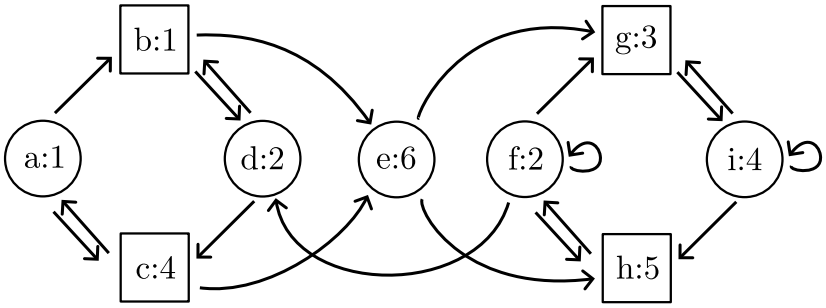
WS 26/27 Wed, 18. Nov. 2026

Exercises for the Lecture
Perfect Information Games
Exercise Sheet 2

Prof. Dr. Roland Meyer,
Eren Keskin Due until Wed, 2. Dec. 2026

Exercise 2.1 (Applying Algorithms, 4 Points)
Consider the parity game G below.



1. Determine the winning regions W_{E} and W_{A} of the respective players using the McNaughton/Zielonka algorithm. (If a subgame has 2 nodes or less, you may just give the winning regions without subcalls.)

2. Consider the play
$$\rho = a.b.d.b.e.g.i.h.f.h.f.h.f$$
in G. Construct the corresponding play ρ' in the safety reduction $G^{\otimes}$ of G that starts from $(a, 0, 0, 0)$, and visits the same positions.

   Example: The play ρ' in the safety reduction of G that starts from $(i, 0, 0, 0)$ and visits the same G-positions as $\rho = i.g.i.g.i$ would be $\rho' = (i, 0, 0, 0).(g, 0, 0, 0).(i, 0, 1, 0).(g, 0, 0, 0).(i, 0, 1, 0)$.

Exercise 2.2 (Slightly More Efficient Counting, 4 Points)
A subset of nodes $S \subseteq Q$ in a (game) graph $G = (Q_{\mathsf{E}}, Q_{\mathsf{A}}, E)$ is *strongly connected*, if for all $q, p \in S$, there is a path from q to p. We call a strongly connected $S \subseteq Q$, a *strongly connected component (SCC) of G*, if there is no $S' \supsetneq S$ that is strongly connected. It is easy to see that SCCs of G partition G. They are disjoint, because otherwise we would have paths that connect the SCCs leading to larger SCCs. They cover all nodes, because $\{q\}$ is strongly connected for all nodes q, so all nodes are in an SCC.

Let $G = (Q_{\mathsf{E}}, Q_{\mathsf{A}}, E, \Omega)$ be a parity game, and $G^{\otimes} = (Q_{\mathsf{E}} \times M_T, Q_{\mathsf{A}} \times M_T, E^{\otimes}, \mathsf{Bad})$ the corresponding safety reduction. Let $k \in \mathbb{N}$ be the size of largest SCC in G, and

let ℓ be the largest number of SCCs that can be visited in one play. We construct $G' = (Q_{\mathsf{E}} \times M_T, Q_{\mathsf{A}} \times M_T, E^{\otimes} \cup E_{new}, \mathsf{Bad})$ from $G^{\otimes}$ by adding edges, where Adam can directly win when a counter exceeds $k\ell$. That is,

$$E_{new} = \{((q,m),(q,\top)) \mid (q,m) \in Q'_{\mathsf{A}}, m[i] > k\ell \text{ for some } i\}$$

Argue that Eve wins $(q,0,\ldots,0)$ in G' iff she wins q in G.

Exercise 2.3 (Centered Cycles, 4 Points)
Show that the Zielonka/McNaughton algorithm runs in polynomial time on the class of game graphs $G = (Q_{\mathsf{E}}, Q_{\mathsf{A}}, E, \Omega)$ (in the size of G), where there is a position $p \in Q$, such that all cycles $q_0 \ldots q_k q_0$ have $i \leqslant k$ with $p = q_i$.

Hint: To find an idea, first consider graphs with maximum priority 1*,* 2*,* 3*, . . . This will remove some complexity of finding a generalizable idea.*