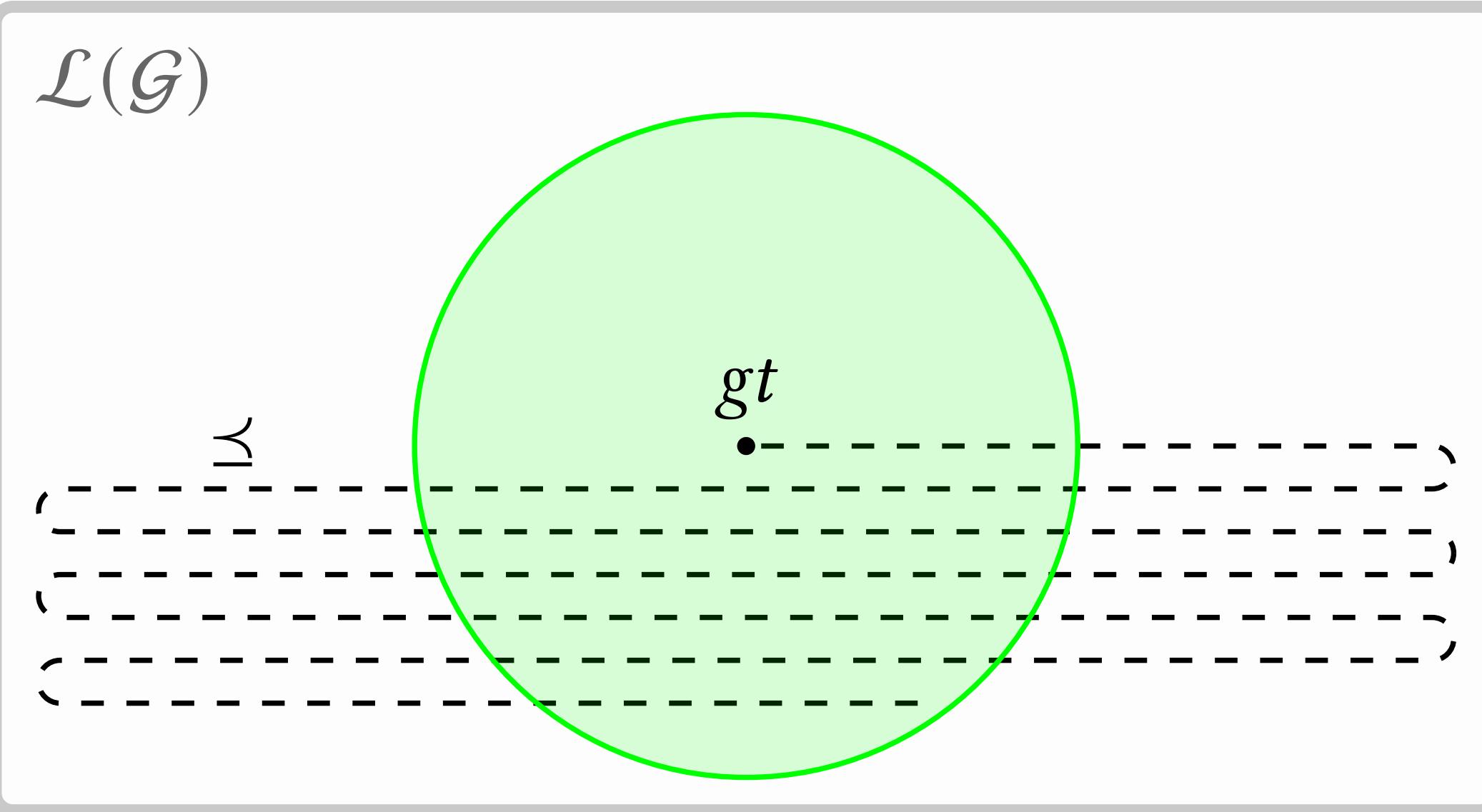
Set radius  $r$  to 100.

# Pruning with an Orimetric

<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
"Rennes_City"	"Rennes"
"PLDI_Conf"	"PLDI"
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$$m_{In}(p, q) = \sum_{i \in In} m(p(i), q(i))$$



"\_City"."\_City"

Set radius  $r$  to 100.

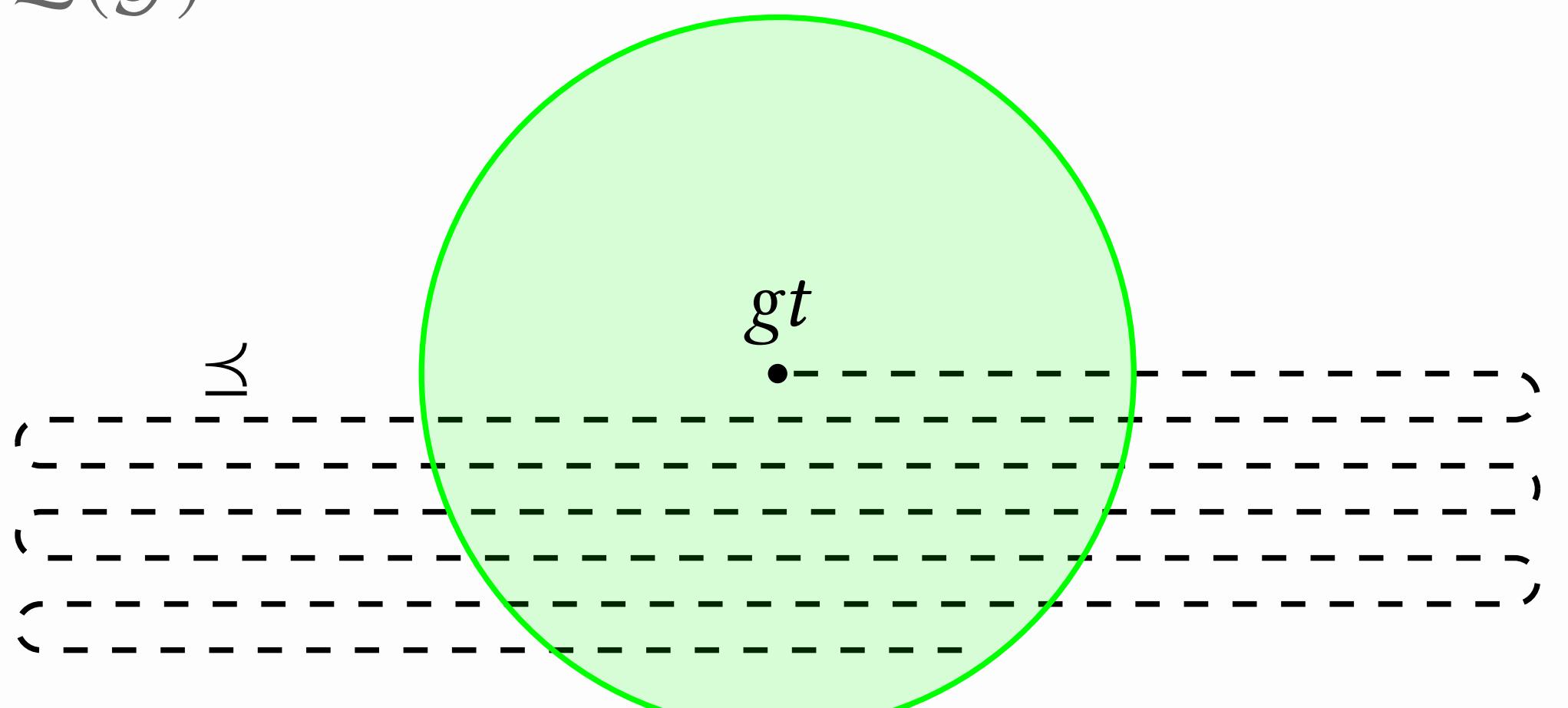
# Pruning with an Orimetric

<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
"Rennes_City"	"Rennes"
"PLDI_Conf"	"PLDI"
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$$m_{In}(p, q) = \sum_{i \in In} m(p(i), q(i))$$

$\mathcal{L}(\mathcal{G})$



Set radius  $r$  to 100.

"\_City".\_City"  
 $m_{In}("City"._City, gt) > 100$

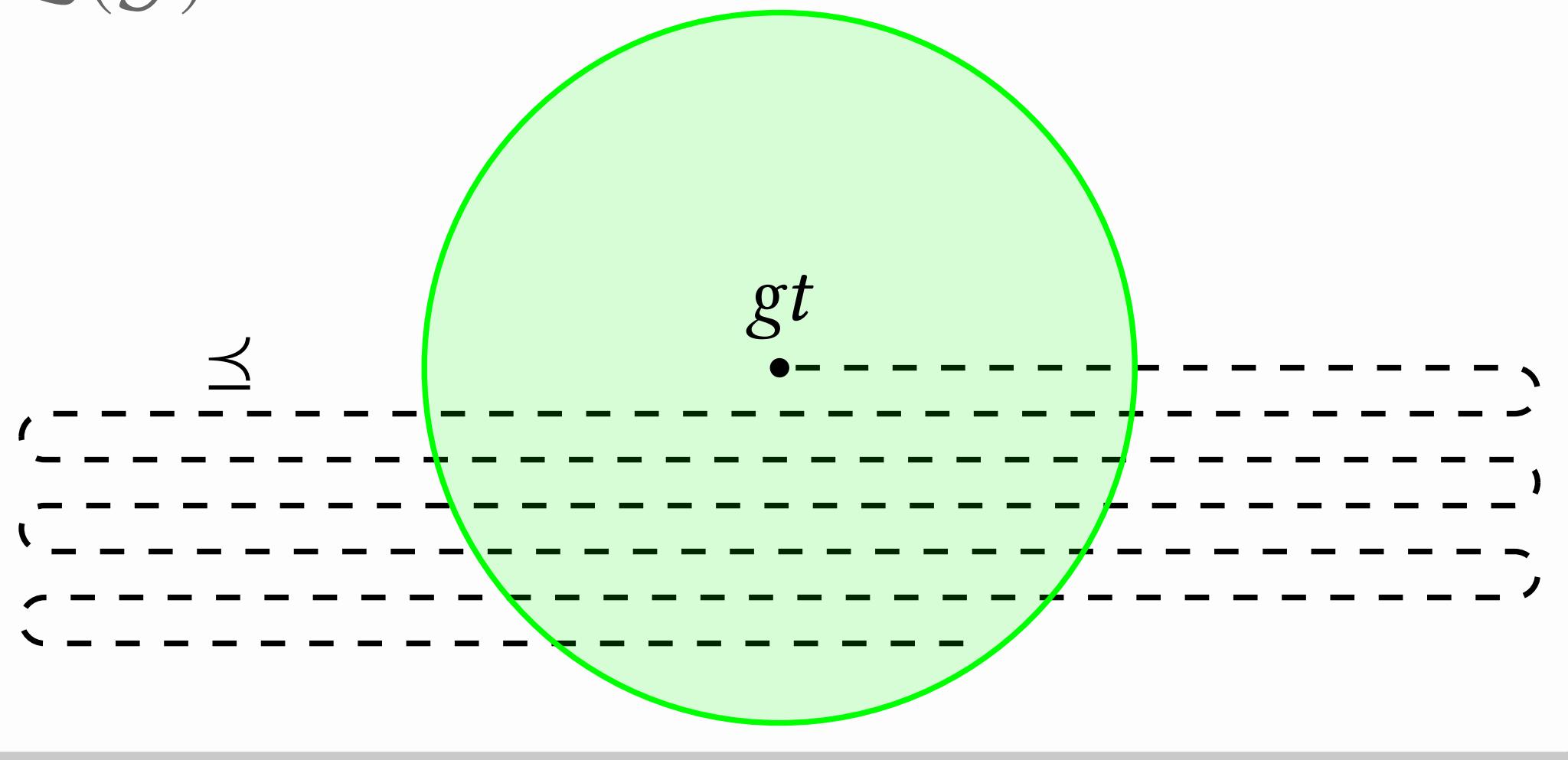
# Pruning with an Orimetric

<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
"Rennes_City"	"Rennes"
"PLDI_Conf"	"PLDI"
"Seoul_City"	"Seoul"

$$m(i, o) = \begin{cases} \text{len}(i) - \text{len}(o) & \text{if } i \text{ is a superstring of } o \\ 100 + |\text{len}(i) - \text{len}(o)| & \text{otherwise} \end{cases}$$

$$m_{In}(p, q) = \sum_{i \in In} m(p(i), q(i))$$

$\mathcal{L}(\mathcal{G})$



Set radius  $r$  to 100.

