Making Programs Memory Safe

Through Program Synthesis

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Program Synthesis

$$\models \{pre\} \quad prog \quad \{post\}$$

$$\in Progs$$

Program Synthesis

$$\models \{pre\}\ prog\ \{post\}$$

$$\in Progs$$

$Progs$ given as a Sketch:

Program Synthesis

$\models \{pre\}\ prog\ \{post\}$

$\in Progs$

$Progs$ given as a Sketch:

```
N;
x++;
N;
y = 1;    +    y = 2;
M;
```

Program Synthesis

$\models \{pre\}\ prog\ \{post\}$

$\in Progs$

N ::= x = 1 | x = 2

M ::= y++ | M;M

$Progs$ given as a Sketch:

```
N;
x++;
N;
y = 1;   +   y = 2;
M;
```

Two Problems:

1: Is synthesis possible?

Verification - Realizability Logic

Program Synthesis

$\models \{pre\}\ prog\ \{post\}$

$\in Progs$

N ::= x = 1 | x = 2

M ::= y++ | M

$Progs$ given as a Sketch:

```
x = 0;
N;
x++;
y = 1;   +   y = 2;
M;
```

Two Problems:

1: Is synthesis possible?

Verification - Realizability Logic

2: What does the solution look like?

Synthesis - Realization Logic

Realizability Logic

$$\{true\}$$

$$\{x = 2\}$$

Realizability Logic

$\{true\}$

N;

x++;

$\{x = 2\}$

Realizability Logic

$\{true\}$

N;

x++;

$\{x = 2\}$

N ::= x = 1 | x = 2

Realizability Logic

$\exists$prog $\in$

$$\left\{ \begin{array}{l} \{true\} \\ \text{N;} \\ \\ \text{x++;} \\ \{x = 2\} \end{array} \right.$$

N ::= x = 1 | x = 2

$\vDash \{true\}\text{prog}\{x = 2\}$

Realizability Logic

$\exists$prog $\in$ $\left\{ \begin{array}{l} \{true\} \\ \texttt{x = 1;} \\ \\ \texttt{x++;} \\ \{x = 2\} \end{array} \right.$

$\vDash \{true\}\text{prog}\{x = 2\}$ ✅

N ::= x = 1 | x = 2

Realizability Logic

$\exists$prog $\in$

$$\left\{ \begin{array}{l} \{true\} \\ \texttt{x = 1;} \\ \{x = 1\} \\ \texttt{x++;} \\ \{x = 2\} \end{array} \right.$$

$\models \{true\}$prog$\{x = 2\}$ ✅

N ::= x = 1 | x = 2

Realizability Logic

$\exists$prog $\in$

$\{true\}$

N;

x++;

$\{x = 2\}$

$\vDash \{true\}$prog$\{x = 2\}$ ✅

N ::= x = 1 | x = 2

Realizability Logic

$\langle true \rangle$

$\exists$prog $\in$ $\left\{ \begin{array}{l} \text{N;} \\ \\ \text{x++;} \end{array} \right.$

$\langle x = 2 \rangle$

$\vDash \{true\}\text{prog}\{x = 2\}$ ✅

N ::= x = 1 | x = 2

Realizability Logic

$\exists$prog $\in$ $\left\{ \begin{array}{l} \langle true \rangle \\ \text{N;} \\ \langle x = 1, x = 2 \rangle \\ \text{x++;} \\ \langle x = 2 \rangle \end{array} \right.$

N ::= x = 1 | x = 2

$\vDash \{true\}\text{prog}\{x = 2\}$ ✅

Realizability Logic

$\exists$prog $\in$ $\left\{ \begin{array}{l} \langle true \rangle \\ \text{N;} \\ \langle x = 1, x = 2 \rangle \\ \text{x++;} \\ \langle x = 2, x = 3 \rangle \end{array} \right.$

$\vDash \{true\}\text{prog}\{x = 2\}$ ✅

N ::= x = 1 | x = 2

Realizability Logic

$\langle \mathit{true} \rangle$

∃prog ∈ $\left\{ \begin{array}{l} \texttt{x = 1;} \quad + \quad \texttt{x = 2;} \\ \\ \texttt{x++;} \end{array} \right.$

N ::= x = 1 | x = 2

$\models \{\mathit{true}\}\mathrm{prog}\{x = 2\}$

Realizability Logic

$\exists$prog $\in$ $\left\{ \begin{array}{l} \langle true \rangle \\ \texttt{x = 1;} \quad + \quad \texttt{x = 2;} \\ \langle x = 1 \lor x = 2 \rangle \\ \texttt{x++;} \end{array} \right.$

N ::= x = 1 | x = 2

$\models \{true\}\text{prog}\{x = 2\}$

Realizability Logic

$\exists$prog $\in$ $\left\{ \begin{array}{l} \langle true \rangle \\ \texttt{x = 1;} \quad + \quad \texttt{x = 2;} \\ \langle x = 1 \lor x = 2 \rangle \\ \texttt{x++;} \\ \langle x = 2 \lor x = 3 \rangle \end{array} \right.$

N ::= x = 1 | x = 2

$\models \{true\}$prog$\{x = 2\}$

Realizability Logic

$\exists$prog $\in$ $\left\{ \begin{array}{l} \langle true \rangle \\ \texttt{x = 1;} \quad + \quad \texttt{x = 2;} \\ \langle x = 1 \lor x = 2 \rangle \\ \texttt{x++;} \\ \langle x = 2 \lor x = 3 \rangle \end{array} \right.$

$\vDash \{true\}\text{prog}\{x = 2\}$ ✗

N ::= x = 1 | x = 2

Realizability Logic

$\langle true \rangle$

$\exists$prog $\in$ $\left\{ \begin{array}{l} \text{N;} \\ \\ \text{x++;} \quad + \quad \text{skip;} \end{array} \right.$

N ::= x = 1 | x = 2

$\models \{true\}\text{prog}\{x = 2\}$

Realizability Logic

$\exists$prog $\in$ $\left\{ \begin{array}{l} \langle true \rangle \\ \text{N;} \\ \langle x = 1, x = 2 \rangle \\ \text{x++;} \quad + \quad \text{skip;} \end{array} \right.$

N ::= x = 1 | x = 2

$\vDash \{true\}\text{prog}\{x = 2\}$

Realizability Logic

$\exists \text{prog} \in \left\{ \begin{array}{l} \langle true \rangle \\ \text{N}; \\ \langle x = 1, x = 2 \rangle \\ \text{x++;} \quad + \quad \text{skip;} \\ \langle x = 1 \vee x = 2, x = 2 \vee x = 3 \rangle \end{array} \right.$

$\vDash \{true\}\text{prog}\{x = 2\}$

N ::= x = 1 | x = 2

Realizability Logic

$\exists$prog $\in$

$\vDash \{true\}\mathrm{prog}\{x = 2\}$ ✗

$$\left\{\begin{array}{l}\langle true \rangle \\ \mathsf{N}; \\ \langle x = 1, x = 2 \rangle \\ \mathsf{x{+}{+};} \quad + \quad \mathsf{skip;} \\ \langle x = 1 \vee x = 2, x = 2 \vee x = 3 \rangle\end{array}\right.$$

N ::= x = 1 | x = 2

Realizability Logic

$\exists$prog $\in$ $\left\{ \begin{array}{l} \langle true \rangle \\ \text{x = 1;} \quad + \quad \text{x = 2;} \\ \\ \text{N;} \end{array} \right.$

N ::= x++ | skip

$\models \{true\}\text{prog}\{x = 2\}$

Realizability Logic

$$\exists \text{prog} \in \left\{ \begin{array}{l} \langle true \rangle \\ \text{x = 1;} \quad + \quad \text{x = 2;} \\ \langle x = 1 \vee x = 2 \rangle \\ \text{N;} \end{array} \right.$$

N ::= x++ | skip

$$\vDash \{true\}\text{prog}\{x = 2\}$$

Realizability Logic

$\exists$prog $\in$

$\models \{true\}\text{prog}\{x = 2\}$

$\langle true \rangle$

x = 1; **+** x = 2;

$\langle x = 1 \vee x = 2 \rangle$

N;

$\langle x = 2 \vee x = 3, x = 1 \vee x = 2 \rangle$

N ::= x++ | skip

Realizability Logic

$\exists$prog $\in$

$\models \{true\}\text{prog}\{x = 2\}$ ❌

$\langle true \rangle$

x = 1; **+** x = 2;

$\langle x = 1 \vee x = 2 \rangle$

N;

$\langle x = 2 \vee x = 3, x = 1 \vee x = 2 \rangle$

N ::= x++ | skip

Realizability Logic

$\langle true \rangle$

$\exists$prog $\in$ $\left\{ \begin{array}{l} \text{M;} \\ \\ \text{N;} \end{array} \right.$

M ::= x = 1 | x = 2

N ::= x++ | skip

$\vDash \{true\}\text{prog}\{x = 2\}$

Realizability Logic

$$\exists \text{prog} \in \left\{ \begin{array}{l} \langle true \rangle \\ \text{M;} \\ \langle x = 1, x = 2 \rangle \\ \text{N;} \end{array} \right.$$

$$\vDash \{true\}\text{prog}\{x = 2\}$$

M ::= x = 1 | x = 2

N ::= x++ | skip

Realizability Logic

$\exists$prog $\in$

$\langle \mathit{true} \rangle$

M;

$\langle x = 1, x = 2 \rangle$

N;

$\langle x = 1, x = 2, x = 3 \rangle$

$\vDash \{\mathit{true}\}\text{prog}\{x = 2\}$

M ::= x = 1 | x = 2

N ::= x++ | skip

Realizability Logic

$\exists$prog $\in$

$\langle \mathit{true} \rangle$
M;
$\langle x = 1, x = 2 \rangle$
N;
$\langle x = 1, x = 2, x = 3 \rangle$

$\vDash \{\mathit{true}\}\mathrm{prog}\{x = 2\}$ ✅

M ::= x = 1 | x = 2

N ::= x++ | skip

Realizability Logic

$\exists$prog $\in$

$$\left\{ \begin{array}{l} \langle true \rangle \\ \text{M;} \\ \langle x = 1, x = 2 \rangle \\ \text{N;} \\ \langle x = 1, x = 2, x = 3 \rangle \end{array} \right.$$

M ::= x = 1 | x = 2

N ::= x++ | skip

$\models \{true\}$prog$\{x = 2\}$ ✅

What semantics?

Realizability Logic

Set of predicates

$$\models_a \langle R \rangle \text{sketch} \langle S \rangle \qquad \Leftrightarrow$$

Realizability Logic

Set of predicates

$\vDash_a \langle R \rangle \text{sketch} \langle S \rangle \quad \Leftrightarrow$

$\vDash_a \langle true \rangle$ M;N $\langle x = 1, x = 2, x = 3 \rangle$

M ::= x = 1 | x = 2

N ::= x++ | skip

Realizability Logic

Set of predicates

$\vDash_a \langle R \rangle \text{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.$

$\vDash_a \langle true \rangle$ M;N $\langle x = 1, x = 2, x = 3 \rangle$

M ::= x = 1 | x = 2

N ::= x++ | skip

Realizability Logic

Set of predicates

$\vDash_a \langle R \rangle \text{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S . \exists r \in R . \exists \text{prog} \in drv(\text{sketch}) .$

$\vDash_a \langle true \rangle$ M;N $\langle x = 1, x = 2, x = 3 \rangle$

M ::= x = 1 | x = 2

N ::= x++ | skip

Realizability Logic

Set of predicates

$\vDash_a \langle R \rangle \text{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \text{prog} \in drv(\text{sketch}).\ \vDash_d \{r\}\text{prog}\{s\}$

$\vDash_a \langle true \rangle$ M;N $\langle x = 1, x = 2, x = 3 \rangle$

M ::= x = 1 | x = 2

N ::= x++ | skip

Realizability Logic

Set of predicates

$\vDash_a \langle R \rangle \text{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S . \exists r \in R . \exists \text{prog} \in drv(\text{sketch}) . \vDash_d \{r\} \text{prog} \{s\}$

demonic (standard Hoare)

$\vDash_a \langle true \rangle$ M;N $\langle x = 1, x = 2, x = 3 \rangle$

M ::= x = 1 | x = 2

N ::= x++ | skip

Realizability Logic

Set of predicates

$$\vDash_a \langle R \rangle \text{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \text{prog} \in drv(\text{sketch}).\ \vDash_d \{r\}\text{prog}\{s\}$$

angelic

demonic (standard Hoare)

$$\vDash_a \langle true \rangle \quad \text{M;N} \quad \langle x = 1, x = 2, x = 3 \rangle$$

M ::= x = 1 | x = 2

N ::= x++ | skip

Realizability Logic

$\langle true \rangle$

N

x++; **+** skip;

N ::= x = 1 | x = 2

Realizability Logic

$$(\textbf{COM})\ \frac{}{\vdash_a \langle \mathit{true} \rangle \texttt{x = 1;} \langle x = 1 \rangle}$$

$\langle \mathit{true} \rangle$

N

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\text{(ANG)}\ \dfrac{\text{(COM)}\ \dfrac{}{\vdash_a \langle true \rangle \mathtt{x\ =\ 1;} \langle x = 1 \rangle}}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle}$$

$\langle true \rangle$

N

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\text{(ANG)}\ \dfrac{\text{(COM)}\ \dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle} \qquad \dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\ \text{(COM)}}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle}$$

$\langle true \rangle$

N

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\text{(ANG)}\ \dfrac{\text{(COM)}\ \dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle} \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\ \text{(COM)}}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}\ \text{(ANG)}$$

