SS 2025 19.04.2025

Exercises to the lecture
Semantics
Sheet 7

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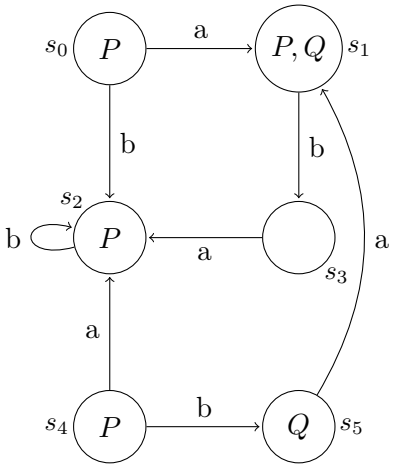
Jan Grünke Delivery until 26.04.2025 at 16:45

Exercise 7.1 (μ-Calculus Formulas)
Consider a transition system with actions $Act = \{a, b\}$ and propositions $Prop = \{P, Q\}$. Write μ-calculus formulas for the following properties:

1. Nodes that have a path to a Q-labeled node.
2. Nodes that have a path to a node v labeled with Q, such that v has a path to a node labeled with P.
3. Nodes from which an infinite path exists where inifitely often P holds.
4. Nodes from which every path eventually reaches a state where Q holds, and from that state onward, P holds globally.

Exercise 7.2 (μ-Calculus)
Consider the following transition system M with propositions P and Q, and actions a and b.



For each of the following formulas, describe its semantics in words and evaluate it on M:

1. $[a]ff$
2. $\mu X.\ ((P \wedge \langle a \rangle X) \vee Q)$
3. $\nu X.\ (\neg Q \wedge [a]X \wedge [b]X)$
4. $\nu X.\mu Y.[a](Y \vee (X \wedge P))$

Exercise 7.3 (μ-Calculus to Parity Games)
Consider the formula $F = \nu X.\mu Y.\ (P \wedge [b]X) \vee \langle b \rangle Y$, and let M be the transition system from the previous exercise. Construct the parity game $G(M, F)$ as described in the lecture, such that

$$M, s_0 \models F \quad \text{if and only if} \quad \text{Eve has a winning strategy from } (s_0, F) \text{ in } G(M, F).$$

You only need to construct positions that are reachable from (s_0, F). Finally, determine whether s_0 satisfies F by solving the parity game.