"\_City\_X\_City"

$$m_{In}("City". "City", gt) > 100$$

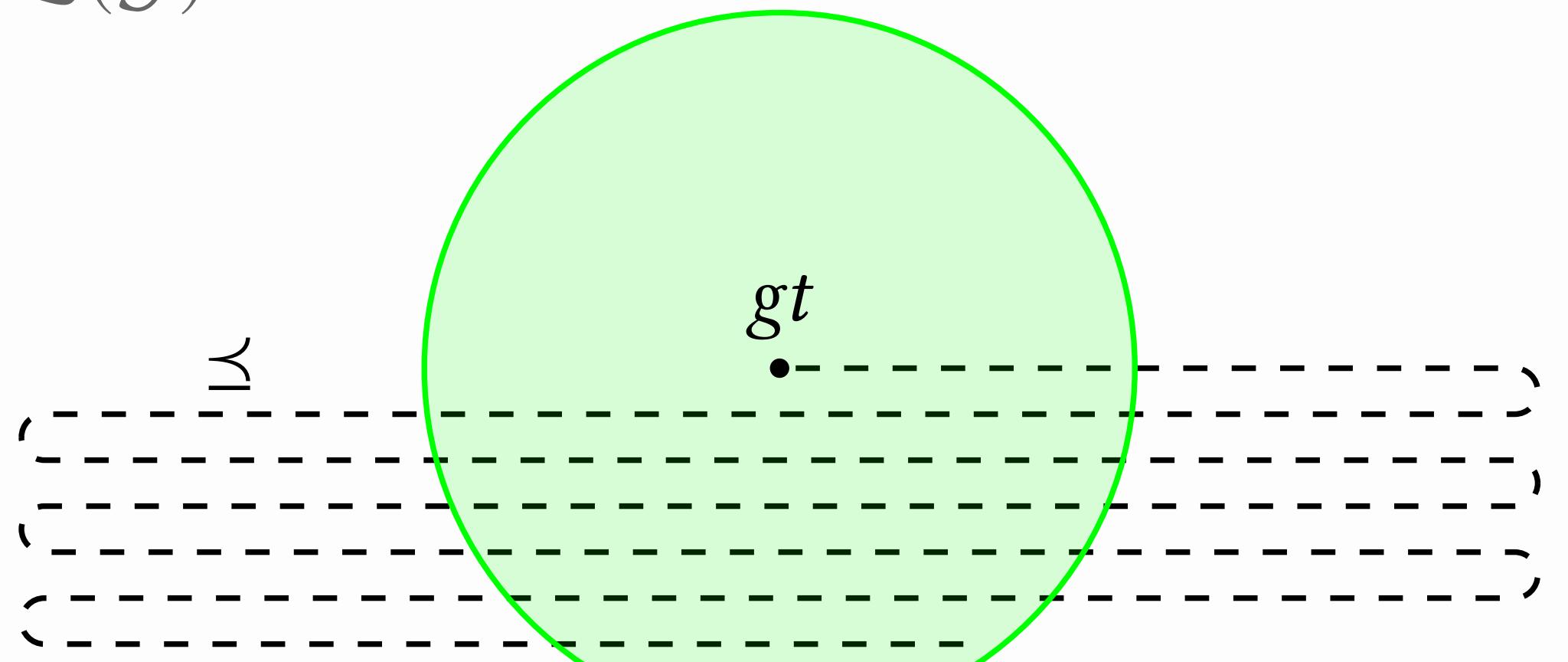
# Pruning with an Orimetric

<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
"Rennes_City"	"Rennes"
"PLDI_Conf"	"PLDI"
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$$m_{In}(p, q) = \sum_{i \in In} m(p(i), q(i))$$

$\mathcal{L}(\mathcal{G})$



Set radius  $r$  to 100.

Method	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$
No Pruning or Factorization	4	-	16	64	128	1280	4352
OE Factorization	4	-	9	6	27	56	119
Orimetric Pruning (OP)	4	-	7	18	56	323	929

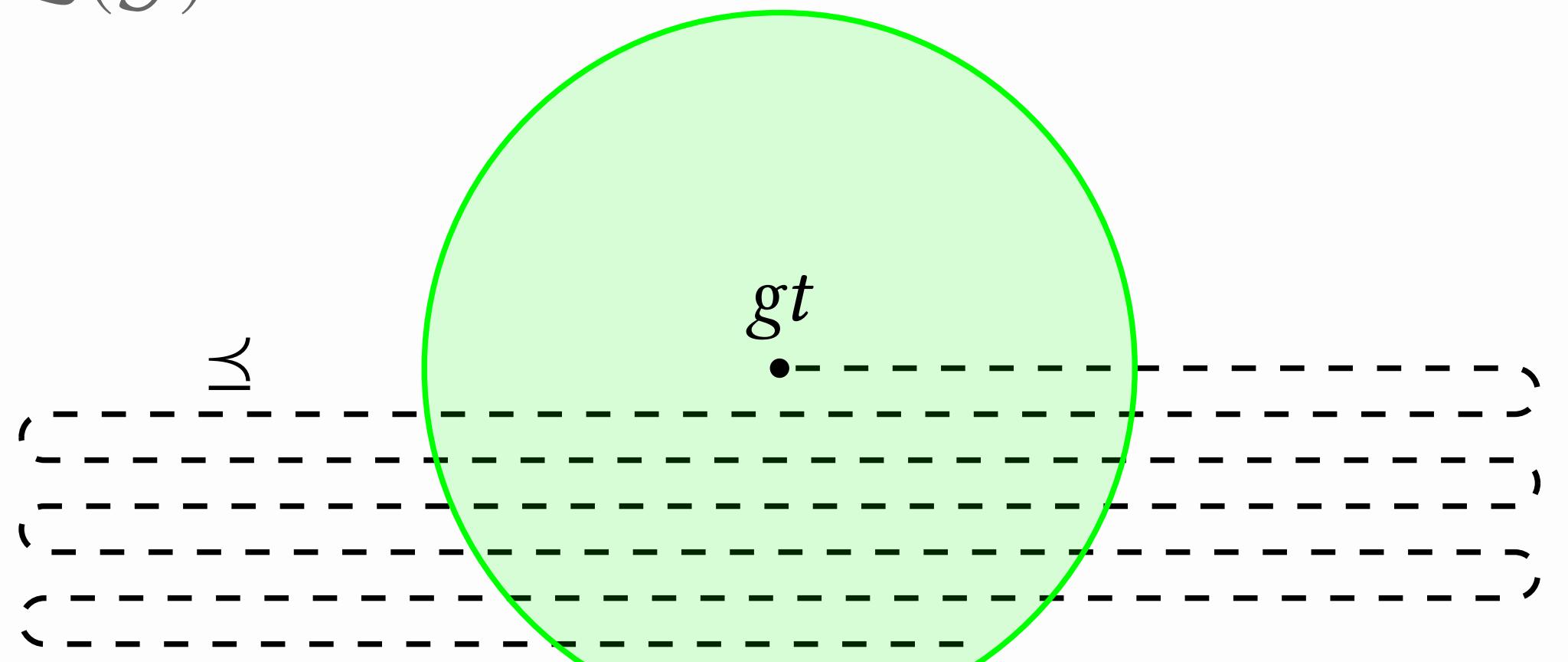
# Factorizing with an Orimetric

<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
"Rennes_City"	"Rennes"
"PLDI_Conf"	"PLDI"
"Seoul_City"	"Seoul"

$$m(i, o) = \begin{cases} \text{len}(i) - \text{len}(o) & \text{if } i \text{ is a superstring of } o \\ 100 + |\text{len}(i) - \text{len}(o)| & \text{otherwise} \end{cases}$$