$\langle true \rangle$

N

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$(\text{ANG})\ \dfrac{(\text{COM})\ \dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle} \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\ (\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}\ (\text{ANG})$$

$\langle true \rangle$

N

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}\ (\text{COM})}{\vdash_a \langle true \rangle \, \mathsf{N} \, \langle x = 1 \rangle}\ (\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\ (\text{COM})}{\vdash_a \langle true \rangle \, \mathsf{N} \, \langle x = 2 \rangle}\ (\text{ANG})$$

$$\dfrac{\vdash_a \langle true \rangle \, \mathsf{N} \, \langle x = 1 \rangle \qquad \vdash_a \langle true \rangle \, \mathsf{N} \, \langle x = 2 \rangle}{\vdash_a \langle true \rangle \, \mathsf{N} \, \langle x = 1 \underline{,}\ x = 2 \rangle}\ (\text{GATHER})$$

1️⃣

$\langle true \rangle$

N

$\langle x = 1 \underline{,}\ x = 2 \rangle$

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 1;} \langle x = 1\rangle}\ (\text{COM})}{\vdash_a \langle true\rangle \, N \, \langle x = 1\rangle}\ (\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 2;} \langle x = 2\rangle}\ (\text{COM})}{\vdash_a \langle true\rangle \, N \, \langle x = 2\rangle}\ (\text{ANG})$$

$$\text{(GATHER)}\ \ \vdash_a \langle true\rangle \, N \, \langle x = 1, x = 2\rangle$$

1

$\langle true\rangle$

N

$\langle x = 1, x = 2\rangle$

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}\,(\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle}\,(\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\,(\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}\,(\text{ANG})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1, x = 2 \rangle}\,(\text{GATHER})$$

1

$$(\text{COM})\ \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \langle x = 2 \rangle}$$

$\langle true \rangle$

N

$\langle x = 1, x = 2 \rangle$

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}\,(\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle}\,(\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\,(\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}\,(\text{ANG})$$

$$\dfrac{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle \qquad \vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1, x = 2 \rangle}\,(\text{GATHER})$$

1

$$(\text{COM})\,\dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \langle x = 2 \rangle} \qquad \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{skip;} \langle x = 1 \rangle}\,(\text{COM})$$

$\langle true \rangle$

N

$\langle x = 1, x = 2 \rangle$

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}\ (\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle}\ (\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\ (\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}\ (\text{ANG})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1, x = 2 \rangle}\ (\text{GATHER}) \quad 1$$

$$\dfrac{\dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \langle x = 2 \rangle}\ (\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{skip;} \langle x = 1 \rangle}\ (\text{COM})}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \ + \ \texttt{skip;} \langle x = 1 \lor x = 2 \rangle}\ (\text{DEM}) \quad 2$$

$\langle true \rangle$

N

$\langle x = 1, x = 2 \rangle$

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}\ (\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle}\ (\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\ (\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}\ (\text{ANG})$$

$$\dfrac{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle \qquad \vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1, x = 2 \rangle}\ (\text{GATHER}) \quad \boxed{1}$$

$$\dfrac{\dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \langle x = 2 \rangle}\ (\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{skip;} \langle x = 1 \rangle}\ (\text{COM})}{\vdash_a \langle x = 1 \rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \vee x = 2 \rangle}\ (\text{DEM}) \quad \boxed{2}$$

$\langle true \rangle$

N

$\langle x = 1, x = 2 \rangle$

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}\ (\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle}\ (\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\ (\text{COM})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}\ (\text{ANG})}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1,\ x = 2 \rangle}\ (\text{GATHER}) \quad 1$$

$$\dfrac{\dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \langle x = 2 \rangle}\ (\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{skip;} \langle x = 1 \rangle}\ (\text{COM})}{\vdash_a \langle x = 1 \rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \lor x = 2 \rangle}\ (\text{DEM}) \quad 2$$

$$\dfrac{}{\vdash_a \langle x = 2 \rangle \texttt{x++;} \langle x = 3 \rangle}\ (\text{COM})$$

$\langle true \rangle$
N
$\langle x = 1,\ x = 2 \rangle$
x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 1;} \langle x = 1\rangle}\,(\text{COM})}{\vdash_a \langle true\rangle\, \mathsf{N}\, \langle x = 1\rangle}\,(\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 2;} \langle x = 2\rangle}\,(\text{COM})}{\vdash_a \langle true\rangle\, \mathsf{N}\, \langle x = 2\rangle}\,(\text{ANG})}{\vdash_a \langle true\rangle\, \mathsf{N}\, \langle x = 1, x = 2\rangle}\,(\text{GATHER})$$

1

$$\dfrac{\dfrac{}{\vdash_a \langle x = 1\rangle \texttt{x++;} \langle x = 2\rangle}\,(\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 1\rangle \texttt{skip;} \langle x = 1\rangle}\,(\text{COM})}{\vdash_a \langle x = 1\rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \vee x = 2\rangle}\,(\text{DEM})$$

2

$$\dfrac{}{\vdash_a \langle x = 2\rangle \texttt{x++;} \langle x = 3\rangle}\,(\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 2\rangle \texttt{skip;} \langle x = 2\rangle}\,(\text{COM})$$

$\langle true\rangle$
N
$\langle x = 1, x = 2\rangle$
x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{\overline{\vdash_a \langle true\rangle \texttt{x = 1;} \langle x = 1\rangle}\ (\text{COM})}{\vdash_a \langle true\rangle\, \mathsf{N}\, \langle x = 1\rangle}\ (\text{ANG}) \qquad \dfrac{\overline{\vdash_a \langle true\rangle \texttt{x = 2;} \langle x = 2\rangle}\ (\text{COM})}{\vdash_a \langle true\rangle\, \mathsf{N}\, \langle x = 2\rangle}\ (\text{ANG})}{\vdash_a \langle true\rangle\, \mathsf{N}\, \langle x = 1, x = 2\rangle}\ (\text{GATHER}) \quad 1$$

$$\dfrac{\overline{\vdash_a \langle x = 1\rangle \texttt{x++;} \langle x = 2\rangle}\ (\text{COM}) \qquad \overline{\vdash_a \langle x = 1\rangle \texttt{skip;} \langle x = 1\rangle}\ (\text{COM})}{\vdash_a \langle x = 1\rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \vee x = 2\rangle}\ (\text{DEM}) \quad 2$$

$$\dfrac{\overline{\vdash_a \langle x = 2\rangle \texttt{x++;} \langle x = 3\rangle}\ (\text{COM}) \qquad \overline{\vdash_a \langle x = 2\rangle \texttt{skip;} \langle x = 2\rangle}\ (\text{COM})}{\vdash_a \langle x = 2\rangle \texttt{x++;} + \texttt{skip;} \langle x = 2 \vee x = 3\rangle}\ (\text{DEM}) \quad 3$$

$\langle true\rangle$
N
$\langle x = 1, x = 2\rangle$
x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 1;} \langle x = 1\rangle}\ \text{(COM)}}{\vdash_a \langle true\rangle\ \mathsf{N}\ \langle x = 1\rangle}\ \text{(ANG)} \qquad \dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 2;} \langle x = 2\rangle}\ \text{(COM)}}{\vdash_a \langle true\rangle\ \mathsf{N}\ \langle x = 2\rangle}\ \text{(ANG)}$$

$$\text{(GATHER)}\ \vdash_a \langle true\rangle\ \mathsf{N}\ \langle x = 1, x = 2\rangle \quad \boxed{1}$$

$$\text{(DEM)}\ \dfrac{\dfrac{}{\vdash_a \langle x = 1\rangle \texttt{x++;} \langle x = 2\rangle}\ \text{(COM)} \qquad \dfrac{}{\vdash_a \langle x = 1\rangle \texttt{skip;} \langle x = 1\rangle}\ \text{(COM)}}{\vdash_a \langle x = 1\rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \lor x = 2\rangle} \quad \boxed{2}$$

$$\text{(DEM)}\ \dfrac{\dfrac{}{\vdash_a \langle x = 2\rangle \texttt{x++;} \langle x = 3\rangle}\ \text{(COM)} \qquad \dfrac{}{\vdash_a \langle x = 2\rangle \texttt{skip;} \langle x = 2\rangle}\ \text{(COM)}}{\vdash_a \langle x = 2\rangle \texttt{x++;} + \texttt{skip;} \langle x = 2 \lor x = 3\rangle} \quad \boxed{3}$$

$\langle true\rangle$
N
$\langle x = 1, x = 2\rangle$
x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 1;} \langle x = 1\rangle}\ (\text{COM})}{\vdash_a \langle true\rangle\ \mathsf{N}\ \langle x = 1\rangle}\ (\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 2;} \langle x = 2\rangle}\ (\text{COM})}{\vdash_a \langle true\rangle\ \mathsf{N}\ \langle x = 2\rangle}\ (\text{ANG})}{\vdash_a \langle true\rangle\ \mathsf{N}\ \langle x = 1,\ x = 2\rangle}\ (\text{GATHER}) \quad 1$$

$$\dfrac{\dfrac{}{\vdash_a \langle x = 1\rangle \texttt{x++;} \langle x = 2\rangle}\ (\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 1\rangle \texttt{skip;} \langle x = 1\rangle}\ (\text{COM})}{\vdash_a \langle x = 1\rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \vee x = 2\rangle}\ (\text{DEM}) \quad 2$$

$$\dfrac{\dfrac{}{\vdash_a \langle x = 2\rangle \texttt{x++;} \langle x = 3\rangle}\ (\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 2\rangle \texttt{skip;} \langle x = 2\rangle}\ (\text{COM})}{\vdash_a \langle x = 2\rangle \texttt{x++;} + \texttt{skip;} \langle x = 2 \vee x = 3\rangle}\ (\text{DEM}) \quad 3$$

2

$\langle true\rangle$
N
$\langle x = 1,\ x = 2\rangle$
x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}\,(\text{COM})}{\vdash_a \langle true \rangle\ \mathsf{N}\ \langle x = 1 \rangle}\,(\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\,(\text{COM})}{\vdash_a \langle true \rangle\ \mathsf{N}\ \langle x = 2 \rangle}\,(\text{ANG})}{\vdash_a \langle true \rangle\ \mathsf{N}\ \langle x = 1, x = 2 \rangle}\,(\text{GATHER}) \quad 1$$

$$(\text{DEM})\ \dfrac{\dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \langle x = 2 \rangle}\,(\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{skip;} \langle x = 1 \rangle}\,(\text{COM})}{\vdash_a \langle x = 1 \rangle \texttt{x++;}\ +\ \texttt{skip;} \langle x = 1 \vee x = 2 \rangle} \quad 2$$

$$(\text{DEM})\ \dfrac{\dfrac{}{\vdash_a \langle x = 2 \rangle \texttt{x++;} \langle x = 3 \rangle}\,(\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 2 \rangle \texttt{skip;} \langle x = 2 \rangle}\,(\text{COM})}{\vdash_a \langle x = 2 \rangle \texttt{x++;}\ +\ \texttt{skip;} \langle x = 2 \vee x = 3 \rangle} \quad 3$$

2 3

$\langle true \rangle$
N
$\langle x = 1, x = 2 \rangle$
x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$
\dfrac{
  \dfrac{
    \dfrac{}{\vdash_a \langle \mathit{true} \rangle \texttt{x = 1;} \langle x = 1 \rangle}\,(\text{COM})
  }{\vdash_a \langle \mathit{true} \rangle\, \mathsf{N}\, \langle x = 1 \rangle}\,(\text{ANG})
  \qquad
  \dfrac{
    \dfrac{}{\vdash_a \langle \mathit{true} \rangle \texttt{x = 2;} \langle x = 2 \rangle}\,(\text{COM})
  }{\vdash_a \langle \mathit{true} \rangle\, \mathsf{N}\, \langle x = 2 \rangle}\,(\text{ANG})
}{\vdash_a \langle \mathit{true} \rangle\, \mathsf{N}\, \langle x = 1,\ x = 2 \rangle}\,(\text{GATHER}) \quad \boxed{1}
$$

$$
(\text{DEM})\,\dfrac{
  \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \langle x = 2 \rangle}\,(\text{COM})
  \qquad
  \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{skip;} \langle x = 1 \rangle}\,(\text{COM})
}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \;+\; \texttt{skip;} \langle x = 1 \vee x = 2 \rangle} \quad \boxed{2}
$$

$$
(\text{DEM})\,\dfrac{
  \dfrac{}{\vdash_a \langle x = 2 \rangle \texttt{x++;} \langle x = 3 \rangle}\,(\text{COM})
  \qquad
  \dfrac{}{\vdash_a \langle x = 2 \rangle \texttt{skip;} \langle x = 2 \rangle}\,(\text{COM})
}{\vdash_a \langle x = 2 \rangle \texttt{x++;} \;+\; \texttt{skip;} \langle x = 2 \vee x = 3 \rangle} \quad \boxed{3}
$$

