Technische Universität Braunschweig
Institute for Partial Differential Equations

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Formulary Mathematics for Engineers B
Calculus 2

Binomial formula

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Directional derivative, $\mathbf{n} \in \mathbb{R}^d$, $||\mathbf{n}||_2 = 1$

$$\frac{\partial}{\partial \mathbf{n}} f(\mathbf{x}) = \lim_{h\to 0} \frac{1}{h} \left[f(\mathbf{x}+h\mathbf{n}) - f(\mathbf{x}) \right]$$
$$\frac{\partial}{\partial \mathbf{n}} f(\mathbf{x}) = \nabla f(\mathbf{x}) \cdot \mathbf{n}$$

Symmetry of 2nd derivatives

$$f_{,x_k x_\ell} \in C \ \forall k,\ell \ \Rightarrow f_{,x_k x_\ell} = f_{,x_\ell x_k}$$

Total derivative $A : \mathbb{R}^d \to \mathbb{R}^1$, $f : \mathbb{R}^d \to \mathbb{R}^1$

$$0 = \lim_{\mathbf{v}\to\mathbf{0}} \frac{f(\mathbf{x}+\mathbf{v}) - f(\mathbf{x}) - A\mathbf{v}}{||\mathbf{v}||_2}$$

Hessian matrix of $f : \mathbb{R}^d \to \mathbb{R}^1$

$$\nabla\nabla f(\mathbf{x}) = (f_{,x_k x_\ell})_{k,\ell=1}^{d}$$

Taylor expansion for $f : \mathbb{R}^1 \to \mathbb{R}^1$

$$T(x_0+h) = f(x_0) + hf'(x_0) + \frac{1}{2!} h^2 f''(x_0) + \ldots$$

Tangential plane $T_1(\mathbf{x}) = T_1(\mathbf{x_0} + \mathbf{v})$

Lagrangian function of f with constraints $g_k(\mathbf{x}) = 0$, $k = 1, \ldots, m$

$$L(\mathbf{x}, \lambda_1, \ldots, \lambda_m) = f(\mathbf{x}) + \lambda_1 g_1(\mathbf{x}) + \ldots + \lambda_m g_m(\mathbf{x})$$

Jacobian matrix of $\mathbf{g} : \mathbb{R}^d \to \mathbb{R}^m$:

$$\nabla \mathbf{g}(\mathbf{x}) \in \mathbb{R}^{m\times d}$$

Chain rule $\mathbf{g} : \mathbb{R}^d \to \mathbb{R}^m$, $\mathbf{f} : \mathbb{R}^m \to \mathbb{R}^k$

$$\frac{\partial}{\partial \mathbf{x}} \mathbf{f}(\mathbf{g}(\mathbf{x})) = \frac{\partial}{\partial \mathbf{g}} \mathbf{f}(\mathbf{g}(\mathbf{x})) \cdot \frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}}$$
$$= \nabla \mathbf{f}(\mathbf{g}(\mathbf{x})) \cdot \nabla \mathbf{g}(\mathbf{x})$$

Divergence of $\mathbf{v} : \mathbb{R}^d \to \mathbb{R}^d$

$$\operatorname{div} \mathbf{v} = \nabla \cdot \mathbf{v}$$

Rotation, curl of $\mathbf{g} : \mathbb{R}^3 \to \mathbb{R}^3$

$$\operatorname{rot} \mathbf{g} = \nabla \times \mathbf{g}$$

Laplacian operator

$$\Delta f = \nabla \cdot \nabla f$$

Curve $\Gamma : \mathbb{R}^1 \to \mathbb{R}^d$

$$\Gamma = \{\boldsymbol{\gamma}(t) \in \mathbb{R}^d, t \in [a,b] \subset \mathbb{R}^1\}$$

Tangential vector

$$\frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{\gamma}(t) = \boldsymbol{\gamma}'(t)$$

Balance point, centre of mass

$$\mathbf{x}_s = \frac{1}{m} \int_\Omega \mathbf{x} \rho(\mathbf{x}) \ \mathrm{d}V \ , \ m = \int_\Omega \rho(\mathbf{x}) \ \mathrm{d}V$$