$$m_{In}(p, q) = \sum_{i \in In} m(p(i), q(i))$$

$\mathcal{L}(\mathcal{G})$



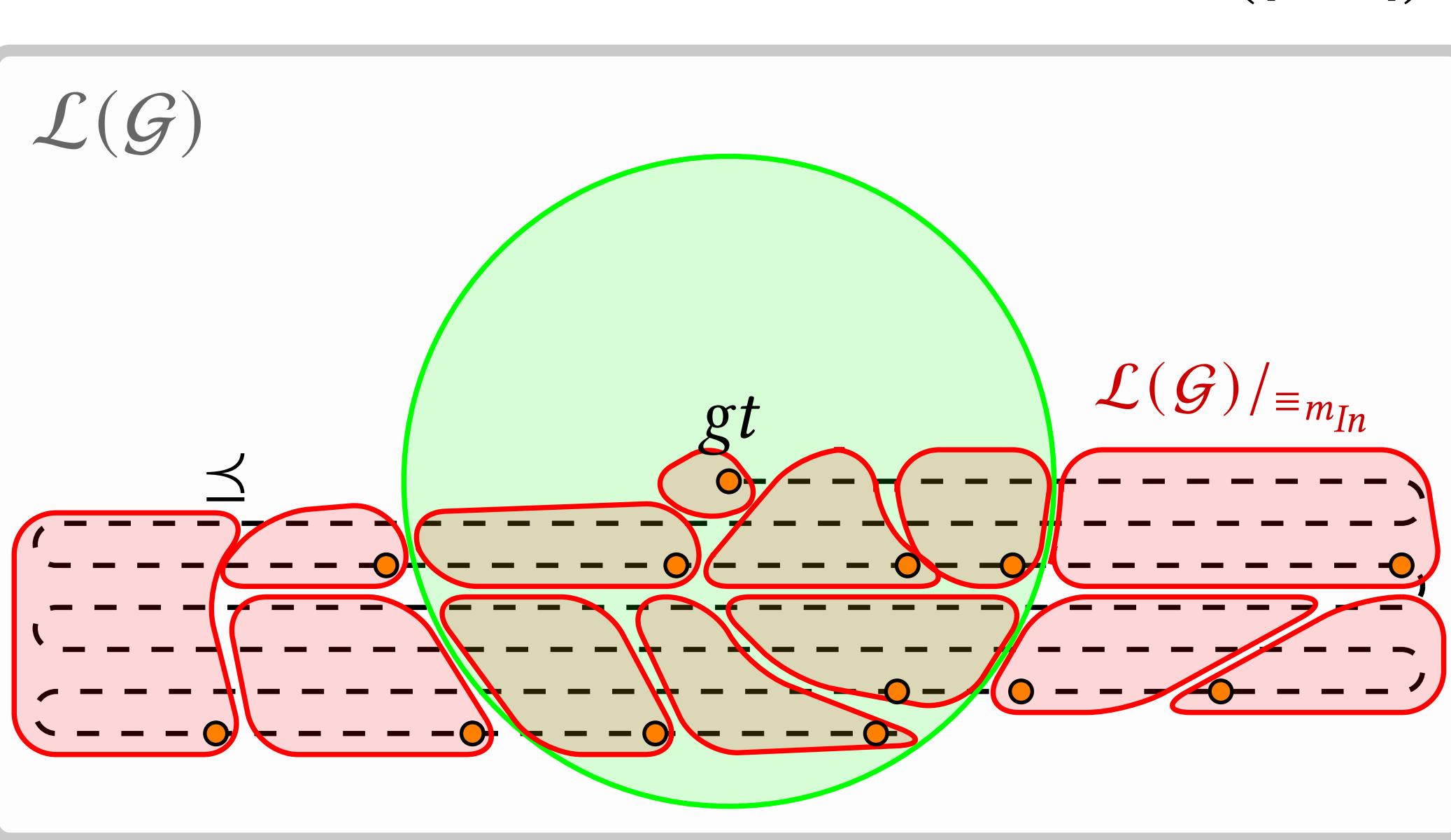
Method	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$
No Pruning or Factorization	4	-	16	64	128	1280	4352
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Orimetric Pruning (OP)	4	-	7	18	56	323	929

# Factorizing with an Orimetric

<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
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$$m(i, o) = \begin{cases} \text{len}(i) - \text{len}(o) & \text{if } i \text{ is a superstring of } o \\ 100 + |\text{len}(i) - \text{len}(o)| & \text{otherwise} \end{cases}$$

$$m_{In}(p, q) = \sum_{i \in In} m(p(i), q(i))$$



Method	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$
No Pruning or Factorization	4	-	16	64	128	1280	4352
OE Factorization	4	-	9	6	27	56	119
Orimetric Pruning (OP)	4	-	7	18	56	323	929

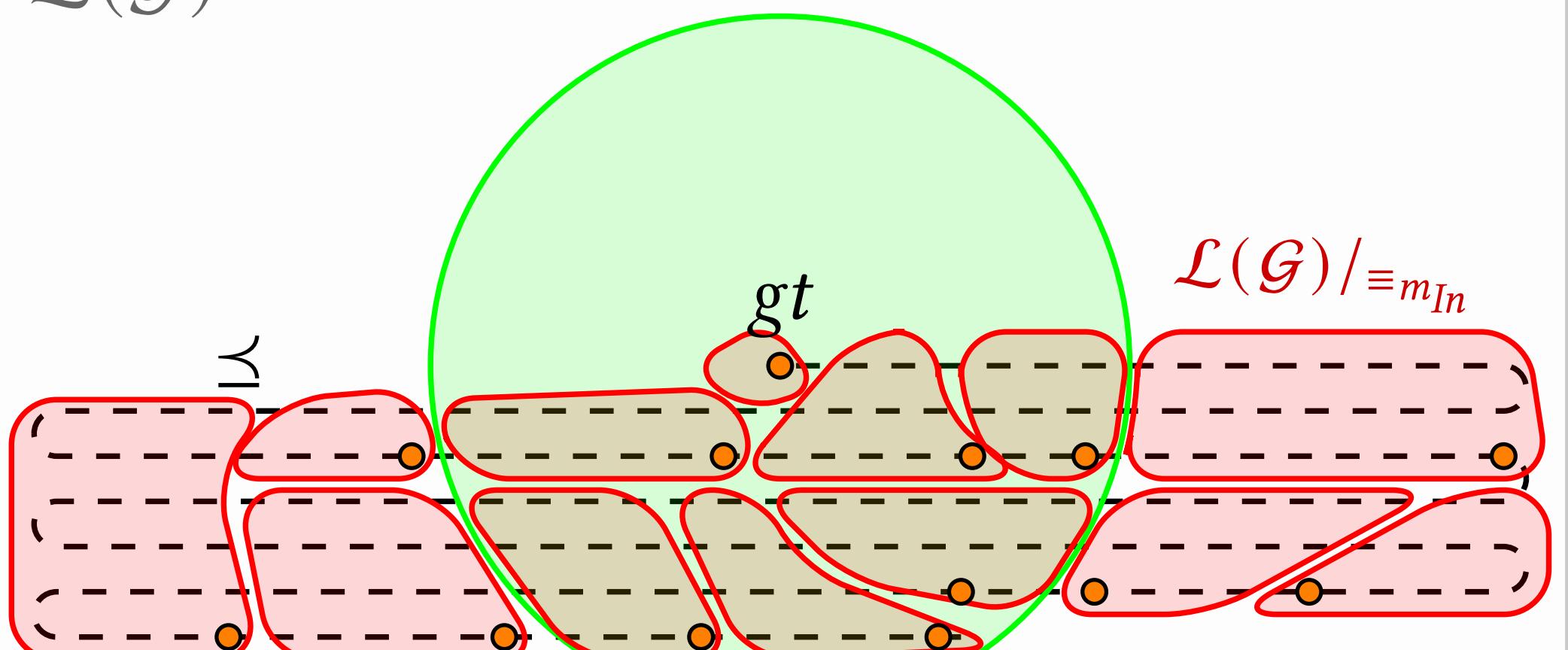
# Factorizing with an Orimetric

<i>In</i>	<i>Out</i>
"POPL_Conf"	"POPL"
"Rennes_City"	"Rennes"
"PLDI_Conf"	"PLDI"
"Seoul_City"	"Seoul"

$$m(i, o) = \begin{cases} \text{len}(i) - \text{len}(o) & \text{if } i \text{ is a superstring of } o \\ 100 + |\text{len}(i) - \text{len}(o)| & \text{otherwise} \end{cases}$$

$$m_{In}(p, q) = \sum_{i \in In} m(p(i), q(i))$$

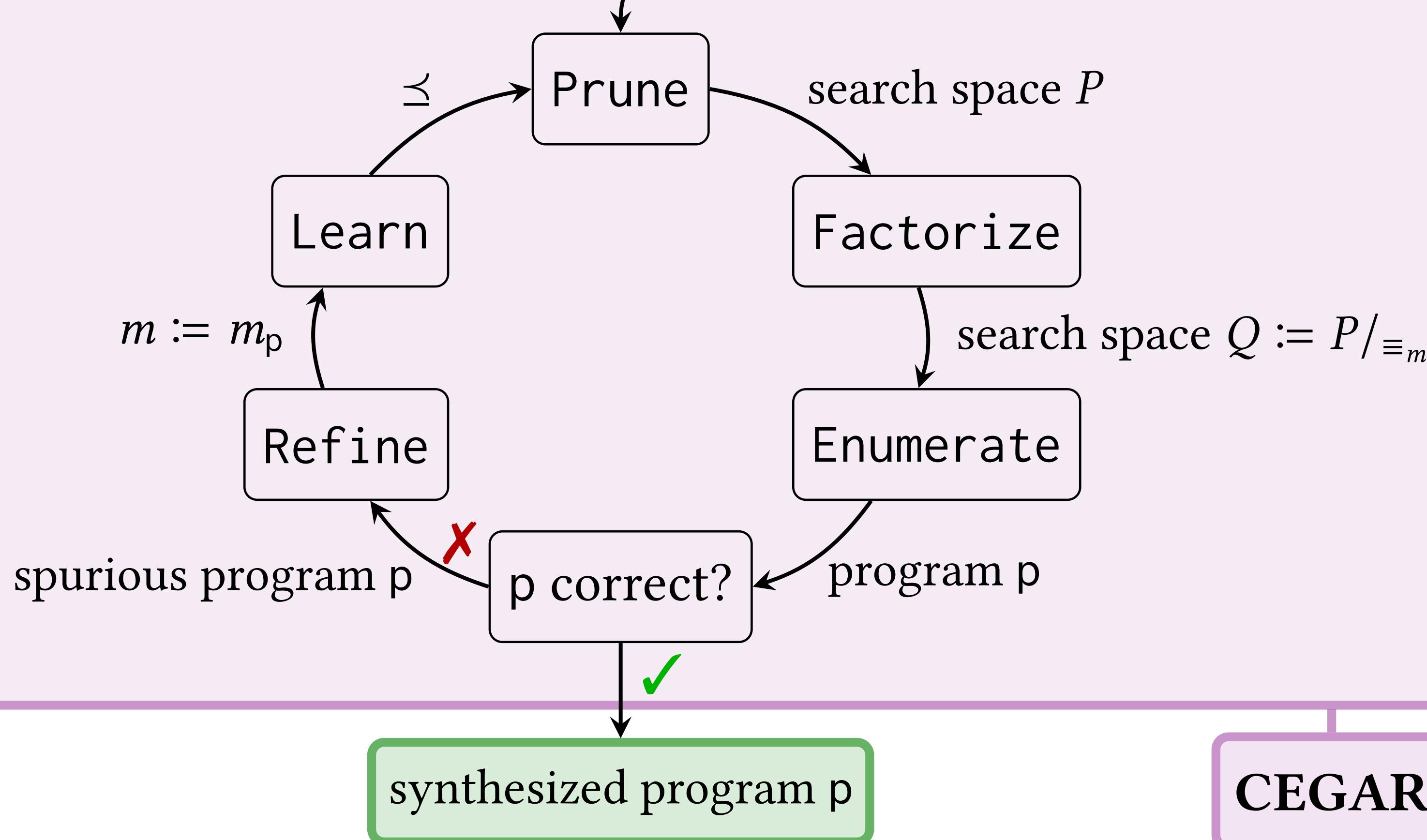
$\mathcal{L}(\mathcal{G})$



Method	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$
No Pruning or Factorization	4	-	16	64	128	1280	4352
OE Factorization	4	-	9	6	27	56	119
Orimetric Pruning (OP)	4	-	7	18	56	323	929
OE Factorization + OP	4	-	5	6	19	50	81