$\boxed{2}$ $\boxed{3}$

$\langle \mathit{true} \rangle$
N
$\langle x = 1,\ x = 2 \rangle$
x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\textbf{(GATHER)}\ \dfrac{\textbf{(ANG)}\ \dfrac{\textbf{(COM)}\ \dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1 \rangle} \qquad \textbf{(ANG)}\ \dfrac{\textbf{(COM)}\ \dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 2 \rangle}}{\vdash_a \langle true \rangle\, \mathsf{N}\, \langle x = 1, x = 2 \rangle}\ \boxed{1}$$

$$\textbf{(DEM)}\ \dfrac{\textbf{(COM)}\ \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \langle x = 2 \rangle} \qquad \textbf{(COM)}\ \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{skip;} \langle x = 1 \rangle}}{\vdash_a \langle x = 1 \rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \vee x = 2 \rangle}\ \boxed{2}$$

$$\textbf{(DEM)}\ \dfrac{\textbf{(COM)}\ \dfrac{}{\vdash_a \langle x = 2 \rangle \texttt{x++;} \langle x = 3 \rangle} \qquad \textbf{(COM)}\ \dfrac{}{\vdash_a \langle x = 2 \rangle \texttt{skip;} \langle x = 2 \rangle}}{\vdash_a \langle x = 2 \rangle \texttt{x++;} + \texttt{skip;} \langle x = 2 \vee x = 3 \rangle}\ \boxed{3}$$

$$\textbf{(GATHER)}\ \dfrac{\boxed{2} \qquad \boxed{3}}{\vdash_a \langle x = 1, x = 2 \rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \vee x = 2, x = 2 \vee x = 3 \rangle}\ \boxed{4}$$

$\langle true \rangle$
N
$\langle x = 1, x = 2 \rangle$
x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$
\text{(GATHER)}\;\dfrac{\text{(ANG)}\;\dfrac{\text{(COM)}\;\dfrac{}{\vdash_a \langle \mathit{true}\rangle \mathtt{x = 1;} \langle x = 1\rangle}}{\vdash_a \langle \mathit{true}\rangle \, N \, \langle x = 1\rangle} \qquad \dfrac{\dfrac{}{\vdash_a \langle \mathit{true}\rangle \mathtt{x = 2;} \langle x = 2\rangle}\;\text{(COM)}}{\vdash_a \langle \mathit{true}\rangle \, N \, \langle x = 2\rangle}\;\text{(ANG)}}{\vdash_a \langle \mathit{true}\rangle \, N \, \langle x = 1, x = 2\rangle}\;\boxed{1}
$$

$$
\text{(DEM)}\;\dfrac{\text{(COM)}\;\dfrac{}{\vdash_a \langle x = 1\rangle \mathtt{x{+}{+};} \langle x = 2\rangle} \qquad \dfrac{}{\vdash_a \langle x = 1\rangle \mathtt{skip;} \langle x = 1\rangle}\;\text{(COM)}}{\vdash_a \langle x = 1\rangle \mathtt{x{+}{+};} \; + \; \mathtt{skip;} \langle x = 1 \lor x = 2\rangle}\;\boxed{2}
$$

$$
\text{(DEM)}\;\dfrac{\text{(COM)}\;\dfrac{}{\vdash_a \langle x = 2\rangle \mathtt{x{+}{+};} \langle x = 3\rangle} \qquad \dfrac{}{\vdash_a \langle x = 2\rangle \mathtt{skip;} \langle x = 2\rangle}\;\text{(COM)}}{\vdash_a \langle x = 2\rangle \mathtt{x{+}{+};} \; + \; \mathtt{skip;} \langle x = 2 \lor x = 3\rangle}\;\boxed{3}
$$

$$
\text{(GATHER)}\;\dfrac{\boxed{2} \qquad \boxed{3}}{\vdash_a \langle x = 1, x = 2\rangle \mathtt{x{+}{+};} \; + \; \mathtt{skip;} \langle x = 1 \lor x = 2, x = 2 \lor x = 3\rangle}\;\boxed{4}
$$

$\langle \mathit{true} \rangle$

N

$\langle x = 1, x = 2\rangle$

x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 1;} \langle x = 1\rangle}\;(\text{COM})}{\vdash_a \langle true\rangle\, \mathsf{N}\, \langle x = 1\rangle}\;(\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true\rangle \texttt{x = 2;} \langle x = 2\rangle}\;(\text{COM})}{\vdash_a \langle true\rangle\, \mathsf{N}\, \langle x = 2\rangle}\;(\text{ANG})}{\vdash_a \langle true\rangle\, \mathsf{N}\, \langle x = 1,\ x = 2\rangle}\;(\text{GATHER})\quad 1$$

$$\dfrac{\dfrac{}{\vdash_a \langle x = 1\rangle \texttt{x++;} \langle x = 2\rangle}\;(\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 1\rangle \texttt{skip;} \langle x = 1\rangle}\;(\text{COM})}{\vdash_a \langle x = 1\rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \vee x = 2\rangle}\;(\text{DEM})\quad 2$$

$$\dfrac{\dfrac{}{\vdash_a \langle x = 2\rangle \texttt{x++;} \langle x = 3\rangle}\;(\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 2\rangle \texttt{skip;} \langle x = 2\rangle}\;(\text{COM})}{\vdash_a \langle x = 2\rangle \texttt{x++;} + \texttt{skip;} \langle x = 2 \vee x = 3\rangle}\;(\text{DEM})\quad 3$$

$$\dfrac{2 \qquad 3}{\vdash_a \langle x = 1, x = 2\rangle \texttt{x++;} + \texttt{skip;} \langle x = 1 \vee x = 2,\ x = 2 \vee x = 3\rangle}\;(\text{GATHER})\quad 4$$

1

$\langle true\rangle$
N
$\langle x = 1, x = 2\rangle$
x++; + skip;

N ::= x = 1 | x = 2

Realizability Logic

$$\dfrac{\dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 1;} \langle x = 1 \rangle}\,(\text{COM})}{\vdash_a \langle true \rangle \, \mathsf{N} \, \langle x = 1 \rangle}\,(\text{ANG}) \qquad \dfrac{\dfrac{}{\vdash_a \langle true \rangle \texttt{x = 2;} \langle x = 2 \rangle}\,(\text{COM})}{\vdash_a \langle true \rangle \, \mathsf{N} \, \langle x = 2 \rangle}\,(\text{ANG})}{\vdash_a \langle true \rangle \, \mathsf{N} \, \langle x = 1, \; x = 2 \rangle}\,(\text{GATHER}) \; \boxed{1}$$

$$\dfrac{\dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \langle x = 2 \rangle}\,(\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 1 \rangle \texttt{skip;} \langle x = 1 \rangle}\,(\text{COM})}{\vdash_a \langle x = 1 \rangle \texttt{x++;} \; + \; \texttt{skip;} \langle x = 1 \vee x = 2 \rangle}\,(\text{DEM}) \; \boxed{2}$$

$$\dfrac{\dfrac{}{\vdash_a \langle x = 2 \rangle \texttt{x++;} \langle x = 3 \rangle}\,(\text{COM}) \qquad \dfrac{}{\vdash_a \langle x = 2 \rangle \texttt{skip;} \langle x = 2 \rangle}\,(\text{COM})}{\vdash_a \langle x = 2 \rangle \texttt{x++;} \; + \; \texttt{skip;} \langle x = 2 \vee x = 3 \rangle}\,(\text{DEM}) \; \boxed{3}$$

$$\dfrac{\boxed{2} \qquad \boxed{3}}{\vdash_a \underbrace{\langle x = 1, x = 2 \rangle}_{\boxed{1}} \texttt{x++;} \; + \; \texttt{skip;} \underbrace{\langle x = 1 \vee x = 2, \; x = 2 \vee x = 3 \rangle}_{\boxed{4}}}\,(\text{GATHER}) \; \boxed{4}$$

$\langle true \rangle$
N
$\langle x = 1, x = 2 \rangle$
$\texttt{x++;} \; + \; \texttt{skip;}$

N ::= x = 1 | x = 2

Realizability Logic

$$\text{(GATHER)}\ \dfrac{\text{(ANG)}\ \dfrac{\text{(COM)}\ \dfrac{}{\vdash_a \langle \mathit{true} \rangle \mathtt{x\ =\ 1;} \langle x = 1 \rangle}}{\vdash_a \langle \mathit{true} \rangle\, \mathsf{N}\, \langle x = 1 \rangle} \qquad \text{(ANG)}\ \dfrac{\text{(COM)}\ \dfrac{}{\vdash_a \langle \mathit{true} \rangle \mathtt{x\ =\ 2;} \langle x = 2 \rangle}}{\vdash_a \langle \mathit{true} \rangle\, \mathsf{N}\, \langle x = 2 \rangle}}{\vdash_a \langle \mathit{true} \rangle\, \mathsf{N}\, \langle x = 1, x = 2 \rangle}\ \boxed{1}$$

$$\text{(DEM)}\ \dfrac{\text{(COM)}\ \dfrac{}{\vdash_a \langle x = 1 \rangle \mathtt{x++;} \langle x = 2 \rangle} \qquad \text{(COM)}\ \dfrac{}{\vdash_a \langle x = 1 \rangle \mathtt{skip;} \langle x = 1 \rangle}}{\vdash_a \langle x = 1 \rangle \mathtt{x++;}\ +\ \mathtt{skip;} \langle x = 1 \lor x = 2 \rangle}\ \boxed{2}$$

$$\text{(DEM)}\ \dfrac{\text{(COM)}\ \dfrac{}{\vdash_a \langle x = 2 \rangle \mathtt{x++;} \langle x = 3 \rangle} \qquad \text{(COM)}\ \dfrac{}{\vdash_a \langle x = 2 \rangle \mathtt{skip;} \langle x = 2 \rangle}}{\vdash_a \langle x = 2 \rangle \mathtt{x++;}\ +\ \mathtt{skip;} \langle x = 2 \lor x = 3 \rangle}\ \boxed{3}$$

$$\text{(GATHER)}\ \dfrac{\boxed{2} \qquad \boxed{3}}{\vdash_a \langle x = 1, x = 2 \rangle \mathtt{x++;}\ +\ \mathtt{skip;} \langle x = 1 \lor x = 2, x = 2 \lor x = 3 \rangle}\ \boxed{4}$$

$$\text{(SEQ)}\ \dfrac{\boxed{1} \qquad \boxed{4}}{\vdash_a \langle \mathit{true} \rangle\, \mathsf{N}; (\mathtt{x++;}\ +\ \mathtt{skip;}) \langle x = 1 \lor x = 2, x = 2 \lor x = 3 \rangle}$$

$\langle \mathit{true} \rangle$

N

$\langle x = 1, x = 2 \rangle$

x++; + skip;

$\langle x = 1 \lor x = 2, x = 2 \lor x = 3 \rangle$

N ::= x = 1 | x = 2

Contribution 1:

Realizability Logic

$$\vdash_a \langle R \rangle \texttt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \models_a \langle R \rangle \texttt{sketch} \langle S \rangle$$

Contribution 1:

Realizability Logic

$$\vdash_a \langle R \rangle \texttt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \models_a \langle R \rangle \texttt{sketch} \langle S \rangle$$

But what makes this efficient?

Realizability Logic - Secret Sauce

$\langle x = 0 \rangle$

N;

N;

N;

N ::= x++ | x--

Realizability Logic - Secret Sauce

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

N;

N ::= x++ | x--

Realizability Logic - Secret Sauce

$$\langle x = 0 \rangle$$

N;

$$\langle x = 1, x = -1 \rangle$$

N;

$$\langle x = 2, x = 0, x = -2 \rangle$$

N;

N ::= x++ | x--

Realizability Logic - Secret Sauce

$$\langle x = 0 \rangle$$

N;

$$\langle x = 1, x = -1 \rangle$$

N;

$$\langle x = 2, x = 0, x = -2 \rangle$$

N;

$$\langle x = 3, x = 1, x = -1, x = -3 \rangle$$

N ::= x++ | x--

Realizability Logic - Secret Sauce

$$\langle x = 0 \rangle$$

N;

$$\langle x = 1, x = -1 \rangle$$

N;

$$\langle x = 2, x = 0, x = -2 \rangle$$

N;

$$\langle x = 3, x = 1, x = -1, x = -3 \rangle$$

N ::= x++ | x--

8 Programs vs. 4 Predicates

Realizability Logic - Secret Sauce

$$\langle x = 0 \rangle$$
N;
$$\langle x = 1, x = -1 \rangle$$
N;
$$\langle x = 2, x = 0, x = -2 \rangle$$
N;
$$\langle x = 3, x = 1, x = -1, x = -3 \rangle$$

N ::= x++ | x--

8 Programs vs. 4 Predicates

Problem: We forgot the program!

Realizability Logic - Secret Sauce

Solution:

<u>Realization Logic</u>

Rewrite proof to derive program

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -$ …

N;

$\langle x = 2, x$ …

N;

$\langle x = 3, x = 1, x = -1, x = -3 \rangle$

N ::= x++ | x--

8 Programs vs. 4 Predicates

Problem: We forgot the program!

Realization Logic

N ::= x++ | x--

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

$\langle x = 2, x = 0, x = -2 \rangle$

N;

$\langle x = 3, x = 1, x = -1, x = -3 \rangle$

Realization Logic

N ::= x++ | x--

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

$\langle x = 2, x = 0, x = -2 \rangle$

N;

$\langle x = 3, x = 1, x = -1, x = -3 \rangle$

Realization Logic

N ::= x++ | x--

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

$\langle x = 2, x = 0, x = -2 \rangle$

N;

$\langle x = 3, x = 1, x = -1, x = -3 \rangle$

$\models_a \langle R \rangle \texttt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \texttt{prog} \in drv(\texttt{sketch}).\ \models_d \{r\}\texttt{prog}\{s\}$.

