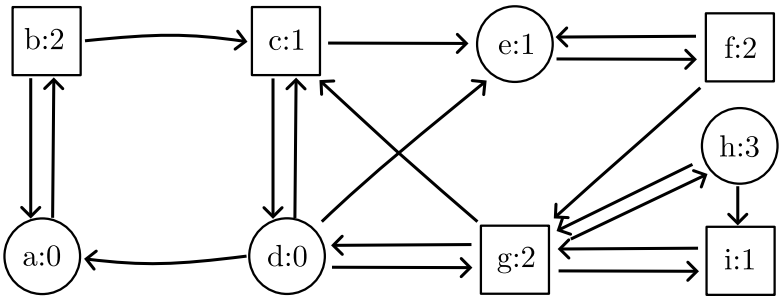
WS 26/27 Mi, 4. Nov. 2026

Exercises for the Lecture
Perfect Information Games
Exercise Sheet 1

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Eren Keskin Due until Mi, 18. Nov. 2026

Exercise 1.1 (Solving Games, 4 Points)
Consider the parity game G below. Determine the winning regions W_{E} and W_{A} of the respective players. Also give positional strategies that justify these winning regions.



Exercise 1.2 (Modulo 5 Reachability Games, 4 Points)
Let $A = (Q_{\mathsf{E}}, Q_{\mathsf{A}}, E)$ be a game arena. For $O \subseteq Q$, we define the winning condition

$$\mathsf{Reach}_5(O) = \{\rho \in \mathsf{MP}(A) \mid \exists i \in 5\mathbb{N}.\ \rho[i] \in O\}$$

that consists of plays that reach O in a number of turns divisible by 5.
Prove or disprove the following statements: For all objectives $O \subseteq Q$ and games $G = (Q_{\mathsf{E}}, Q_{\mathsf{A}}, E, \mathsf{Reach}_5(O))$ with such a winning condition,

a) the game G is determined.

b) the game G is positionally determined.

Exercise 1.3 (NP and coNP membership, 4 Points)
The goal of the exercise is to show $\textbf{PARITY} \in \mathsf{NP} \cap \mathsf{coNP}$. This is broken into steps that make the task easier, where the last step (c) is showing the desired membership.

a) Let $G = (Q_{\mathsf{E}}, Q_{\mathsf{A}}, E, \Omega)$ be a parity game, and $\mathbf{P} \in \{\mathsf{E}, \mathsf{A}\}$. For all $q \in Q_{\mathbf{P}}$, let there be exactly one $q' \in Q$ with $(q, q') \in E$. Show that $\mathbf{P}$ wins G from $q \in Q$ if and only if there is no cycle $q_0.q_1 \dots q_n.q_0$ reachable from q with $\max\{\Omega(q_i) \mid i \leqslant n\} \in 2\mathbb{N} + \mathsf{par}(\overline{\mathbf{P}})$, where $\mathsf{par}(\mathsf{E}) = 0$ and $\mathsf{par}(\mathsf{A}) = 1$.

b) Let $\mathbf{P} \in \{\mathsf{E}, \mathsf{A}\}$, $G = (Q_{\mathsf{E}}, Q_{\mathsf{A}}, E, \Omega)$ be a parity game, and let σ be a positional $\mathbf{P}$-strategy in G. We write $G|_\sigma$ to denote the game where $\mathbf{P}$'s choices are restricted by σ. Formally, $G|_\sigma = (Q_{\mathsf{E}}, Q_{\mathsf{A}}, E|_\sigma, \Omega)$ with

$$E|_\sigma = \{(q, q') \in E \mid q \in Q_{\overline{\mathbf{P}}}\} \cup \{(q, \sigma(q)) \in E \mid q \in Q_{\mathbf{P}}\}.$$

Show that there is a polynomial time algorithm that decides the problem below.

> **SOLVE-PARITY-FIXED**
> **Given:** A parity game $G = (Q_{\mathsf{E}}, Q_{\mathsf{A}}, E, \Omega)$, a player $\mathbf{P} \in \{\mathsf{E}, \mathsf{A}\}$, a positional $\mathbf{P}$-strategy σ, and a position $q \in Q$.
> **Decide:** Does $\mathbf{P}$ win from q in $G|_\sigma$?

c) Show that there is an NP algorithm that decides **PARITY**, and an NP algorithm that decides $\overline{\textbf{PARITY}}$.

Hint: Remember that parity games are positionally determined.