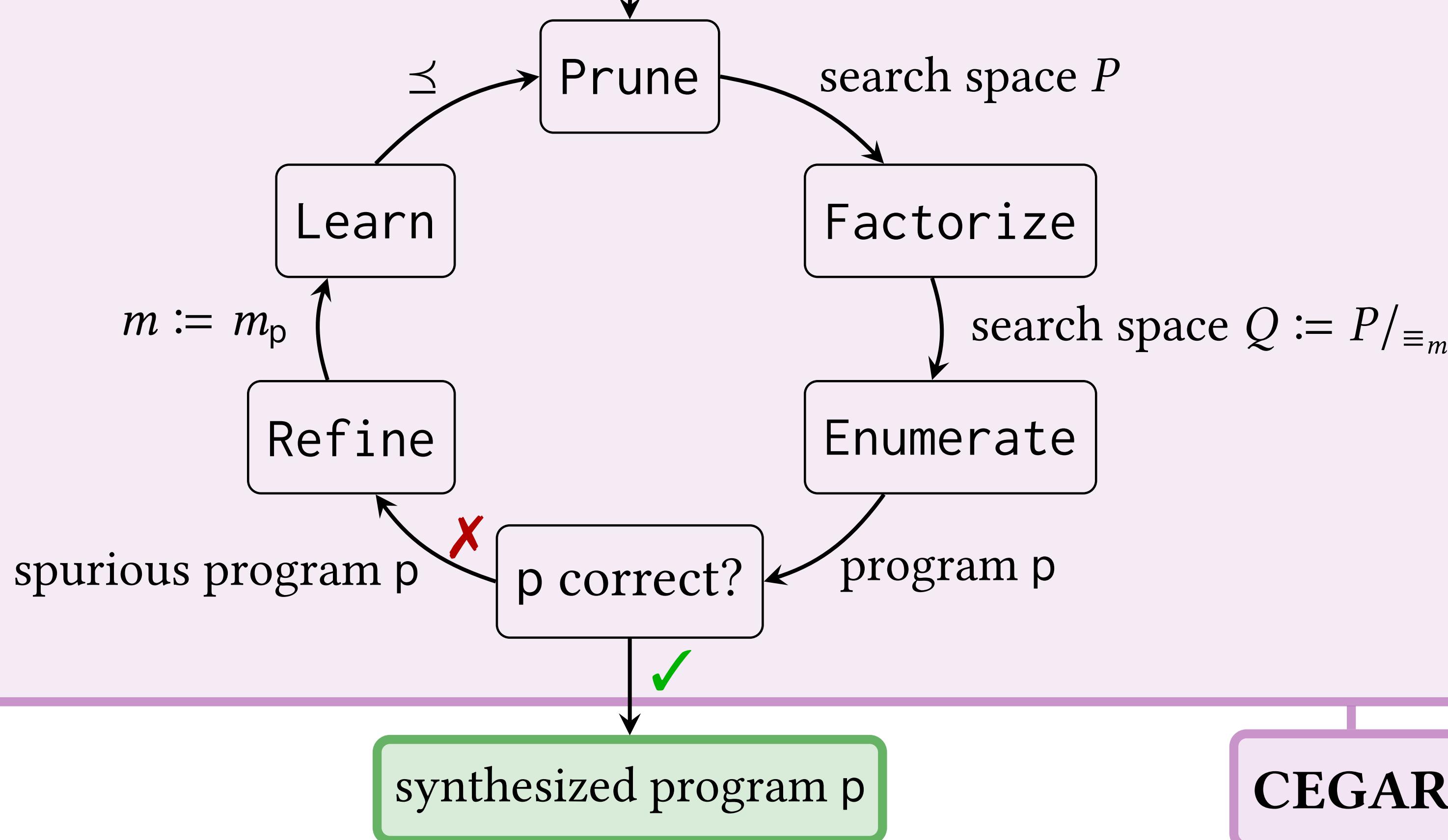
initial enumeration order  $\preceq$ , initial orimetric  $m$

grammar  $\mathcal{G}$ , ground truth  $gt$



initial enumeration order  $\preceq$ , initial orimetric  $m$

grammar  $\mathcal{G}$ , ground truth  $gt$



In practice: concurrent instances employing different orimetrics

# Evaluation of Merlin

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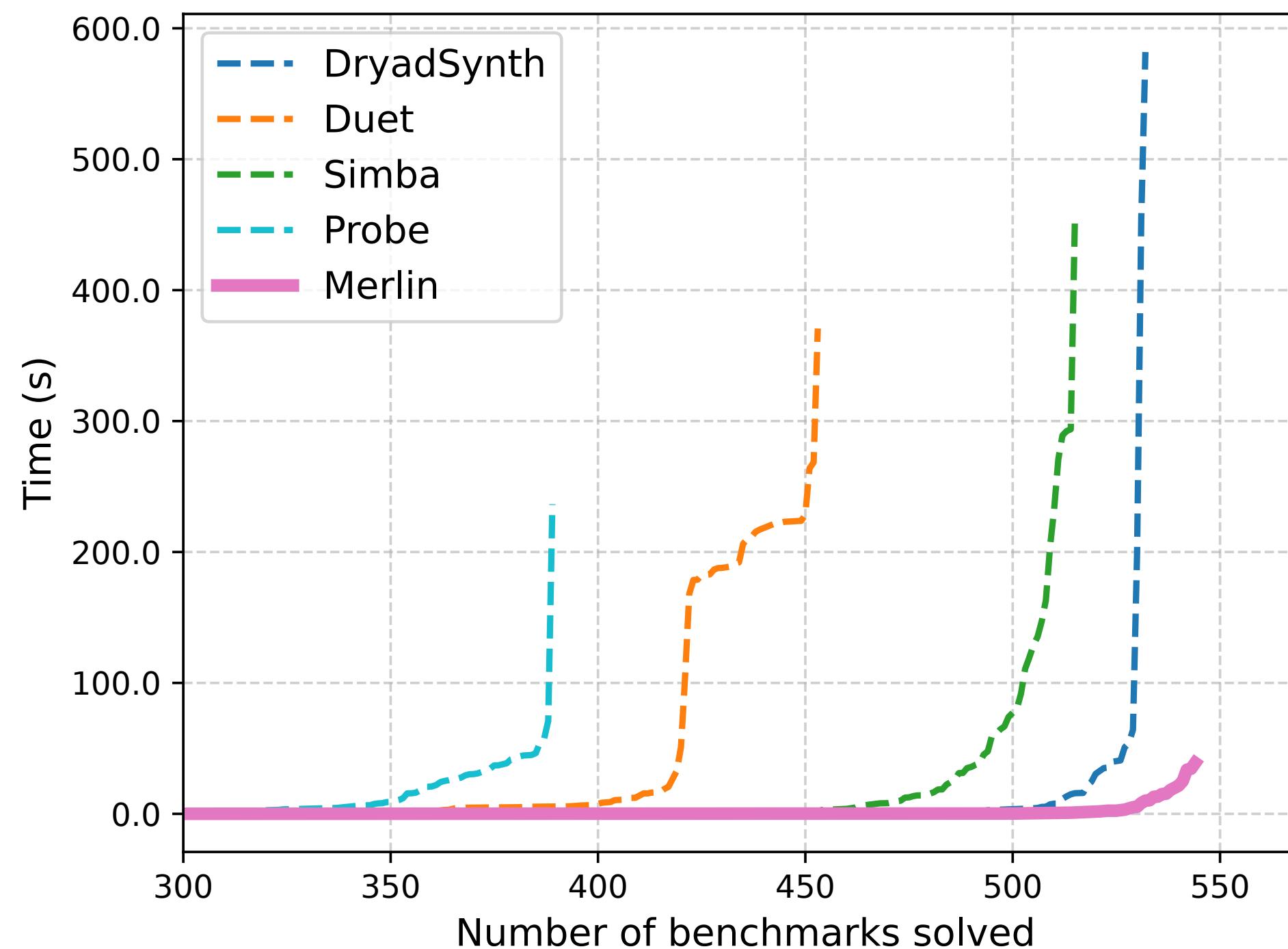


# Evaluation of Merlin



## SyGuS-Bitvector

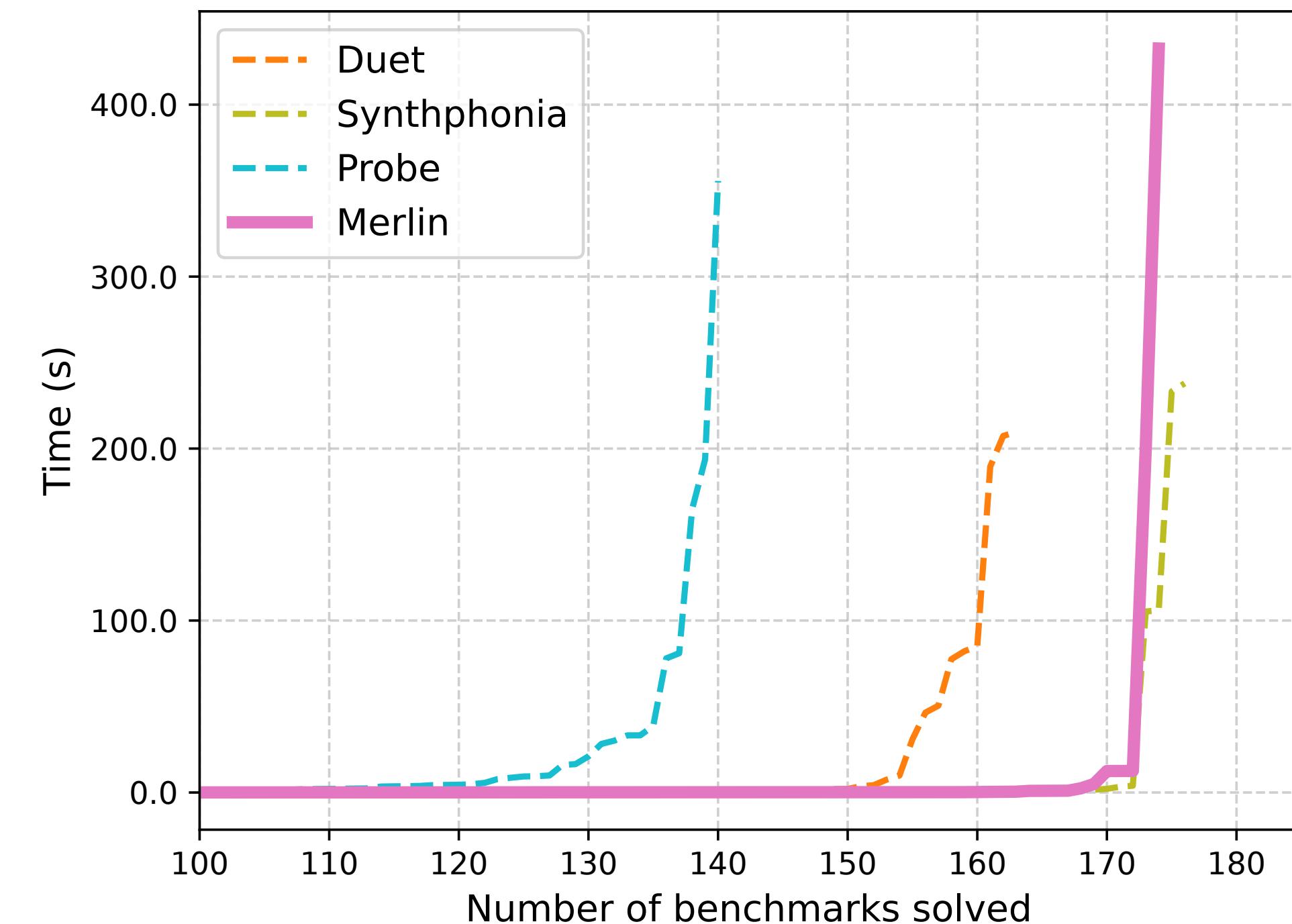
500 Deobfusc [Yoon et. al 2023] 49 Hacker's Delight [Warren 2013]



