Exercises to the lecture
Concurrency Theory
Sheet 1

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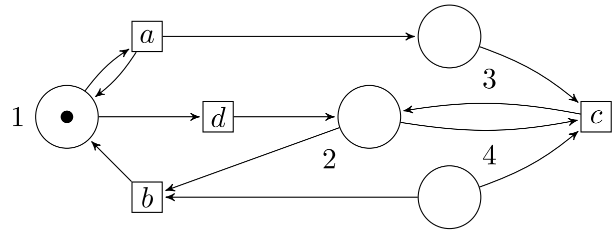
Exercise 1.1 (Rackoff's Theorem)

Consider the Petri net $N = (\{1, 2, 3, 4\}, \{a, b, c, d\}, W)$ with weights as depicted below. The initial marking of interest is $M_0 = (1, 0, 0, 0)^T$ and the final marking is $M_f = (1, 0, 10, 100)^T$.



Compute $minSeq(3, M_0)$, $ball(M_f)$ and $f(3)$ from the proof of Rackoff's Theorem and argue why they are correct.

Exercise 1.2 (Well-Quasi Orderings)

Prove or disprove that the following are well quasi orderings:

a) The lexicographical order $(\{0, 1\}^*, \leq_{lex})$ over binary words:

$u \leq_{lex} v$ if and only if u is a prefix of v or the first symbol $u[\ell]$ that does not coincide with $v[\ell]$ satisfies $u[\ell] < v[\ell]$.

Note that $u[\ell]$ refers to the ℓ-th symbol of u.

b) The colexicographical order $(\{0, 1\}^*, \leq_{colex})$ defined by:

$u \leq_{colex} v$ if and only if u is a postfix of v or the last symbol $u[\ell]$ that does not coincide with $v[\ell]$ satisfies $u[\ell] < v[\ell]$.

c) The radix order $(\{0, 1\}^*, \leq_{radix})$ over binary words:

$$u \leq_{radix} v \text{ if and only if } |u| < |v| \text{ or } |u| = |v| \wedge u \leq_{lex} v$$

d) The quasi ordering $(\mathbb{N}, |)$, where $a \mid b$ means that a divides b.

e) The quasi ordering $(P \uplus Q, \leq_+)$ where $(P, \leq_P)$ and $(Q, \leq_Q)$ are well quasi orderings with $P \cap Q = \emptyset$, and

$$t \leq_+ t' \text{ if and only if } (t, t' \in P \text{ and } t \leq_P t') \text{ or } (t, t' \in Q \text{ and } t \leq_Q t').$$

Exercise 1.3 (König's Lemma)

A tree $T = (V, \rightarrow)$ is a directed graph with a distinguished root node such that every node (except the root) has exactly one predecessor and there are no cycles. A tree is called *finitely branching* if every node has only finitely many successors. Prove the following: An infinite, finitely branching tree contains an infinite path, that starts from the root.

Hint: Recall that if $v \rightarrow v'$ we call v' a *successor* of a node v and v a *predecessor* of v'. A path is a (finite or infinite) sequence of nodes $v_0, v_1, v_2, \ldots$ such that v_{i+1} is a successor of v_i. A cycle is a path $v_0, v_1, \ldots, v_n$ such that $v_0 = v_n$.

Exercise 1.4 (Termination for WSTS)

Let $S = (Q, \leq, I, \rightarrow)$ be a WSTS. Assume that

- $I = \{q_0\}$ is a singleton set of initial states,
- $q \leq q'$ is decidable for each $q, q' \in Q$,
- the set $post(q) = \{q' \mid q \rightarrow q'\}$ is finite for each $q \in Q$, and *post* is computable.

Then, S *terminates* if every transition sequence starting in I is finite.

Show that the *termination problem* is decidable. That is, given a WSTS S, decide whether S terminates.

Hint: Compute a tree representing all transition sequences starting in q_0 and use König's Lemma and both assumtions to decide whether there is an infinite path in this tree.

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