Moment of inertia

$$J = \int_\Omega r(\mathbf{x})^2 \rho(\mathbf{x}) \ \ \mathrm{d}V$$

Steiner's theorem $J_p = J_s + ||\mathbf{y}||_2^2 \cdot m$

Line integral, curve integral

$$\int_\Gamma f(\mathbf{x}) \ \mathrm{d}s = \int_a^b f(\boldsymbol{\gamma}(t)) \cdot ||\boldsymbol{\gamma}'(t)||_2 \ \mathrm{d}t$$

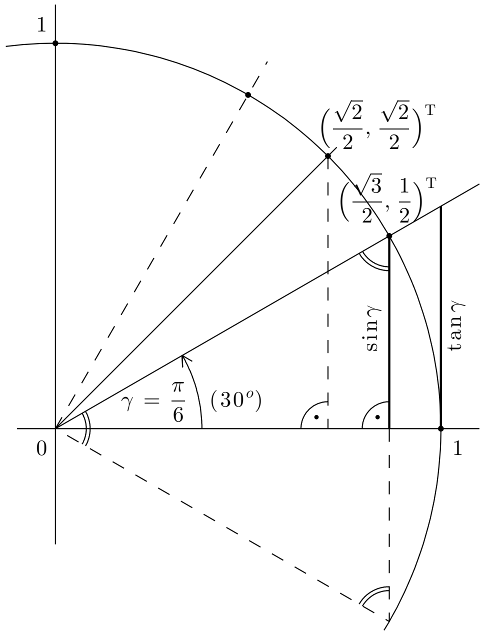
Line integral, work integral

$$\int_\Gamma \mathbf{g}(\mathbf{x}) \cdot \mathrm{d}\mathbf{x} = \int_a^b \mathbf{g}(\boldsymbol{\gamma}(s)) \cdot \boldsymbol{\gamma}'(s) \ \mathrm{d}s$$

Euler's identity

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

Trigonometric functions



Trigonometric relations

$$1 = \sin^2 x + \cos^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$\cos t = \frac{1}{2}\left(e^{it} + e^{-it}\right)$$

$$\sin t = \frac{1}{2i}\left(e^{it} - e^{-it}\right)$$

Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos kx + b_k \sin kx = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$$

with **Fourier coefficients**

$$a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx \; dx$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx \; dx$$

$$c_k = \frac{1}{2\pi}\int_0^{2\pi} f(x) e^{-ikx} \; dx$$

Hyperbolic functions

$$\cosh t = \frac{1}{2}(e^t + e^{-t})$$

$$\sinh t = \frac{1}{2}(e^t - e^{-t})$$

$$\operatorname{arsinh} t = \ln\left(t + \sqrt{t^2+1}\right)$$

$$\operatorname{arcosh} t = \ln\left(t + \sqrt{t^2-1}\right)$$

Partial fraction decomposition

$$\frac{p(x)}{(x-x_0)(x-x_1)} = \frac{A}{x-x_0} + \frac{B}{x-x_1}$$

$$\frac{p(x)}{(x-x_0)^k} = \frac{A_1}{x-x_0} + \frac{A_2}{(x-x_0)^2} + \dots + \frac{A_k}{(x-x_0)^k}$$

Differentiation

product rule $\quad (uv)' = u'v + uv'$

chain rule $\quad \frac{du}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$

$(x^n)' = n \cdot x^{n-1}, \; n \neq 0$

$(a^x)' = \ln a \cdot a^x, \; a > 0$

Partial integration

$$\int u(x)v'(x) \; dx = u(x)v(x) - \int u'(x)v(x) \; dx$$

Special integrals

$$\int_{-\infty}^{\infty} e^{-x^2} \; dx = \sqrt{\pi}$$

$$\int_0^{\pi} \sin^2 x \; dx = \frac{\pi}{2}$$

$$\int \ln x \; dx = x \cdot \ln x - x + c \;, \; c \in \mathbb{R}$$

$$\int x \sin kx \; dx = \frac{\sin kx - kx\cos kx}{k^2} + c$$

$$\int x \cos kx \; dx = \frac{kx \sin kx + \cos kx}{k^2} + c$$

$$\int x^2 \sin kx \; dx = \frac{(2-k^2x^2)\cos kx + 2kx \sin kx}{k^3} + c$$

$$\int x^2 \cos kx \; dx = \frac{(k^2x^2-2)\sin kx + 2kx \cos kx}{k^3} + c$$

p-q-formela for $x^2 + px + q = 0$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$