25x faster than DryadSynth

## SyGuS-String

181 Duet [Lee 2021]



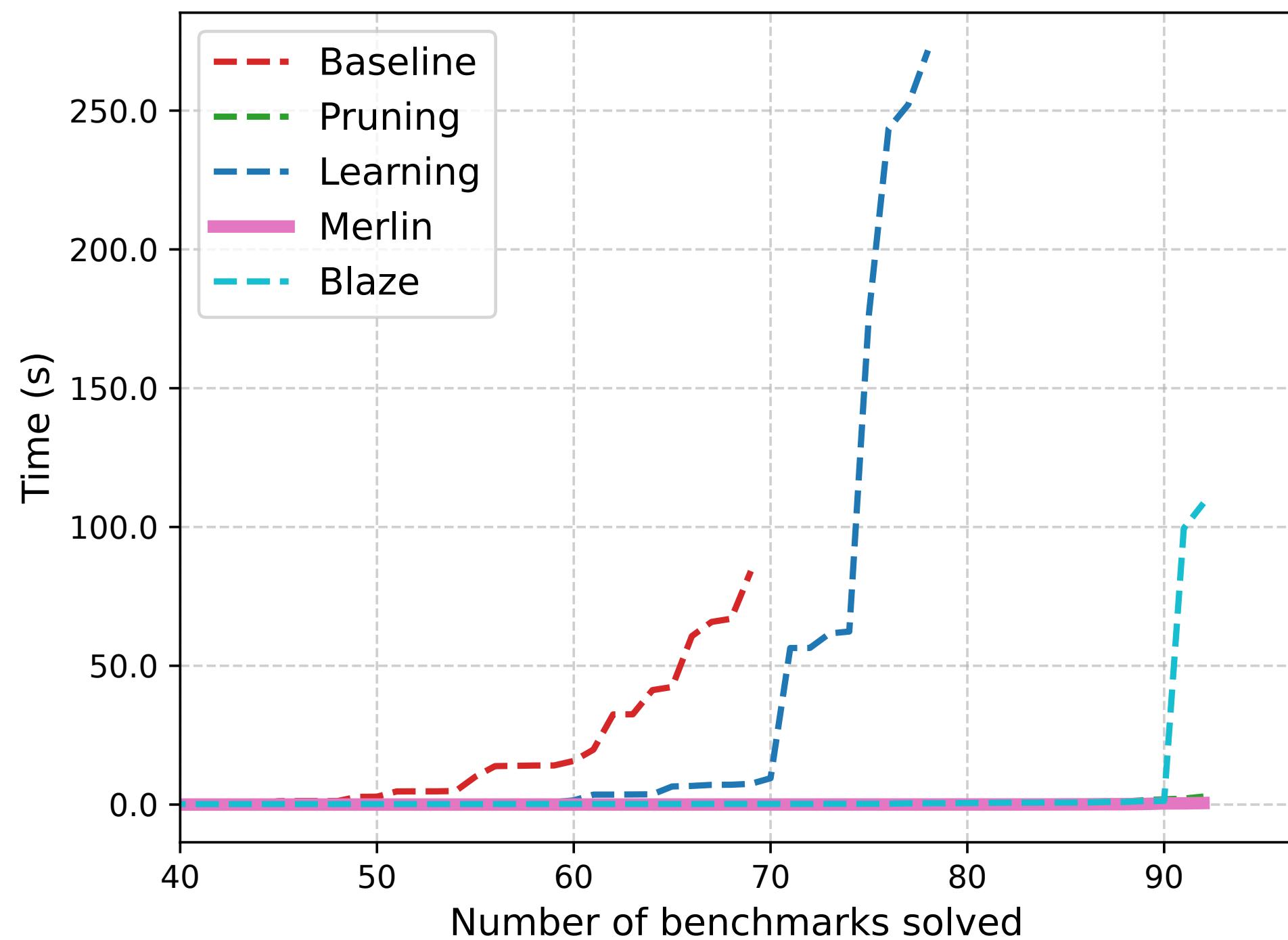
Competitive with Synthphonia

# Evaluation of Merlin



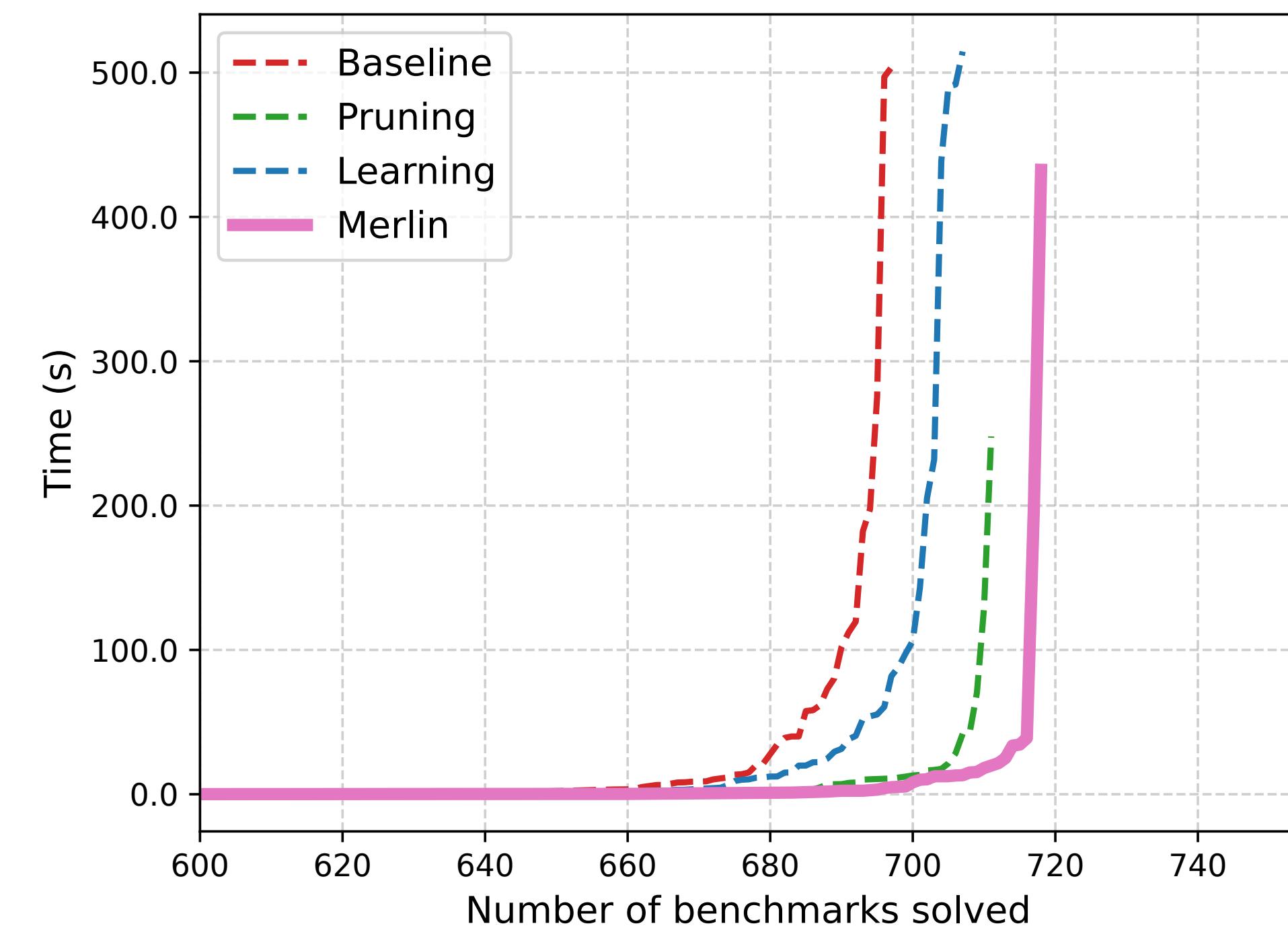
## Blaze (String)

