Exercises to the lecture
Concurrency Theory
Sheet 5

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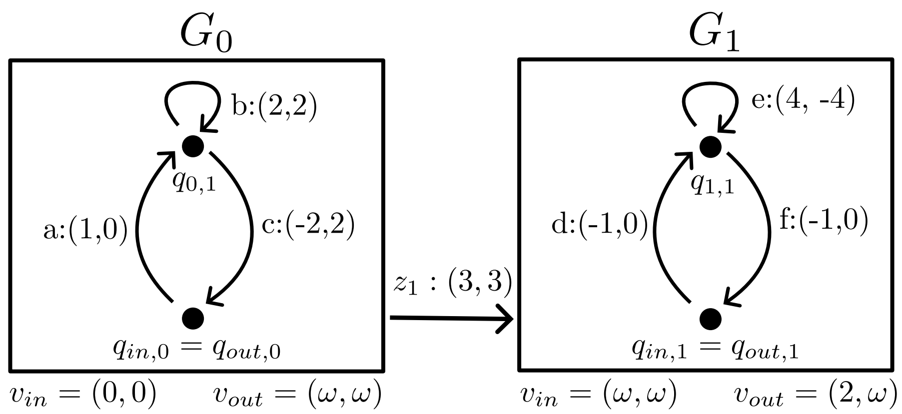
Eren Keskin & Jan Grünke Delivery until 09.07.2026 at 15:00

Exercise 5.1 (Characteristic Equations of MGTS)

Consider the MGTS $\mathcal{M}$ below.



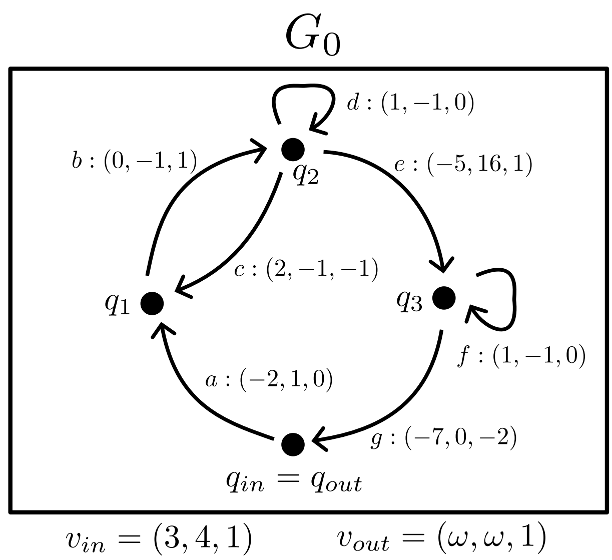
$$\Omega(G_0) = \Omega(G_1) = \{1, 2\}$$

a) Write out the characteristic equation of $\mathcal{M}$ explicitly.

b) Find a full-support solution to the homogenous characteristic equation system. Are (CSUP) and (ESUP) satisfied?

Exercise 5.2 (Pumping Sequences)

Consider the MGTS $\mathcal{M}$ below.



$$\Omega(G_0) = \{1, 2\}$$

$\lambda(q_{in}) = (\omega, \omega, 1)$

$\lambda(q_1) = (\omega, \omega, 1)$

$\lambda(q_2) = (\omega, \omega, 2)$

$\lambda(q_3) = (\omega, \omega, 3)$

a) Determine whether G_0 has an up-pumping sequence.

b) Determine whether G_0 has a down-pumping sequence. Is $\mathcal{M}$ perfect?

Exercise 5.3 (Repeated Enabledness)

The proof of the iteration lemma uses the following fact. If we execute a cycle n times, and we start from, and end at high enough counter values, then the counter value is also in $\mathbb{N}$ along the run. This is encapsulated in the lemma below. Give a formal proof for it.

Lemma 1 *Let* $u_0, \ldots, u_{k-1} \in \mathbb{Z}$, *let* $\mathbf{s} = \sum_{i<k} |u_i|$, $x \in \mathbb{N}$, *and* $n \in \mathbb{N}$. *Let* $t_0, t_1, \ldots, t_{n \cdot k-1} \in \mathbb{Z}$ *with* $t_a = u_r$ *for all* $a \in \{0, \ldots, n \cdot k - 1\}$, *and* $r \in \{0, \ldots, k-1\}$ *with* $a \equiv r \pmod{k}$. *Let*

$$v_i = x + \sum_{j \in \{0, \ldots,\, i-1\}} t_j$$

for all $i \in \{0, \ldots, n \cdot k\}$. *If* $v_0 \geq \mathbf{s}$ *and* $v_{n \cdot k} \geq \mathbf{s}$, *then* $v_i \in \mathbb{N}$ *for all* $i \in \{0, \ldots, n \cdot k\}$.

Exercise 5.4 (Even Runs)

Develop an algorithm to decide the problem below. *Do not* use the decomposition procedure from the lecture. Argue that your algorithm is correct.

EVEN-RUNS

Given: Perfect MGTS $\mathcal{M}$

Decide: Is there a run $\rho \in \mathsf{Runs}_{\mathbb{N}}(\mathcal{M})$ of even length?

Delivery until 09.07.2026 at 15:00 to jan.gruenke@tu-bs.de.