Realization Logic

N ::= x++ | x--

$$\langle x = 2, x = 0, x = -2 \rangle$$

N;

$$\langle x = 3, x = 1, x = -1, x = -3 \rangle$$

$$\langle x = 0 \rangle$$

N;

$$\langle x = 1, x = -1 \rangle$$

N;

$$\langle x = 2, x = 0, x = -2 \rangle$$

N;

$$\langle x = 3, x = 1, x = -1, x = -3 \rangle$$

$$\models_a \langle R \rangle \mathtt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \mathtt{prog} \in drv(\mathtt{sketch}).\ \models_d \{r\}\mathtt{prog}\{s\}\ .$$

Realization Logic

N ::= x++ | x--

$$\langle x = 2, x = 0, x = -2 \rangle$$
N;
$$\langle x = 3, x = 1, x = -1, x = -3 \rangle$$
$$\mid\!\sim$$

$$\langle x = 0 \rangle$$
N;
$$\langle x = 1, x = -1 \rangle$$
N;
$$\langle x = 2, x = 0, x = -2 \rangle$$
N;
$$\langle x = 3, x = 1, x = -1, x = -3 \rangle$$

$$\models_a \langle R \rangle \mathtt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \mathtt{prog} \in drv(\mathtt{sketch}).\ \models_d \{r\}\mathtt{prog}\{s\}\ .$$

Realization Logic

N ::= x++ | x--

$\langle x = 2, x = 0, x = -2 \rangle$

N;

$\langle x = 3, x = 1, x = -1, x = -3 \rangle$

$\mid\!\sim$

$\langle x = 2, x = 0, x = -2 \rangle$

N;

$\langle x = 1 \rangle$

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

$\langle x = 2, x = 0, x = -2 \rangle$

N;

$\langle x = 3, x = 1, x = -1, x = -3 \rangle$

$\models_a \langle R \rangle \mathtt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \mathtt{prog} \in drv(\mathtt{sketch}).\ \models_d \{r\}\mathtt{prog}\{s\}$.

Realization Logic

N ::= x++ | x--

$$\langle x = 2\underline{,}\ x = 0\underline{,}\ x = -2 \rangle$$

N;

$$\langle x = 3\underline{,}\ x = 1\underline{,}\ x = -1\underline{,}\ x = -3 \rangle$$

$$\mathrel{|\!\!\sim}$$

$$\langle x = 2\underline{,}\ x = 0\underline{,}\ x = -2 \rangle$$

N;

$$\langle x = 1 \rangle$$

$$\langle x = 0 \rangle$$

N;

$$\langle x = 1\underline{,}\ x = -1 \rangle$$

N;

$$\langle x = 2\underline{,}\ x = 0\underline{,}\ x = -2 \rangle$$

N;

$$\langle x = 1 \rangle$$

$$\models_a \langle R \rangle \mathtt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \mathtt{prog} \in drv(\mathtt{sketch}).\ \models_d \{r\}\mathtt{prog}\{s\}\ .$$

Realization Logic

N ::= x++ | x--

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

$\langle x = 2, x = 0, x = -2 \rangle$

N;

$\langle x = 1 \rangle$

$\models_a \langle R \rangle \mathtt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \mathtt{prog} \in drv(\mathtt{sketch}).\ \models_d \{r\} \mathtt{prog} \{s\}\ .$

Realization Logic

N ::= x++ | x--

$\langle x = 2, x = 0, x = -2 \rangle$

N;

$\langle x = 1 \rangle$

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

$\langle x = 2, x = 0, x = -2 \rangle$

N;

$\langle x = 1 \rangle$

$$\models_a \langle R \rangle \texttt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \texttt{prog} \in drv(\texttt{sketch}).\ \models_d \{r\}\texttt{prog}\{s\}\ .$$

Realization Logic

N ::= x++ | x--

$$\langle x = 2, x = 0, x = -2 \rangle$$

N;

$$\langle x = 1 \rangle$$

$$\mid\!\sim$$

$$\langle x = 2, x = 0, x = -2 \rangle$$

x++;

$$\langle x = 1 \rangle$$

$$\langle x = 0 \rangle$$

N;

$$\langle x = 1, x = -1 \rangle$$

N;

$$\langle x = 2, x = 0, x = -2 \rangle$$

N;

$$\langle x = 1 \rangle$$

$$\models_a \langle R \rangle \mathtt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \mathtt{prog} \in drv(\mathtt{sketch}).\ \models_d \{r\}\mathtt{prog}\{s\}\ .$$

Realization Logic

N ::= x++ | x--

$\langle x = 2 \underline{,} \; x = 0 \underline{,} \; x = -2 \rangle$

N;

$\langle x = 1 \rangle$

$\mid\!\sim$

$\langle x = 2 \underline{,} \; x = 0 \underline{,} \; x = -2 \rangle$

x++;

$\langle x = 1 \rangle$

$\langle x = 0 \rangle$

N;

$\langle x = 1 \underline{,} \; x = -1 \rangle$

N;

$\langle x = 2 \underline{,} \; x = 0 \underline{,} \; x = -2 \rangle$

x++;

$\langle x = 1 \rangle$

$$\models_a \langle R \rangle \mathtt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \mathtt{prog} \in drv(\mathtt{sketch}).\ \models_d \{r\}\mathtt{prog}\{s\}\ .$$

Realization Logic

N ::= x++ | x--

$\langle x = 0\rangle$

N;

$\langle x = 1, x = -1\rangle$

N;

$\langle x = 2, x = 0, x = -2\rangle$

x++;

$\langle x = 1\rangle$

$$\models_a \langle R\rangle \texttt{sketch}\langle S\rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \texttt{prog} \in drv(\texttt{sketch}).\ \models_d \{r\}\texttt{prog}\{s\}\ .$$

Realization Logic

N ::= x++ | x--

$\langle x = 2, x = 0, x = -2 \rangle$

x++;

$\langle x = 1 \rangle$

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

$\langle x = 2, x = 0, x = -2 \rangle$

x++;

$\langle x = 1 \rangle$

$\models_a \langle R \rangle \texttt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \texttt{prog} \in drv(\texttt{sketch}).\ \models_d \{r\}\texttt{prog}\{s\}\ .$

Realization Logic

N ::= x++ | x--

$\langle x = 2, x = 0, x = -2 \rangle$

x++;

$\langle x = 1 \rangle$

$\mathrel{|\!\sim}$

$\langle x = 0 \rangle$

x++;

$\langle x = 1 \rangle$

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

$\langle x = 2, x = 0, x = -2 \rangle$

x++;

$\langle x = 1 \rangle$

$$\models_a \langle R \rangle \mathtt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \mathtt{prog} \in drv(\mathtt{sketch}).\ \models_d \{r\}\mathtt{prog}\{s\}\ .$$

Realization Logic

N ::= x++ | x--

$\langle x = 2, x = 0, x = -2 \rangle$

x++;

$\langle x = 1 \rangle$

$\mid\!\sim$

$\langle x = 0 \rangle$

x++;

$\langle x = 1 \rangle$

$\langle x = 0 \rangle$

N;

$\langle x = 1, x = -1 \rangle$

N;

$\langle x = 0 \rangle$

x++;

$\langle x = 1 \rangle$

$\models_a \langle R \rangle \texttt{sketch} \langle S \rangle \quad \Leftrightarrow \quad \forall s \in S.\ \exists r \in R.\ \exists \texttt{prog} \in drv(\texttt{sketch}).\ \models_d \{r\} \texttt{prog} \{s\}\ .$

Realization Logic

N ::= x++ | x--

$\langle x = 0 \rangle$

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$\langle x = 1, x = -1 \rangle$

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Contribution 2:

Realization Logic

$$\vdash_a \langle R \rangle \mathsf{po} \langle S \rangle \quad \wedge \quad \langle R \rangle \mathsf{po} \langle S \rangle \mathrel{|\!\!\sim} \langle R' \rangle \mathsf{po}' \langle S' \rangle$$
$$\Longrightarrow \quad \vdash_a \langle R' \rangle \mathsf{po}' \langle S' \rangle$$

Sound

Realization Logic

N ::= x++ | x--

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Sound and Complete

$$\vdash_a \langle R \rangle \mathsf{po} \langle S \rangle \quad \wedge \quad \vdash_a \langle R' \rangle \mathsf{po}' \langle S \rangle \quad \wedge \quad \langle R \rangle \mathsf{po} \langle S \rangle \preceq_p \langle R' \rangle \mathsf{po}' \langle S' \rangle$$
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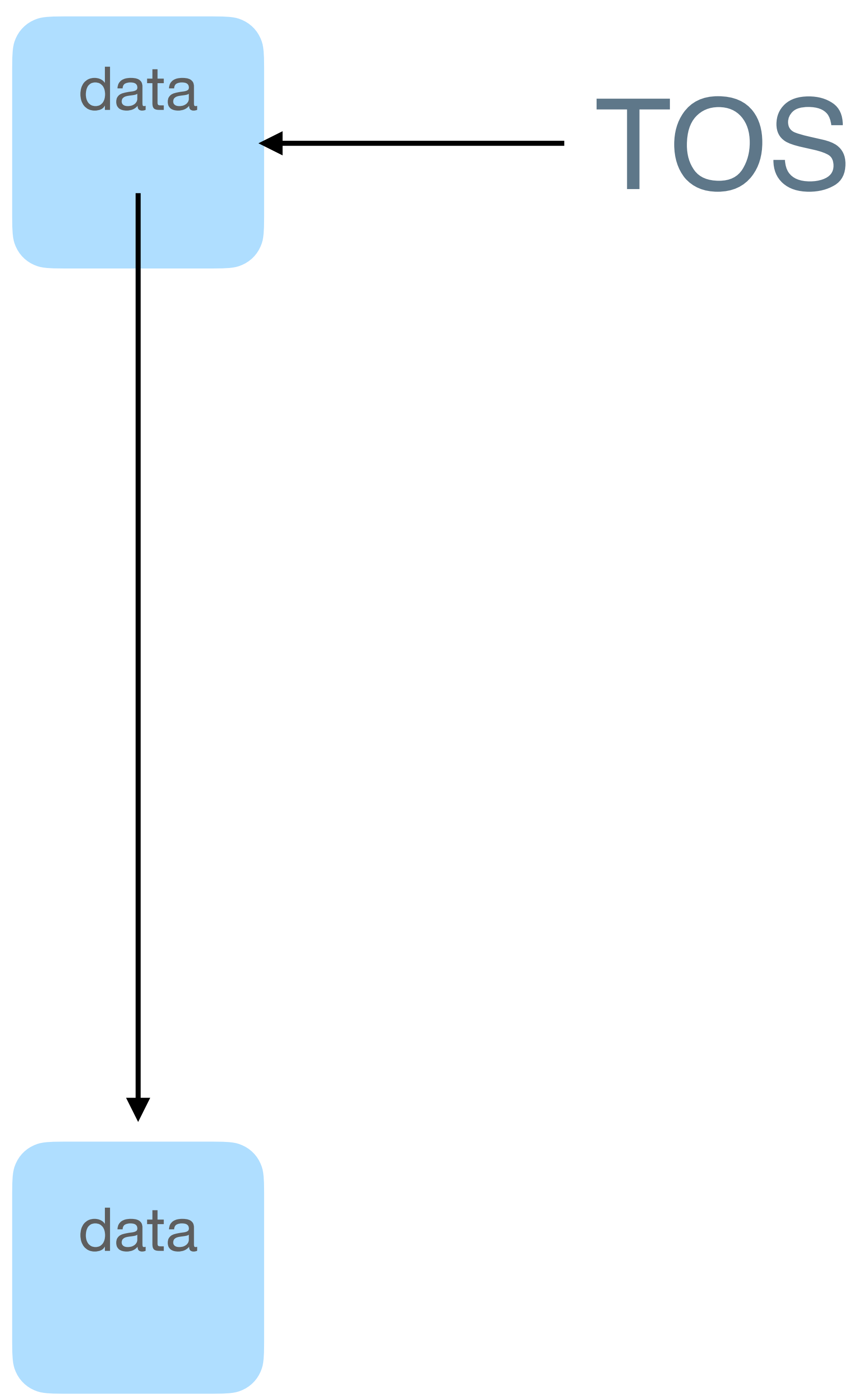
Memory Reclamation

```
top = TOS;
next = top.next;
CAS(TOS, top, next);
free(top);
```