108 Blaze [Wang 2018]



75x faster than Blaze

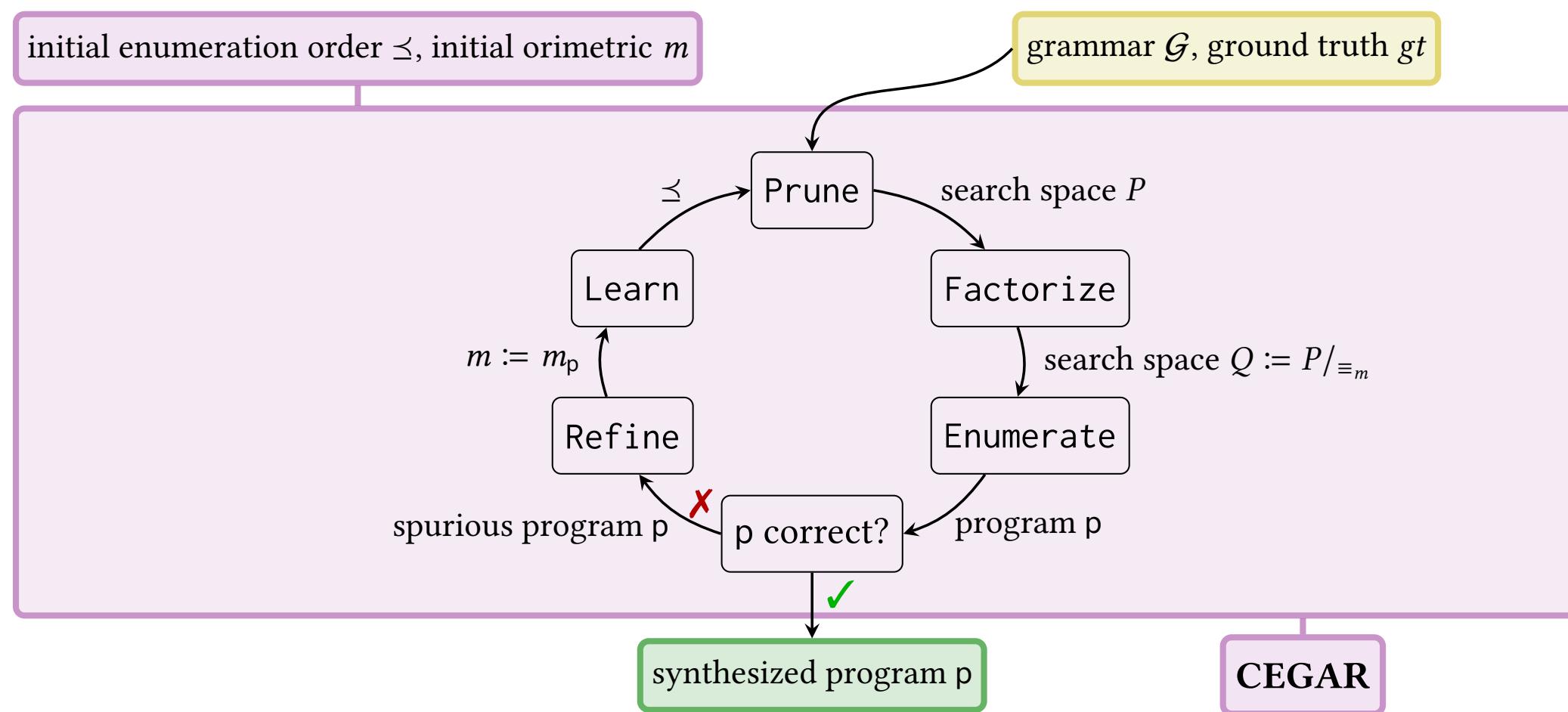
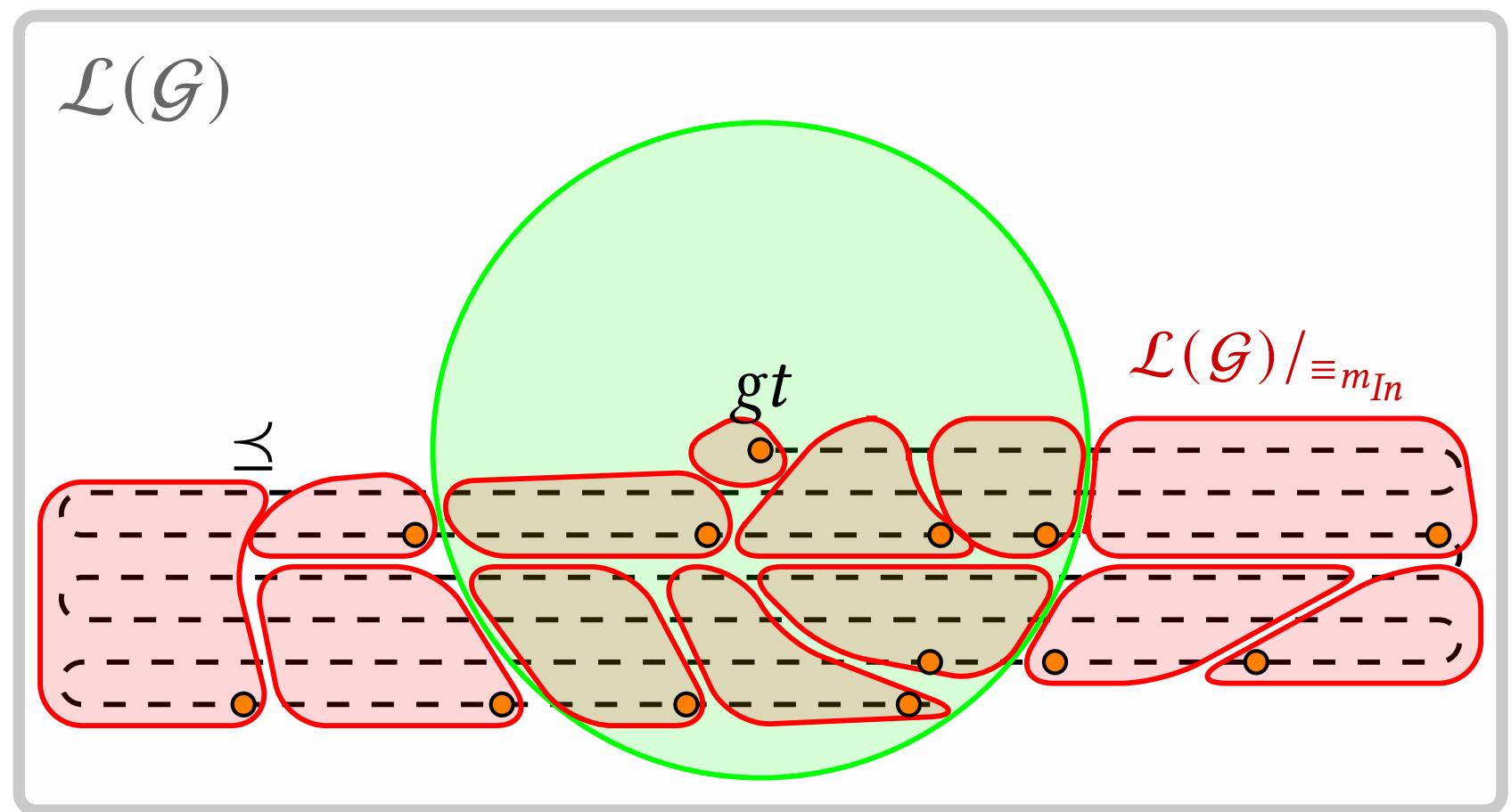
## SyGuS-Ablation



42x faster than Baseline

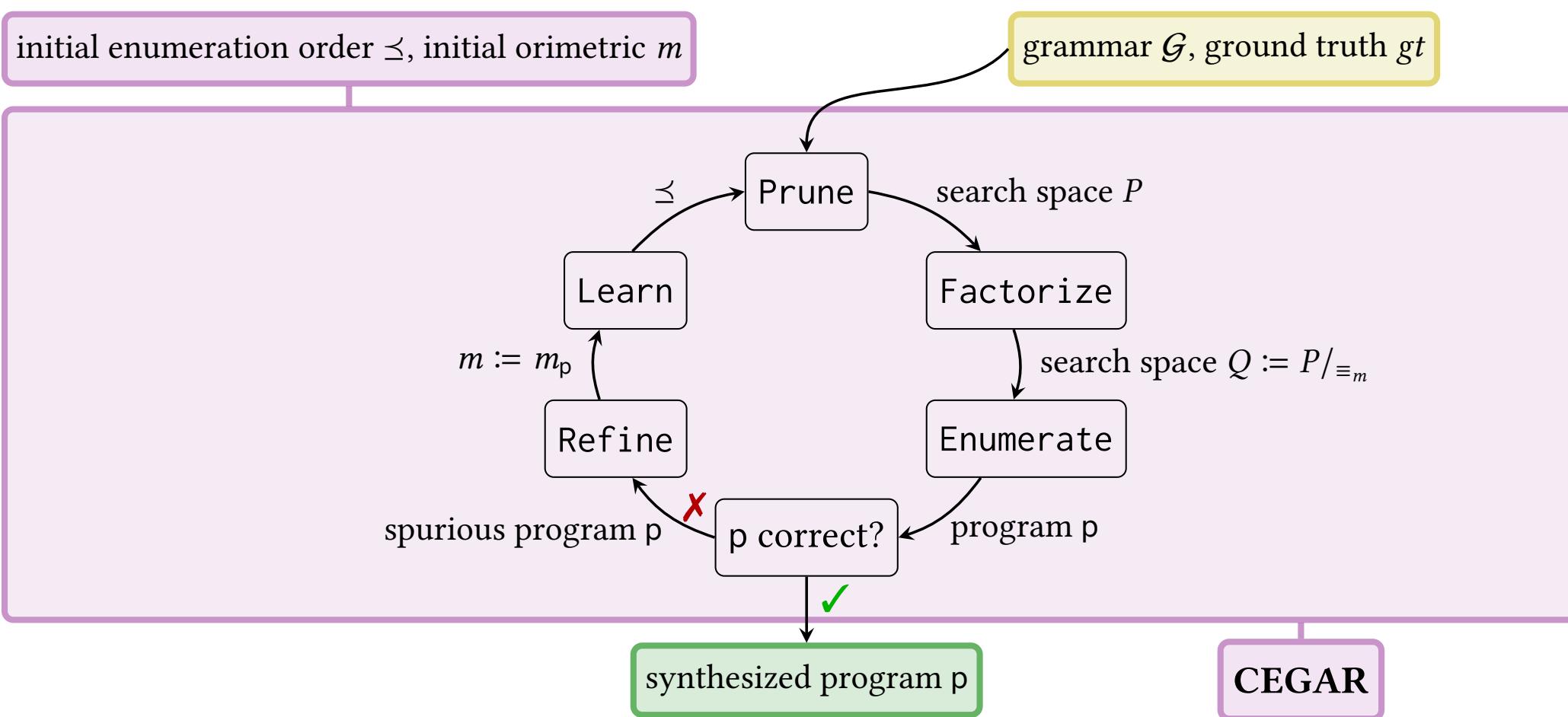
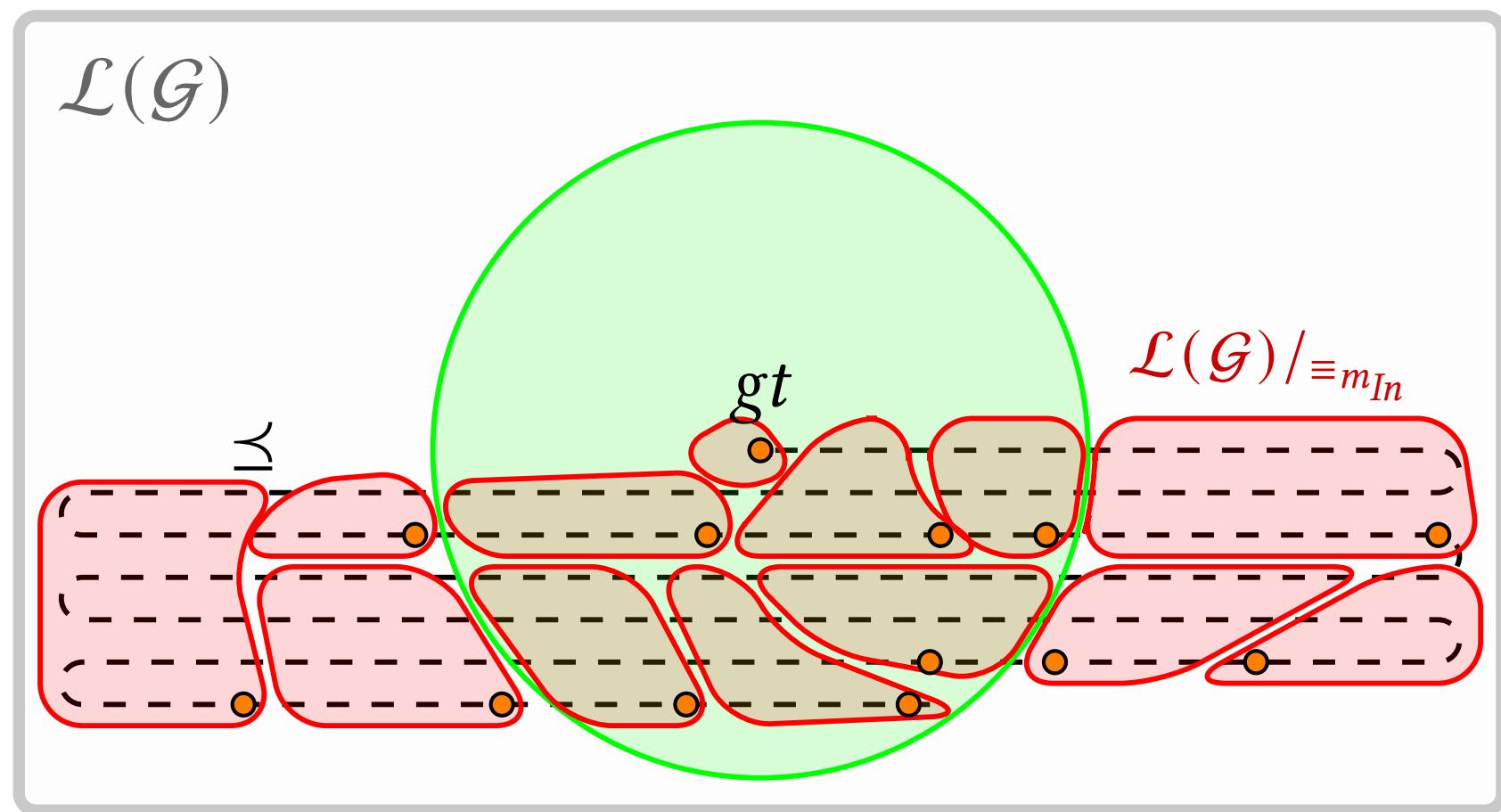
# Conclusion

