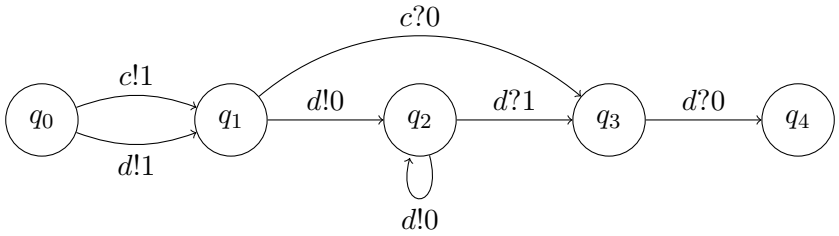
SS 2026 07.05.2026

Exercises to the lecture
Concurrency Theory
Sheet 2

Prof. Dr. Roland Meyer
Eren Keskin & Jan Grünke Delivery until 21.05.2026 at 15:00

Exercise 2.1 (Abdulla's Backward Search for LCS)
Consider the following lossy channel system.



Use Abdulla's backward search to decide whether the configuration $(q_4, \begin{pmatrix} c \mapsto 0 \\ d \mapsto \varepsilon \end{pmatrix})$ is coverable from $(q_0, \begin{pmatrix} c \mapsto \varepsilon \\ d \mapsto \varepsilon \end{pmatrix})$.

Exercise 2.2 (Petri Net Ideals)
Let $N = (S, T, W)$ be a Petri net. Derive that the ideals of $\mathbb{N}^S$ are of the form $\downarrow M$, where $M \in (\mathbb{N} \cup \{\omega\})^S$ is a generalized marking. For this, show the following two properties:

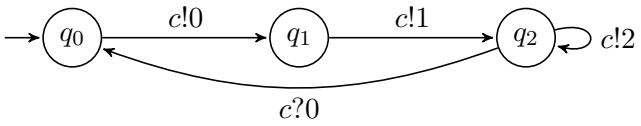
(a) The ideals of $(\mathbb{N}, \leq)$ are $\mathbb{N}$ itself and all sets of the form $\downarrow n$ for $n \in \mathbb{N}$.

(b) Let $(A, \leq_A)$ and $(B, \leq_B)$ be two wqos and $(A \times B, \leq)$ their product. Show that a subset $I \subseteq A \times B$ is an ideal if and only if $I = I_A \times I_B$ where I_A, I_B are ideals in A and B, respectively.

Hint: For one direction you need to show that $I = \pi_1(I) \times \pi_2(I)$, where $\pi_1(I)$ and $\pi_2(I)$ denote the projection of I to its first and second component respectively.

Exercise 2.3 (Forward Uncoverability for LCS)

Consider the following LCS with a single channel c and messages $M = \{0, 1, 2\}$.



(a) Show that ideals can be represented as $\downarrow(q, L)$ where $q \in Q$ and L is a regular language over M.

Hint: Rely on the previous exercise (part (b)) and show that the downward closure $\downarrow L$ is regular for any language $L \subseteq \Sigma^*$ over an alphabet Σ.

(b) Prove uncoverability of $(q_0, c \mapsto 22)$ by defining a downward closed set D that is an inductive invariant wrt. $\downarrow post$:

(i) $(q_0, c \mapsto \varepsilon) \in D$

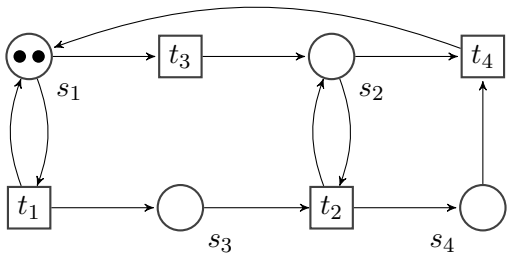
(ii) $\downarrow post(D) \subseteq D$

(iii) $(q_0, c \mapsto 22) \notin D$.

Represent D as a finite union of ideals, i.e. $D = I_1 \cup \ldots \cup I_n$ where each I_i is an ideal represented as in (a).

Exercise 2.4 (Coverability Graphs)

Use the algorithm from the lecture to compute the coverability graph of the Petri net:



Is $(1, 2, 42, 0)^T$ coverable?

Delivery until 21.05.2026 at 15:00 to jan.gruenke@tu-bs.de.