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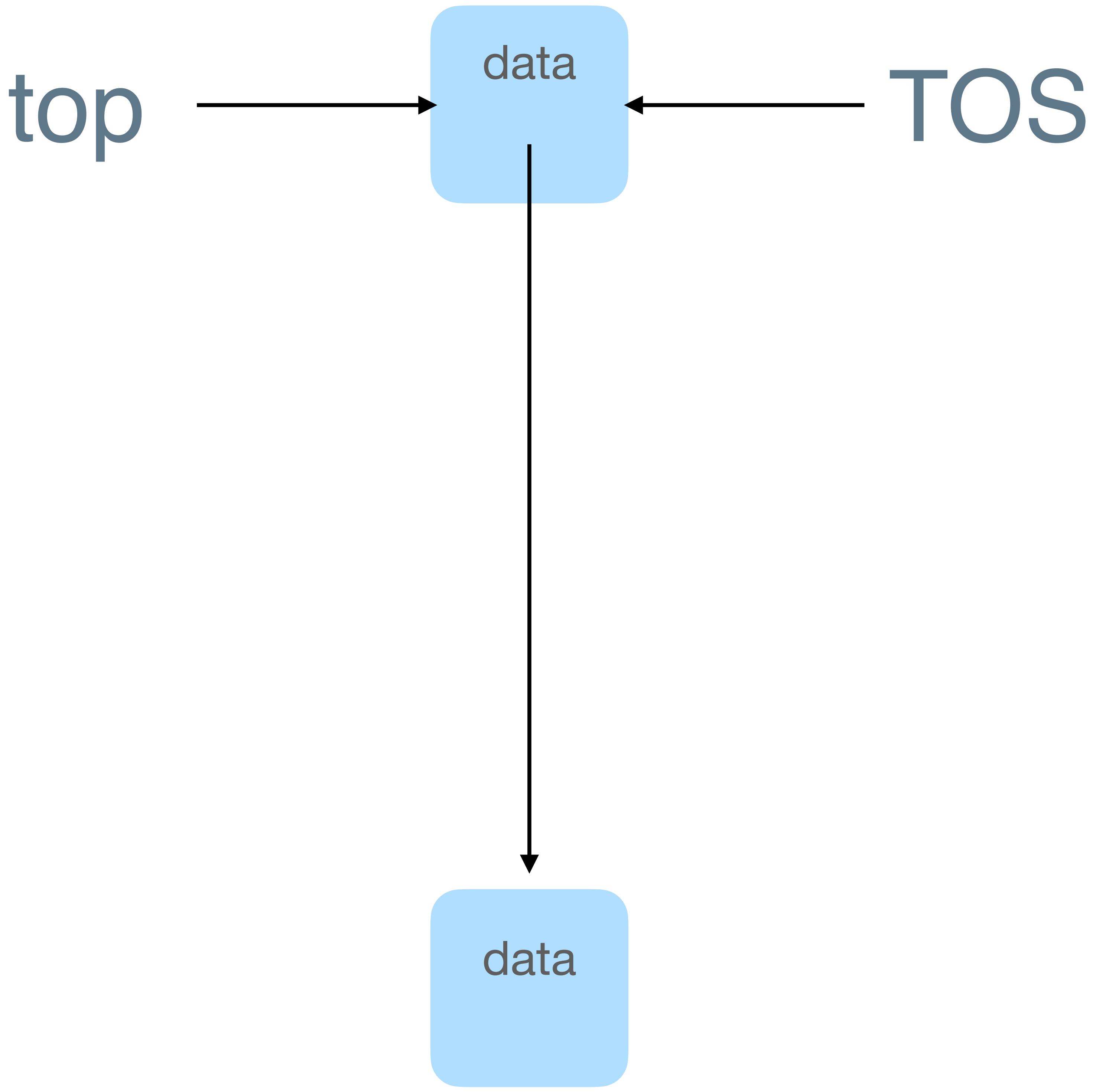