## Framework for bottom-up enum. synthesis



# Conclusion

## Framework for bottom-up enum. synthesis

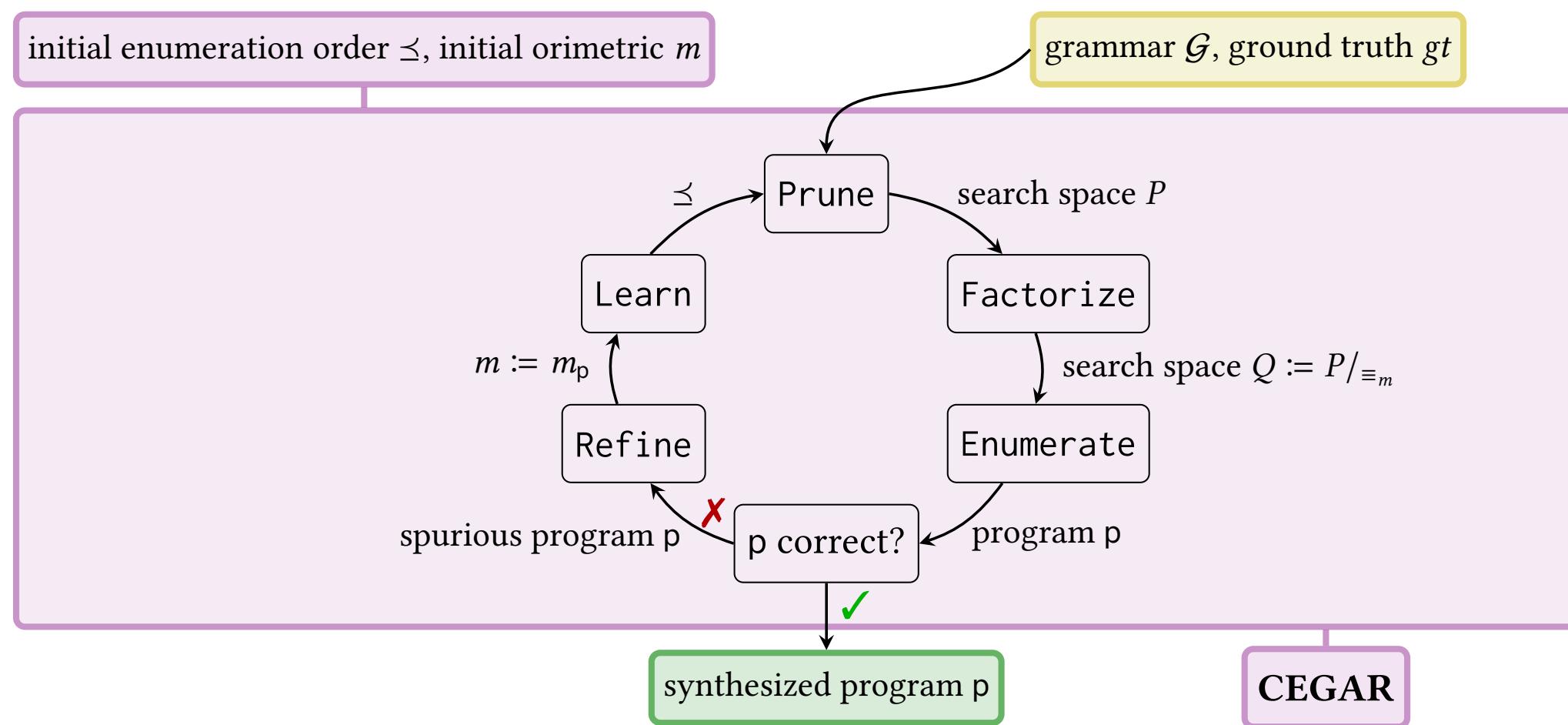
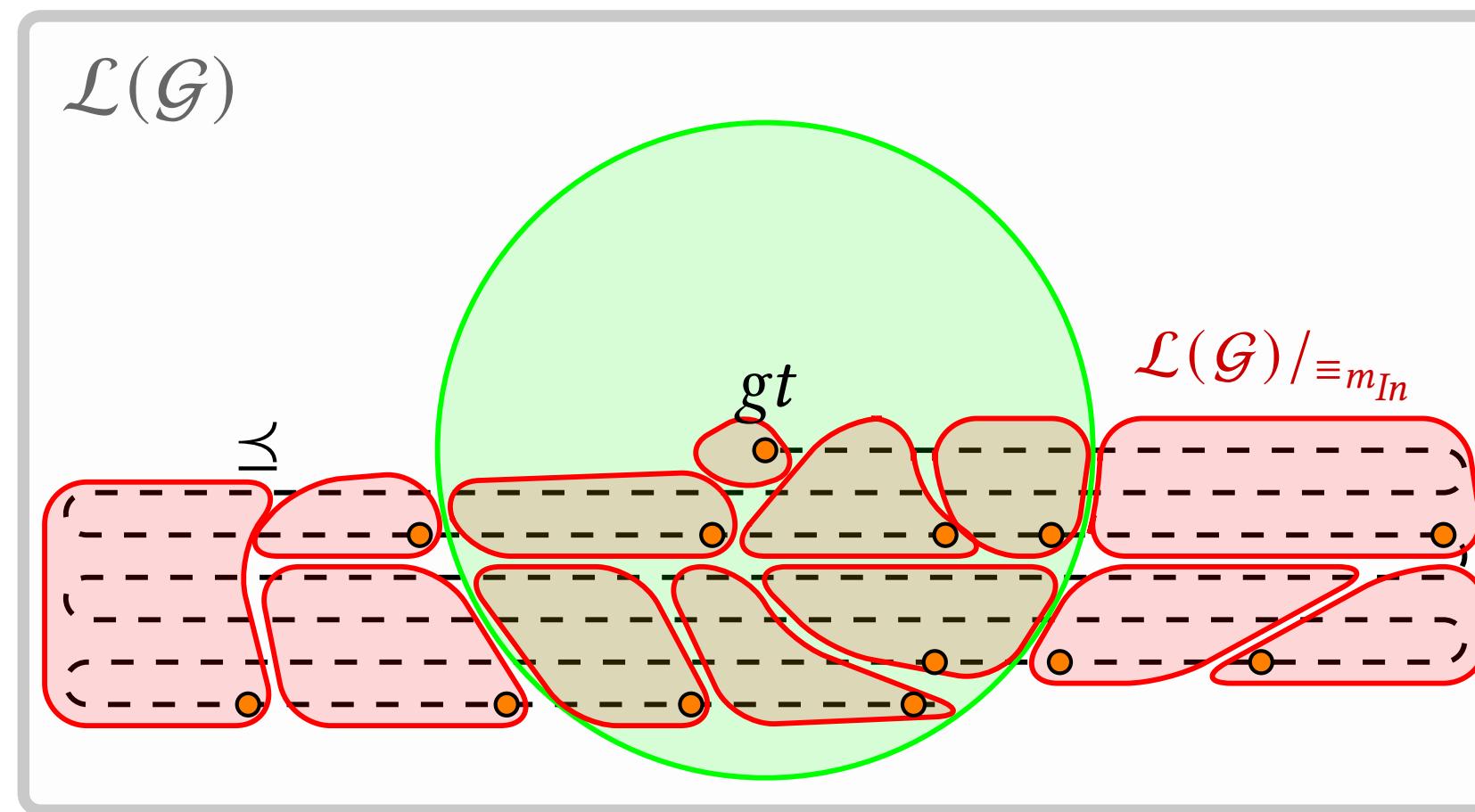


## Oriented Metrics

$$\begin{aligned} m(a, a) &= 0 && \text{(reflexivity)} \\ m(b, a) = 0 \Rightarrow m(a, b) &= 0 && \text{(symmetry at zero)} \\ m(a, c) \leq m(a, b) + m(b, c) &&& \text{( $\Delta$ -inequality)} \end{aligned}$$

