SS 2026 | 12.06.2026

Exercises to the lecture
Concurrency Theory
Sheet 4

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Exercise 4.1 (Parikh Image of Regular Languages)

For a word $w \in \Sigma^*$, the *Parikh image* $\Psi(w) \in \mathbb{N}^\Sigma$ counts the number of occurrences of each letter in w. For example, $\Psi(aabbb)[a] = 2$ and $\Psi(aabbb)[b] = 3$. The Parikh image of a language $L \subseteq \Sigma^*$ is defined as $\Psi(L) := \{\Psi(w) \mid w \in L\}$.
Show that the Parikh image of regular languages are precisely the semi-linear sets, i.e., show the following two properties:

(a) For a regular language L, the Parikh image $\Psi(L)$ is semi-linear.

Hint: Use induction and regular expressions.

(b) For each semi-linear set $S \subseteq \mathbb{N}^d$, there is a regular language L over $\Sigma = \{a_1, \ldots, a_d\}$ with $S = \Psi(L)$.

Exercise 4.2 (Quantifier Elimination for Presburger Arithmetic)

Eliminate the quantifiers and simplify the following formulas in Presburger arithmetic:

(a) $\exists x.\, 2x + 3y \leq z \;\wedge\; x \geq 1$

(b) $\exists x.\, [x + u \geq 0 \vee x \equiv_5 2] \;\rightarrow\; [3x + t \leq 1 \;\wedge\; 1 \leq 2x - u]$

(c) $\neg\forall x.\, 3x < 2y \vee y < 2x$

Exercise 4.3 (Control-State Reachability)

Let $\mathcal{V} = (Q, C, T)$ be a VASS. The *control-state reachability problem* asks: given $q_0, q_f \in Q$ and $v_0 \in \mathbb{N}^C$, decide whether there exists $v_f \in \mathbb{N}^C$ such that $(q_0, v_0) \to^* (q_f, v_f)$. Give an algorithm to solve this problem. What is the computational complexity of the control-state reachability problem?

Exercise 4.4 ($\mathbb{Z}$-Reachability)

Let $\mathcal{V} = (Q, C, T)$ be a VASS and $\Delta \in \mathbb{Z}^{C \times T}$ so that $\Delta[c, t] = z[c]$ for all counter $c \in C$ and transition $t = (q, z, q') \in T$. Disprove the following claim:
There is a $\mathbb{Z}$-run from (q_0, v_0) to (q_f, v_f) if and only if there is a parikh vector $x \in \mathbb{N}^T$ that satisfies

$$\textbf{(Kirchhoff)} \quad \forall q \in Q.\ \sum_{t=(q,z,q')\in T} x[t] - \sum_{t=(q',z,q)\in T} x[t] = \begin{cases} -1 & \text{if } q = q_0 \neq q_f \\ 1 & \text{if } q = q_f \neq q_0 \\ 0 & \text{otherwise} \end{cases}$$

$$\textbf{(Marking Eq.)} \quad v_f = v_0 + \Delta \cdot x$$

Delivery until 25.06.2026 at 15:00 to jan.gruenke@tu-bs.de.