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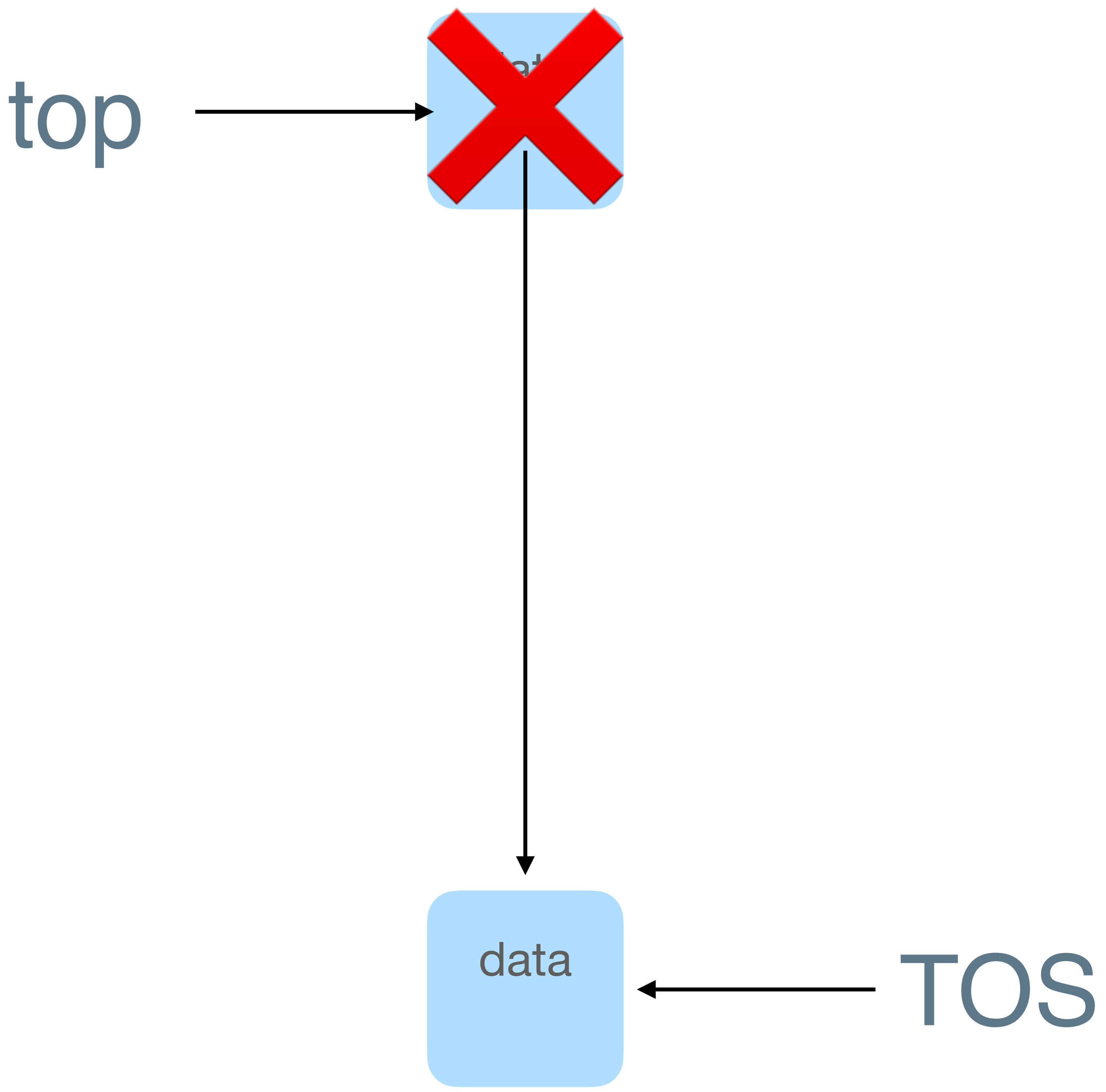
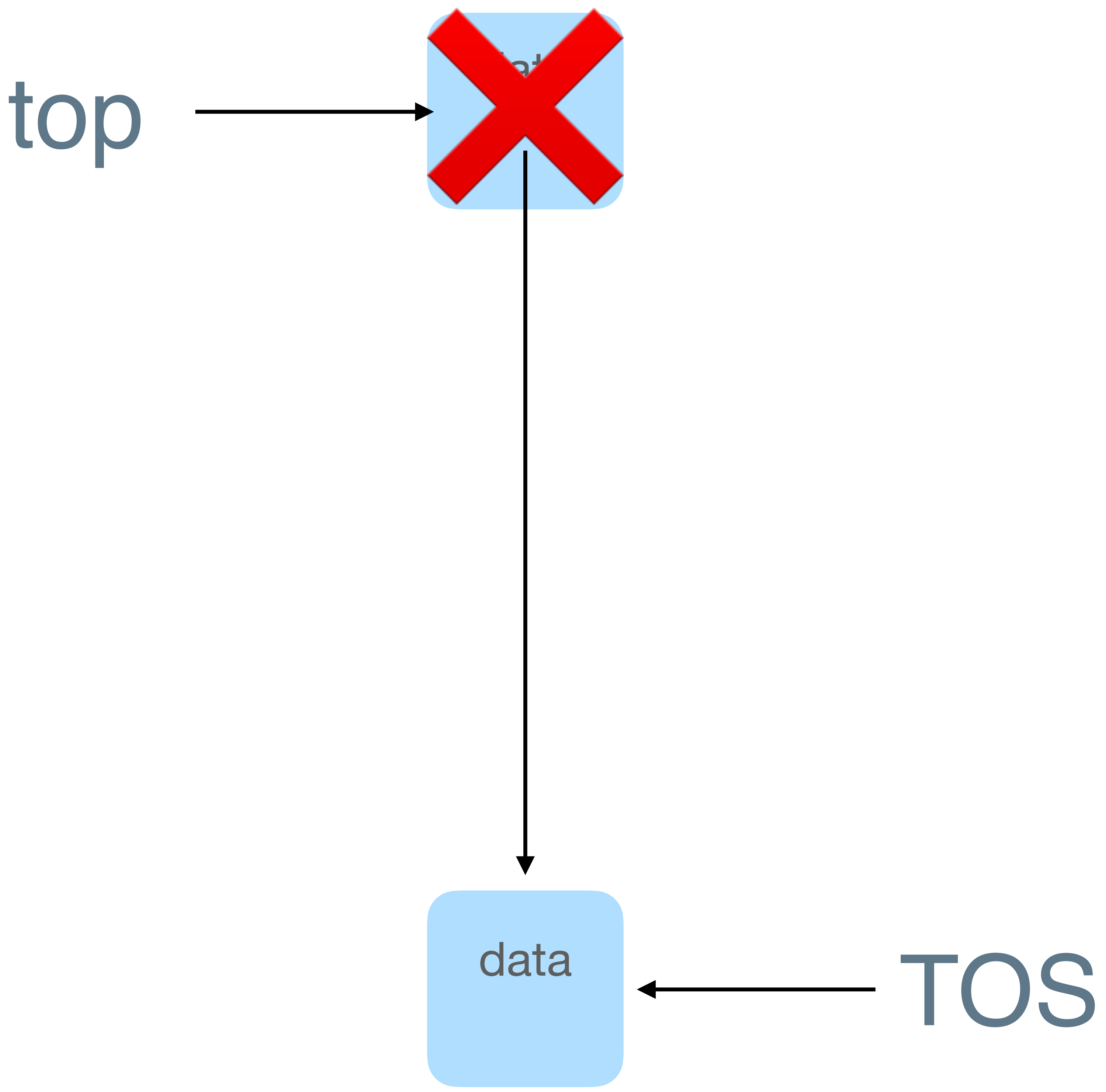
top = TOS;
next = top.next;
CAS(TOS, top, next);
free(top);
```



Memory Reclamation

top = TOS;
next = top.next; Unsafe Dereference
CAS(TOS, top, next);
free(top);



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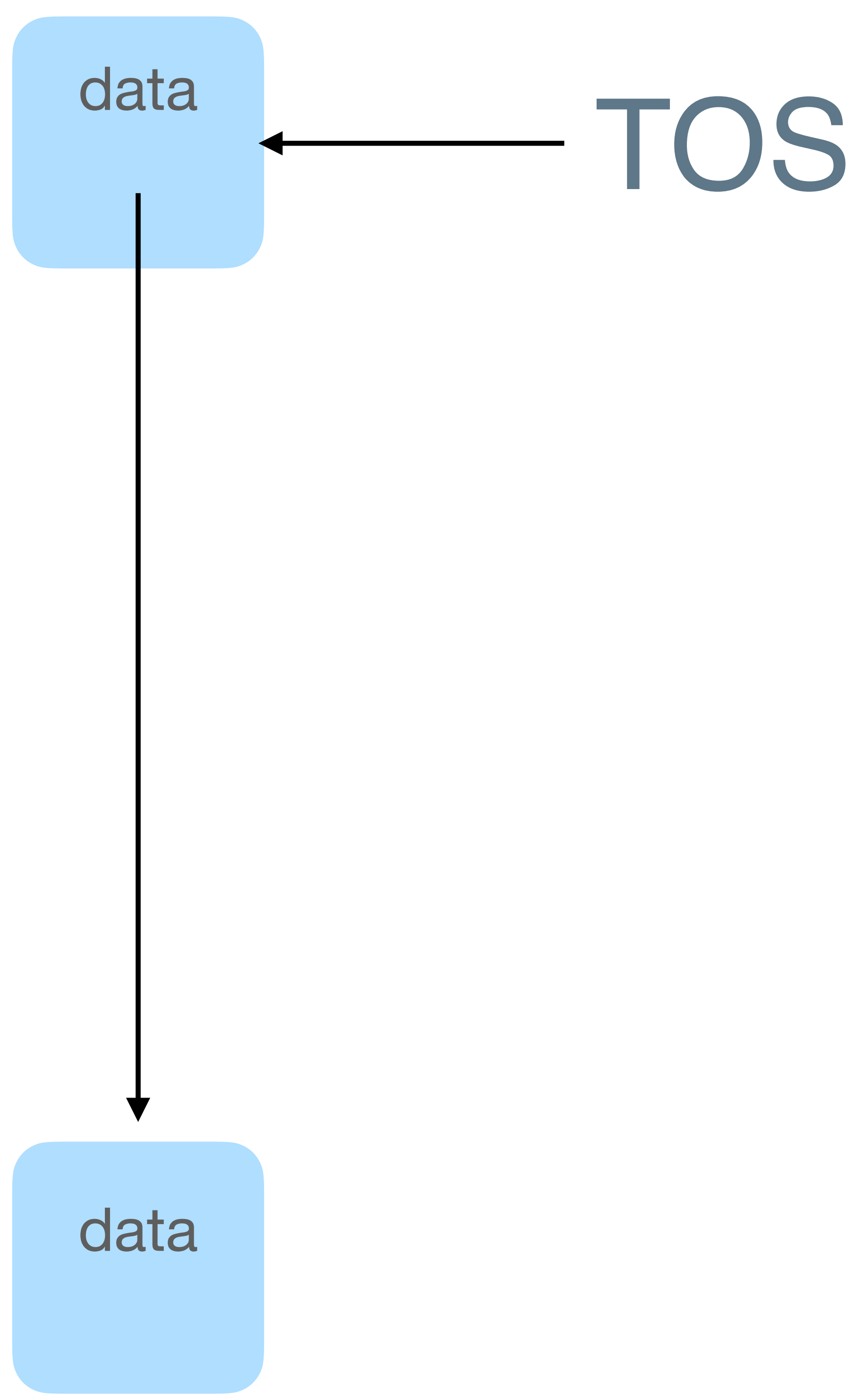

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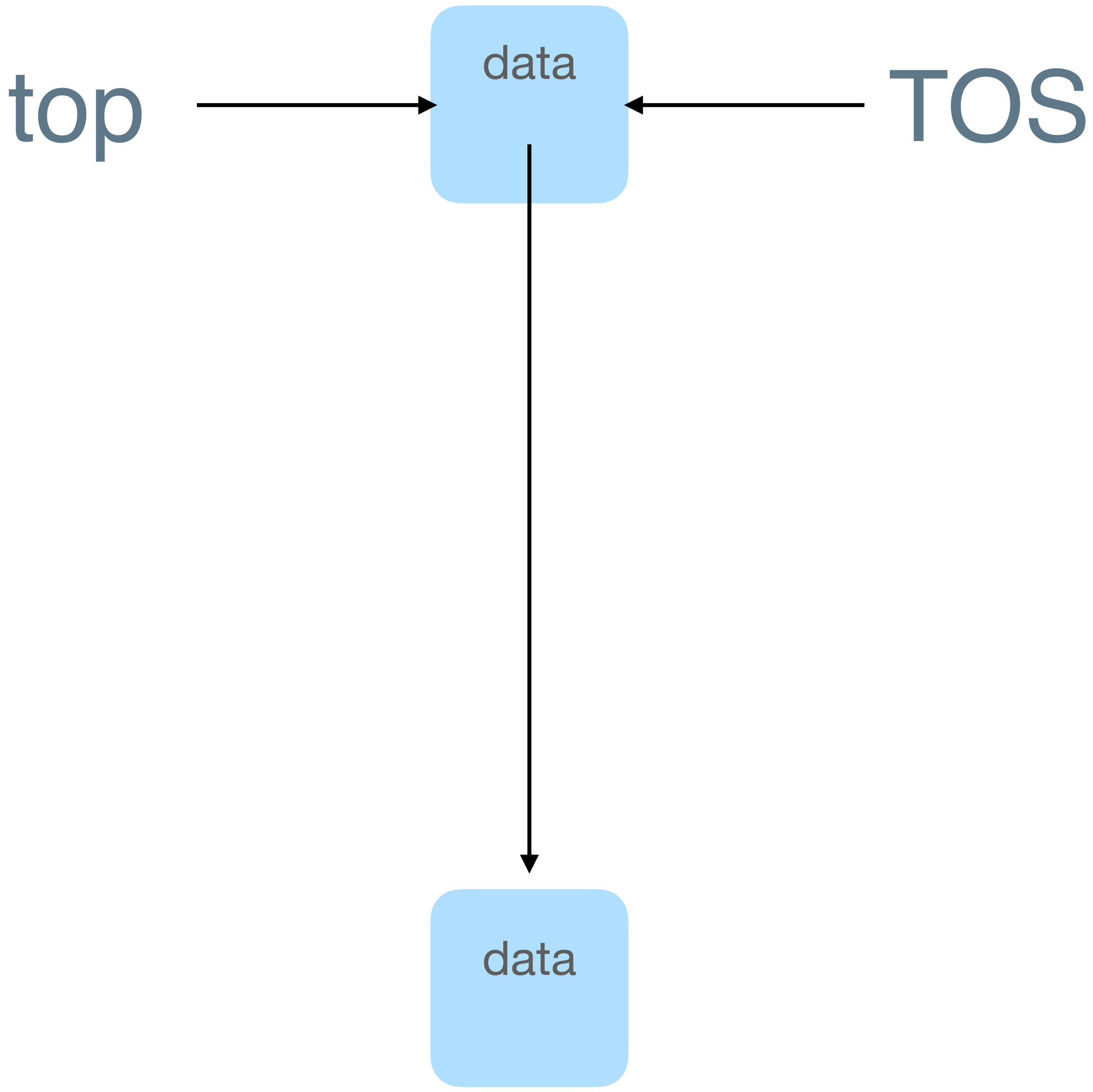

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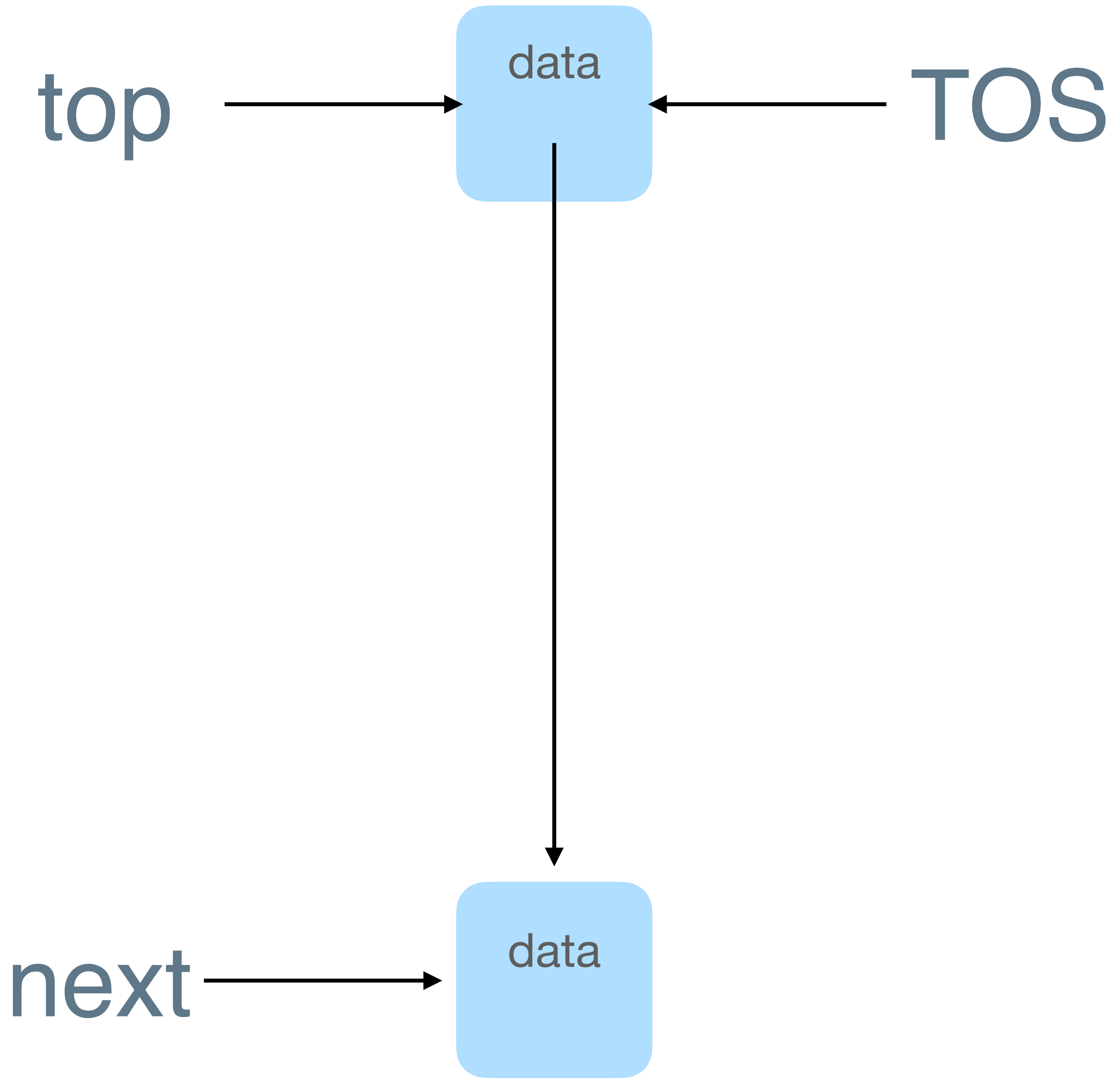

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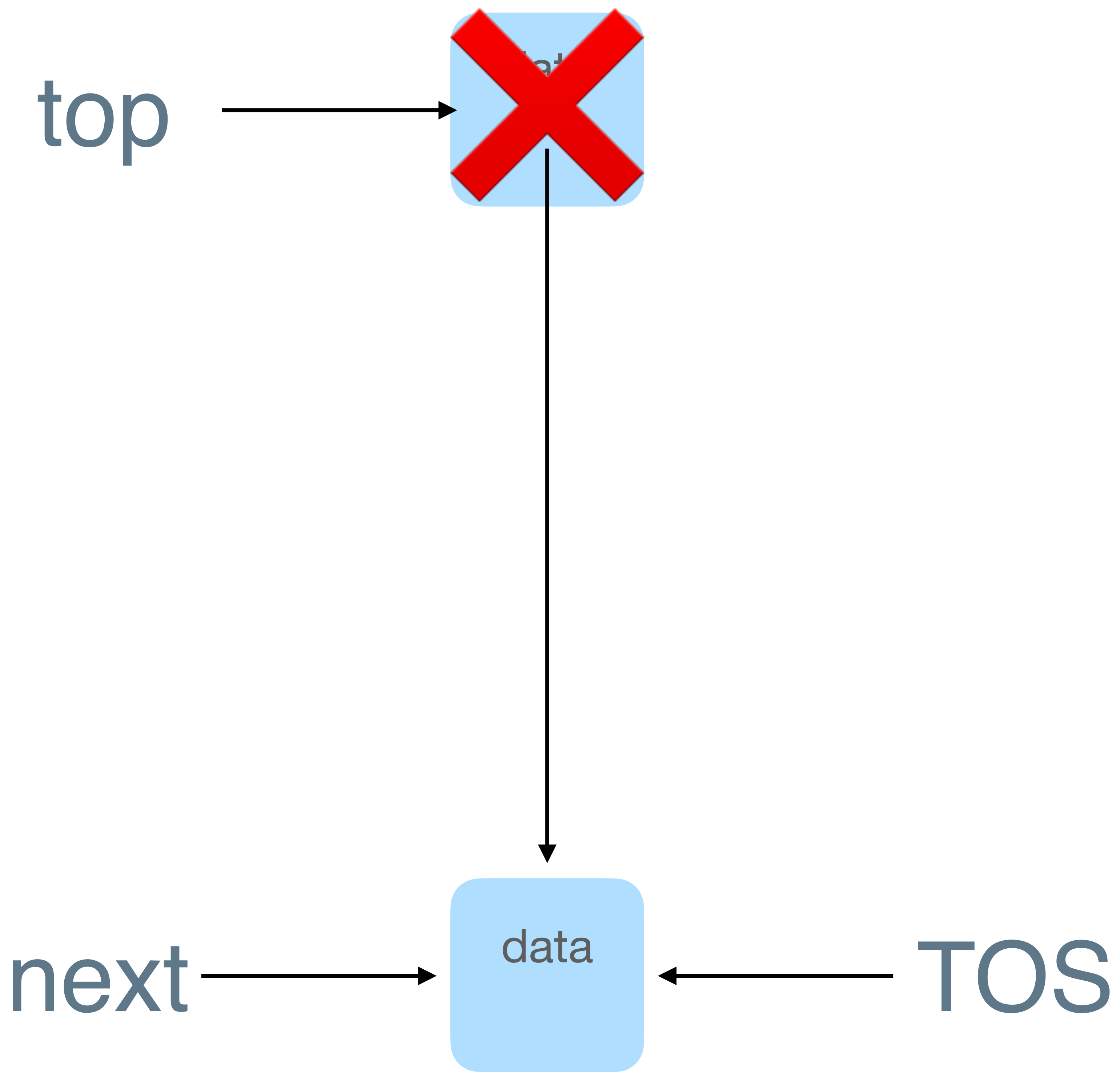

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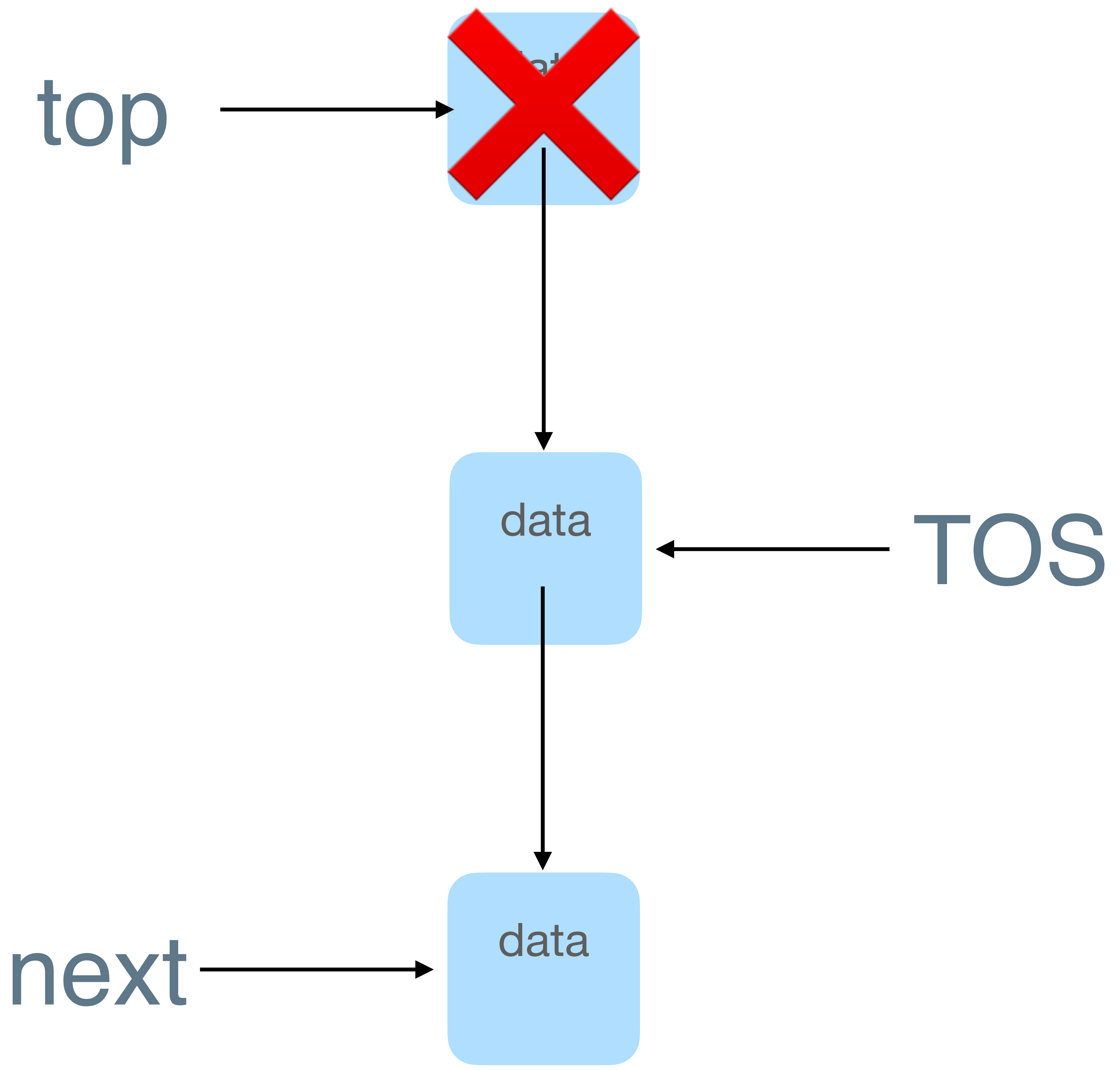

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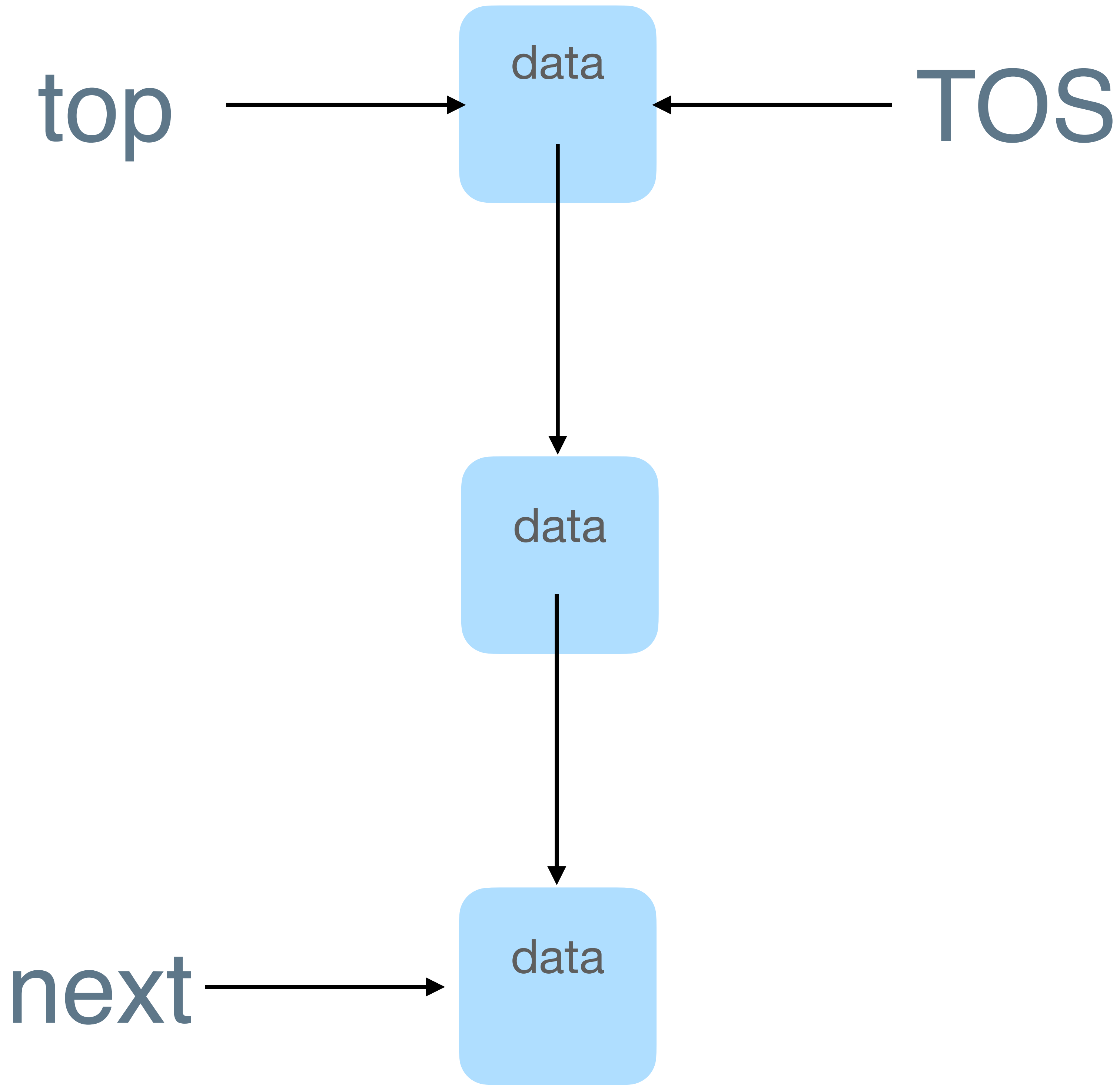

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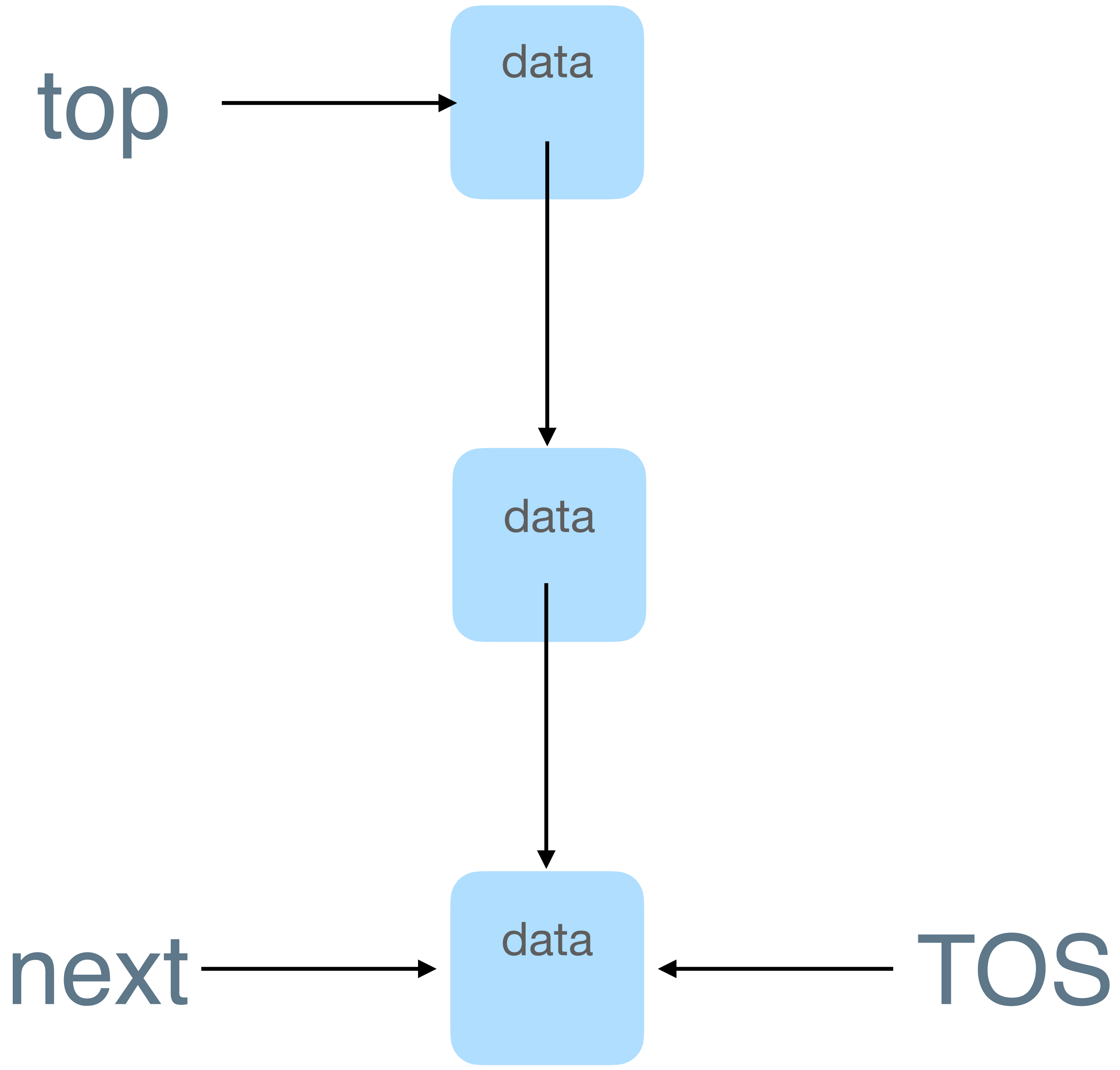