# Conclusion

# Framework for bottom-up enum. synthesis



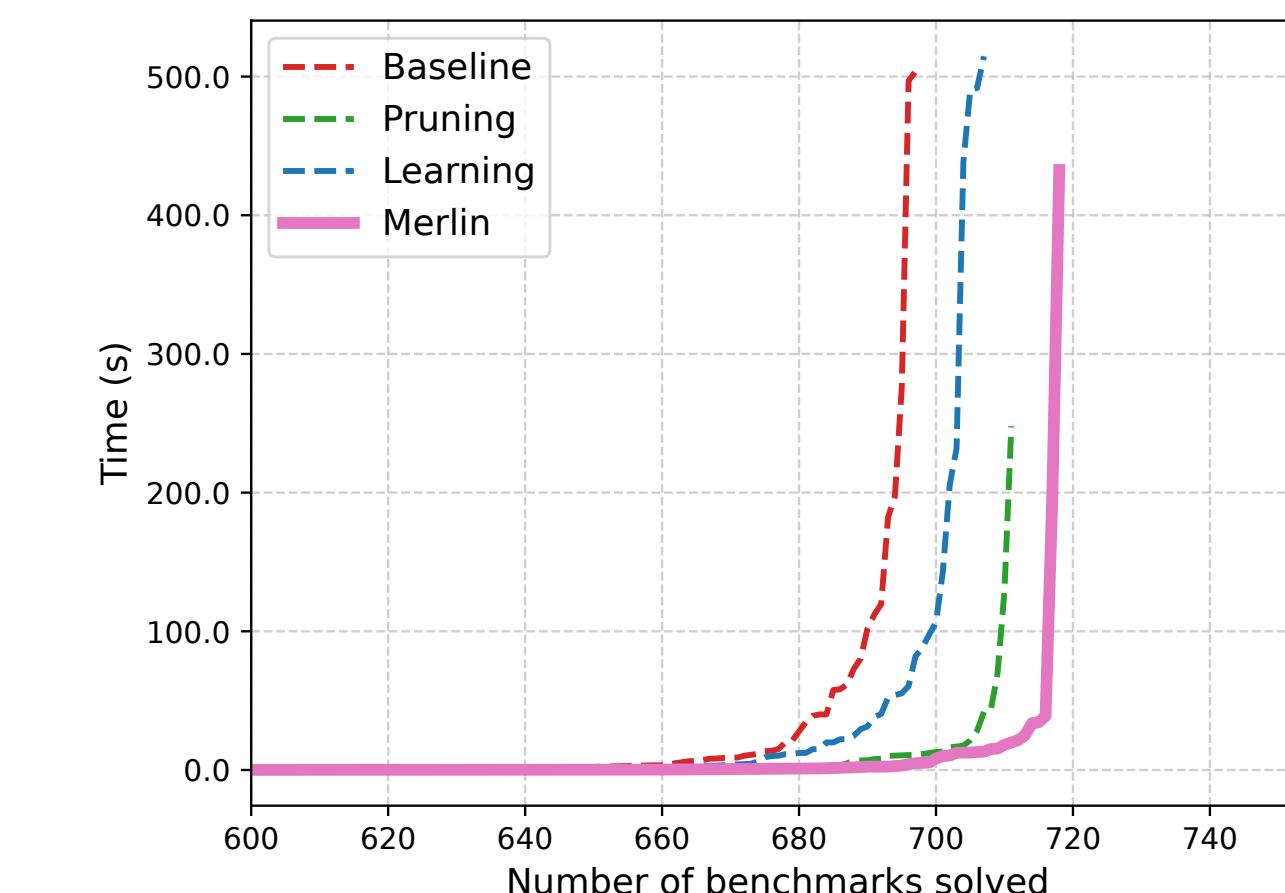
# Oriented Metrics

$$m(a, a) = 0 \quad \text{(reflexivity)}$$

$$m(b, a) = 0 \Rightarrow m(a, b) = 0 \quad (\text{symmetry at zero})$$

$$m(a, c) \leq m(a, b) + m(b, c) \quad (\Delta\text{-inequality})$$

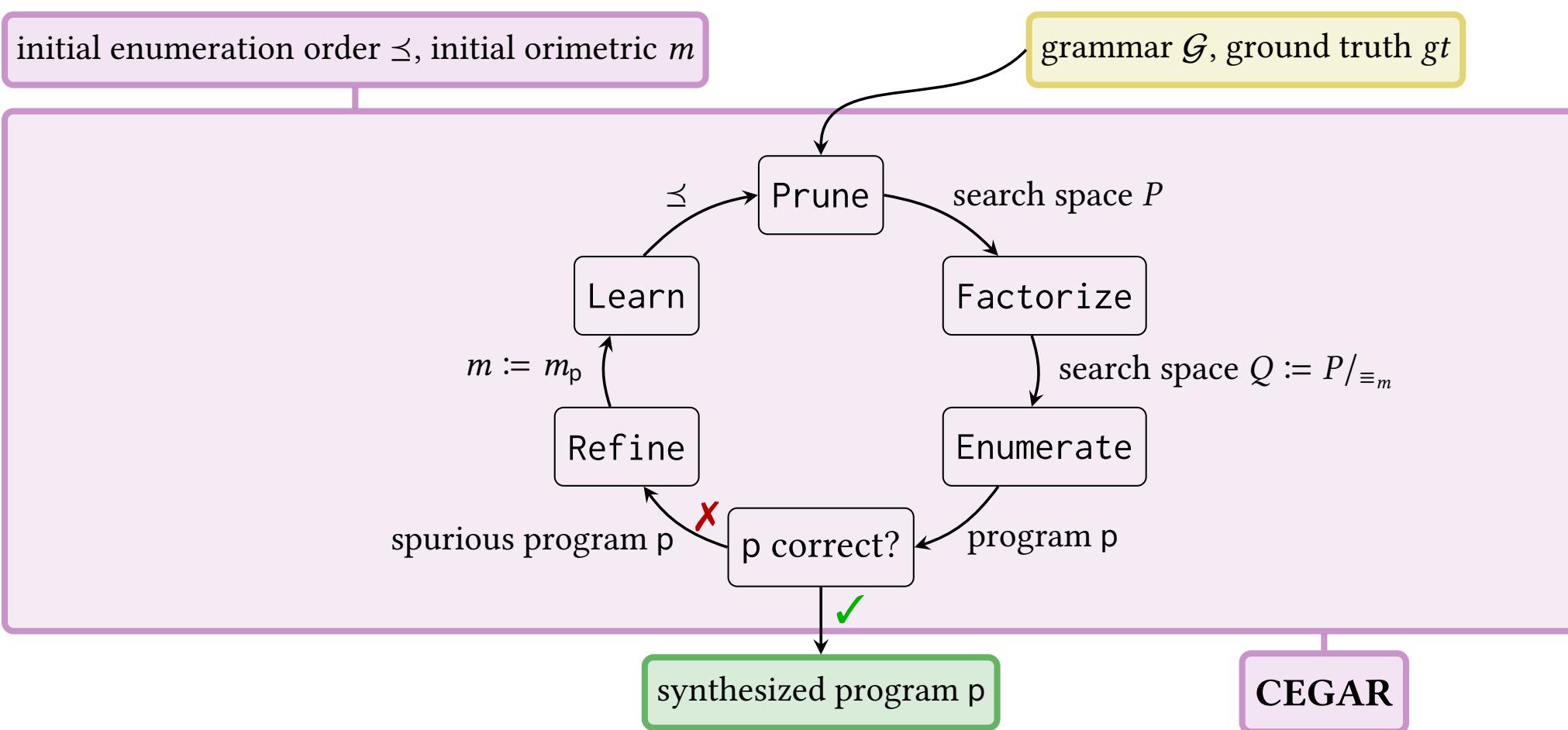
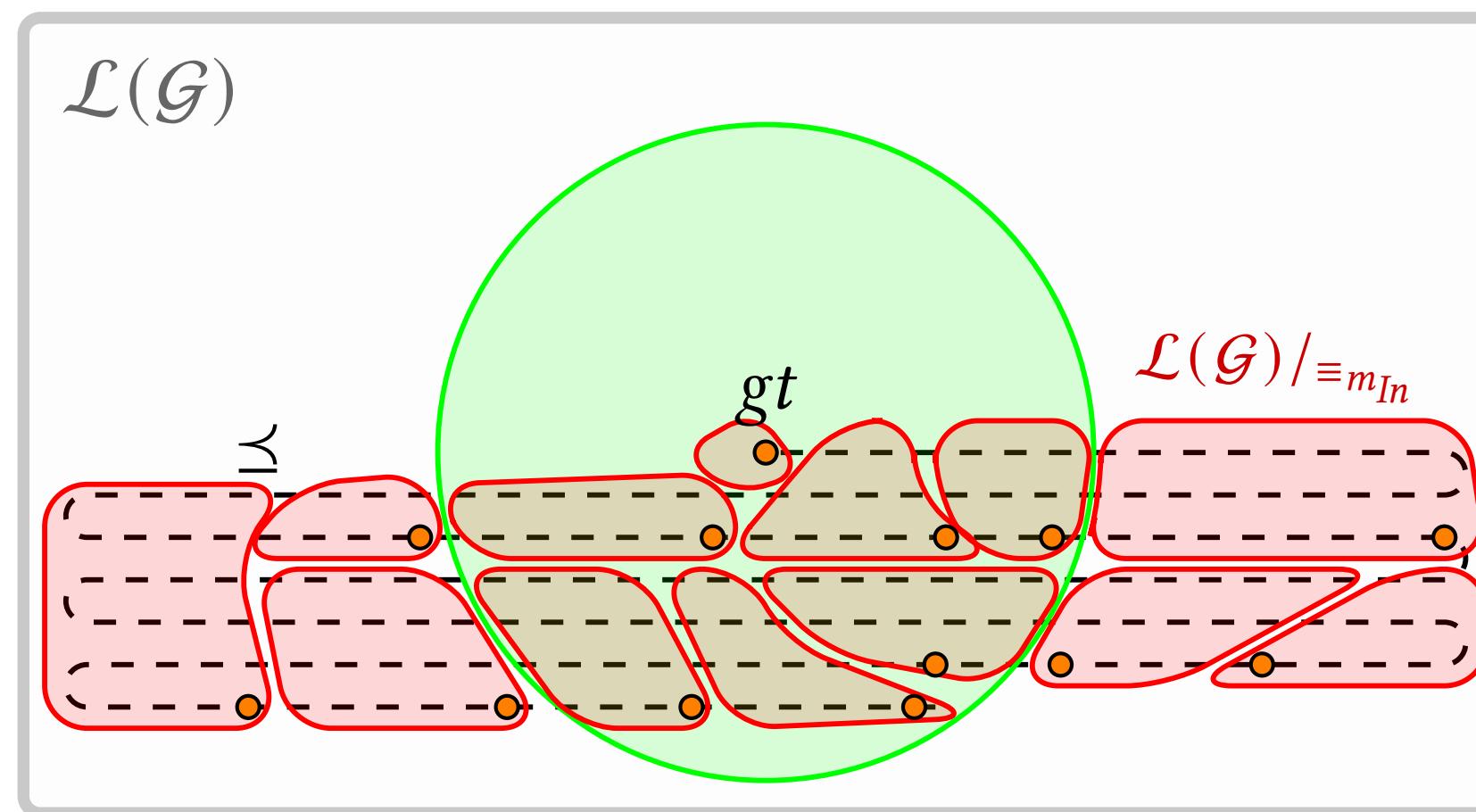
# Substantial impact on performance



# Conclusion

Thank you! Questions?

## Framework for bottom-up enum. synthesis



## Oriented Metrics

$$\begin{aligned}
 m(a, a) &= 0 && \text{(reflexivity)} \\
 m(b, a) = 0 \Rightarrow m(a, b) &= 0 && \text{(symmetry at zero)} \\
 m(a, c) \leq m(a, b) + m(b, c) && \text{(\triangle-inequality)}
 \end{aligned}$$

Substantial impact on performance