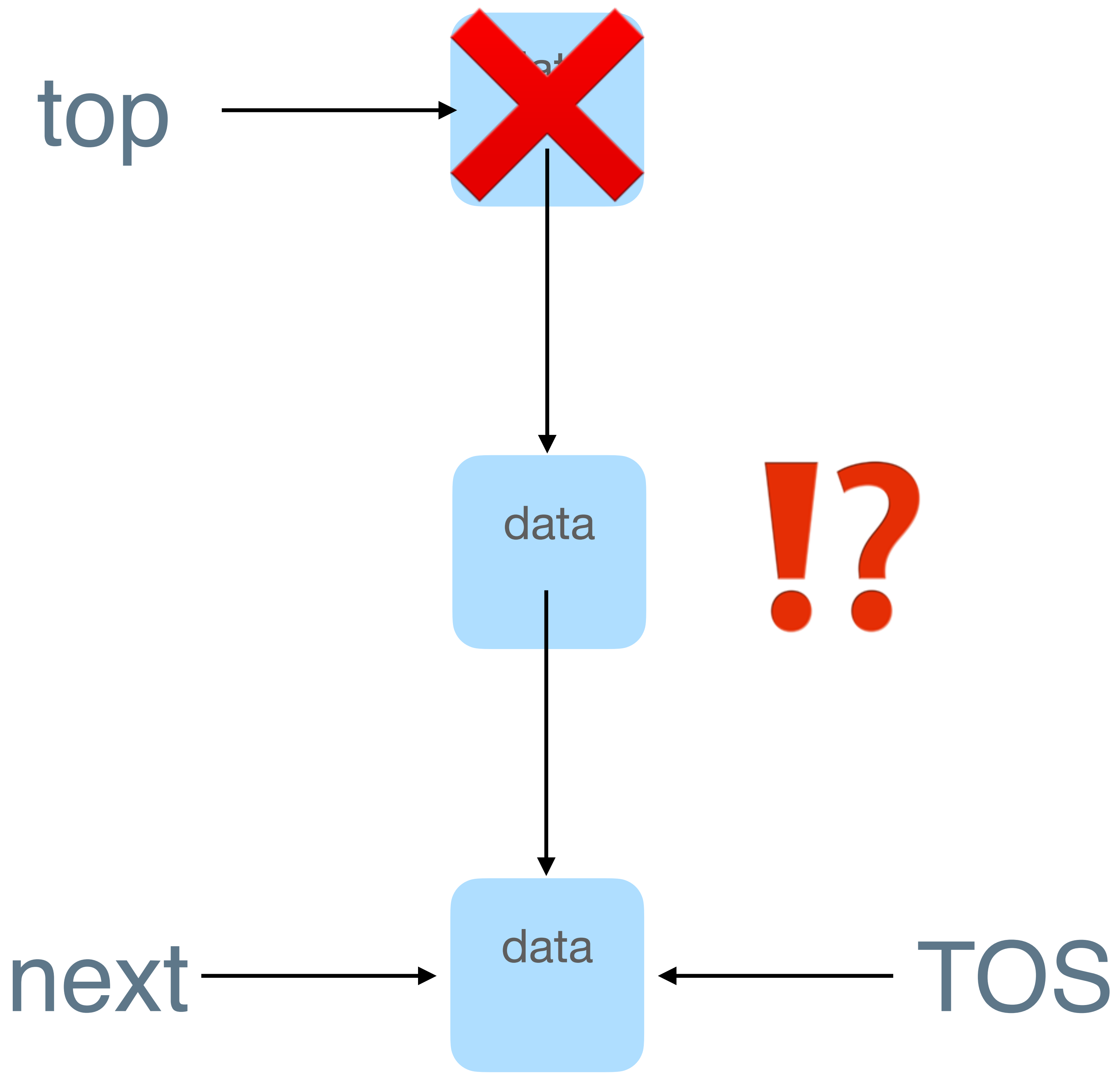


Memory Reclamation

top = TOS;
next = top.next;
CAS(TOS, top, next);   ABA
free(top);



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- Manual Memory Reclamation - very hard

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$$Predicates \quad = \quad Vars \rightarrow \mathbb{T}$$

Hazard Pointer: 5 base types

Memory Reclamation

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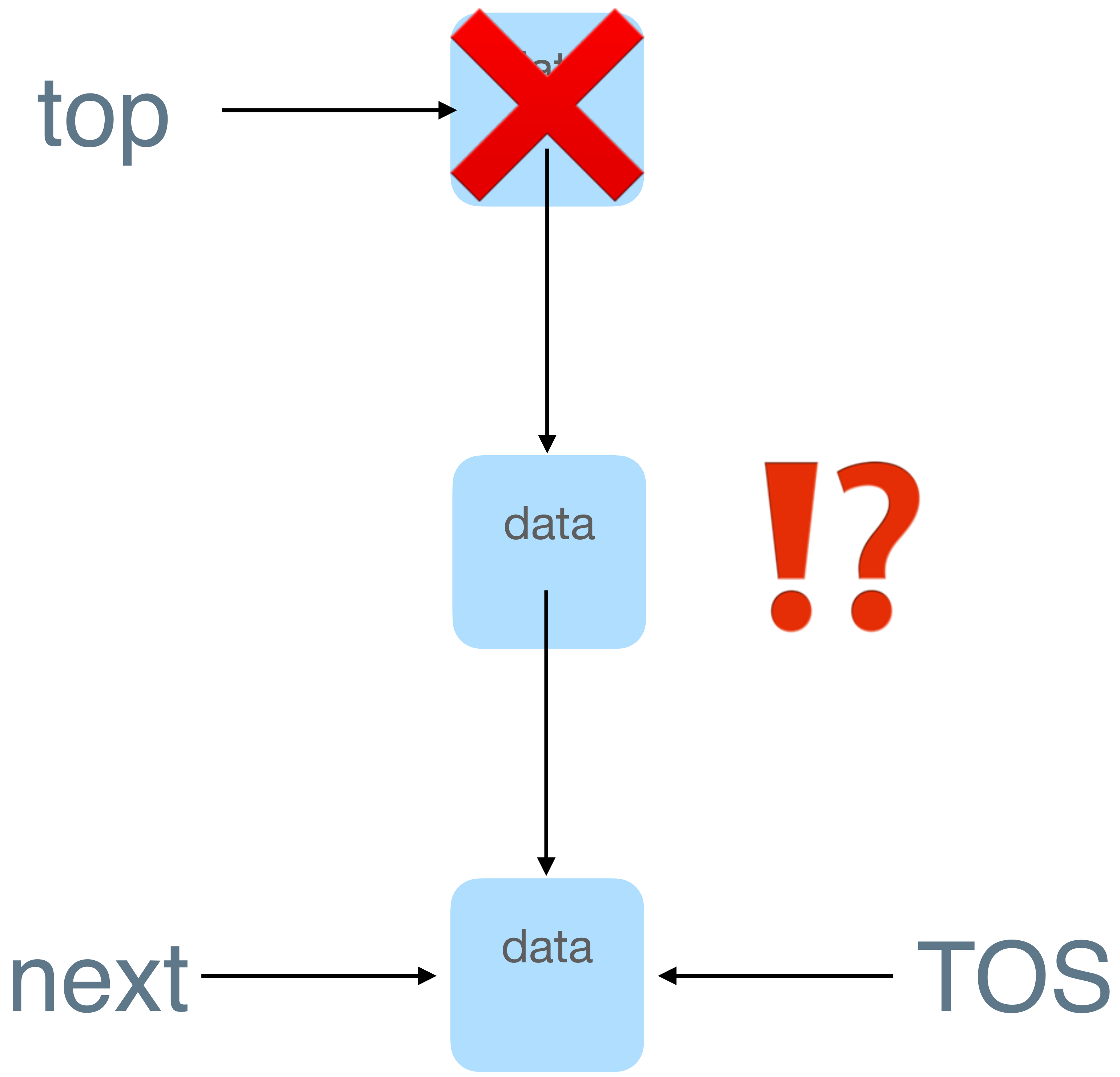
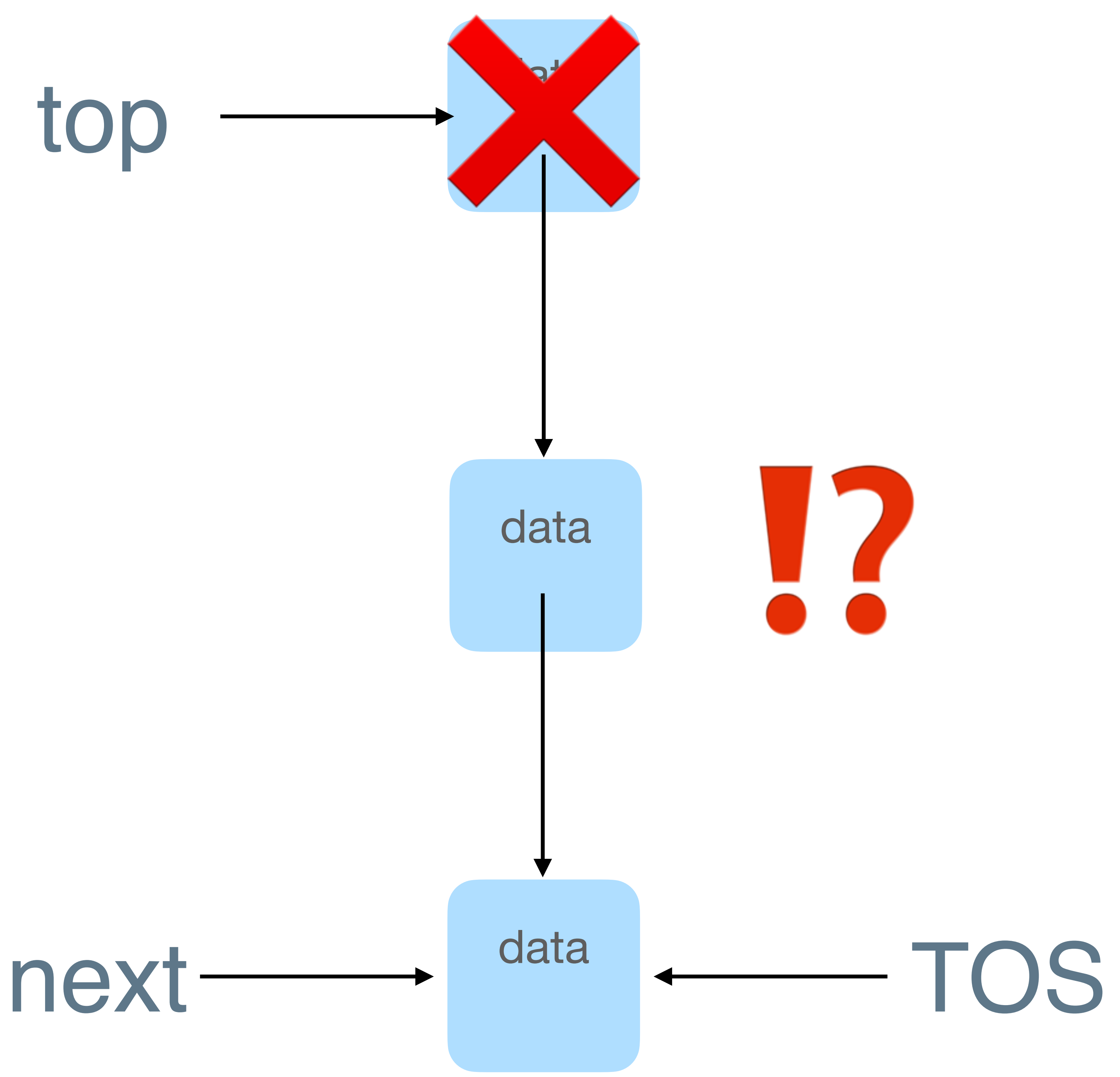


Memory Reclamation

N;
top = TOS;
N;
next = top.next;
N;
CAS(TOS, top, next);
N;
retire(top);
N;

ABA



N ::= protect(top) | @inv active (TOS) | skip

Memory Reclamation

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N;
CAS(TOS, top, next);
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retire(top);
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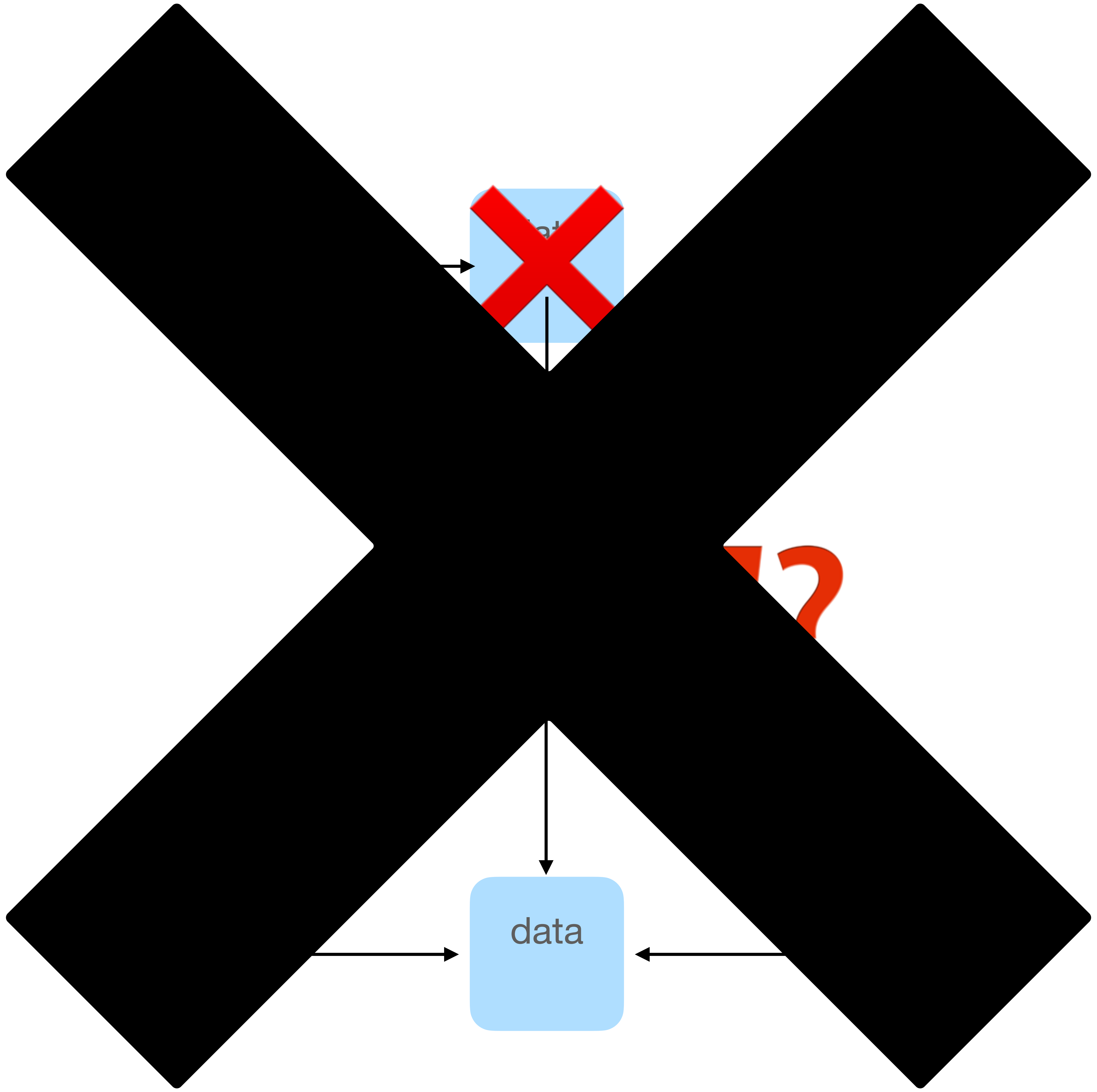

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Contribution 3:

Instantiation on SMR Setting

Data Structure	Treiber's Stack Pop	Treiber's Stack Push	Michael and Scott's Queue Enqueue	Michael and Scott's Queue Dequeue	ORVYY Set Add	ORVYY Set Remove
SMR Algorithm	HP1 (5 Base Types)	HP1 (5 Base Types)	HP1 (5 Base Types)	HP2 (8 Base Types)	HP2 (8 Base Types)	HP2 (8 Base Types)
Time (PO + Synth)	< 0.1s	< 0.1s	< 0.1s	< 7.5s	< 0.9s	< 0.4s
Max / Avg |R|	6 / 1.4	6 / 1.7	5 / 1.4	90 / 1.4	28 / 1.7	30 / 1.3

Conclusion

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- 1: Realizability Logic
- 2: Realization Logic
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Future Work

- SyGuS benchmarks
- Assertion Language

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Thanks for your attention!

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