Technische Universität Braunschweig
Institute for Partial Differential Equations

Prof. Dr. D. Langemann, Dr. M. Stautz, Dr. C. Reisch, K. Tüting, M. Sc., A. Klopp, M. Sc.

Formulary Mathematics for Engineers
Linear Algebra

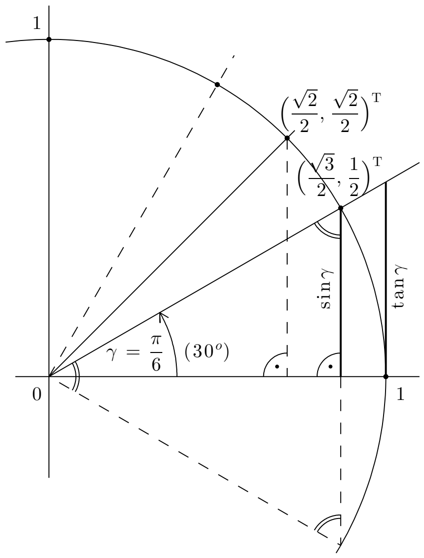
Residue class

$$\bar{x} = \{y \in \mathbb{Z} : x \equiv y, \mathrm{mod}(m)\} \text{ , for fixed } m$$

Complex conjugate

$$\overline{z} = x - \mathrm{i}y \in \mathbb{C} \text{ of } z = x + \mathrm{i}y \in \mathbb{C}$$

Trigonometric functions



Euler's identity

$$\mathrm{e}^{\mathrm{i}\varphi} = \cos\varphi + \mathrm{i}\sin\varphi$$

Trigonometric relations

$$1 = \sin^2 x + \cos^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$\cos t = \frac{1}{2}\left(\mathrm{e}^{\mathrm{i}t} + \mathrm{e}^{-\mathrm{i}t}\right)$$

$$\sin t = \frac{1}{2\mathrm{i}}\left(\mathrm{e}^{\mathrm{i}t} - \mathrm{e}^{-\mathrm{i}t}\right)$$

n-th complex roots of unity

$$\epsilon_k = \mathrm{e}^{2\pi\mathrm{i}\frac{k}{n}} \text{ , } k = 0, \ldots, n-1$$

Linear combination of $\mathbf{v_1}, \ldots, \mathbf{v_r}$

$$\mathbf{v} = \lambda_1 \mathbf{v_1} + \cdots + \lambda_r \mathbf{v_r} = \sum_{k=1}^{r} \lambda_k \mathbf{v_k}$$

Norm map $||\cdot|| : V \to \mathbb{R}$ with

(N1) $\forall \lambda \in \mathbb{R}$, $\forall \mathbf{x} \in V : ||\lambda \mathbf{x}|| = |\lambda| \cdot ||\mathbf{x}||$

(N2) $\forall \mathbf{x} \in V : ||\mathbf{x}|| = 0 \Rightarrow \mathbf{x} = \mathbf{0}$

(N3) $\forall \mathbf{x}, \mathbf{y} \in V : ||\mathbf{x} + \mathbf{y}|| \leq ||\mathbf{x}|| + ||\mathbf{y}||$

p-Norm

$$V = \mathbb{C}^n \text{ , } ||\mathbf{x}||_p = \left(\sum_{k=1}^{n} |x_k|^p\right)^{\frac{1}{p}} \text{ , } p \geq 1$$

Function space $C([a,b])$

set of continuous functions $f : [a,b] \to \mathbb{R}$

Function space $L_2([a,b])$

set of all $f : [a,b] \to \mathbb{C}$ with $\int\limits_a^b |f(x)|^2 \, \mathrm{d}x < \infty$

L_2-Skalar product of $f, g : [a,b] \to \mathbb{C}$

$$\langle f, g \rangle = \int\limits_a^b f(x)\overline{g(x)} \, \mathrm{d}x$$

Induced norm

$$||\mathbf{v}|| = \sqrt{\langle \mathbf{v}, \mathbf{v} \rangle}$$

Projection of $\mathbf{x}$ onto $\mathbf{u}$

Lot $\quad \mathbf{w} = \mathbf{x} - \dfrac{\langle \mathbf{x}, \mathbf{u} \rangle}{||\mathbf{u}||^2} \cdot \mathbf{u}$,

Projection $\quad \lambda \mathbf{u} = \dfrac{\langle \mathbf{x}, \mathbf{u} \rangle}{||\mathbf{u}||^2} \cdot \mathbf{u}$

Cauchy-Schwarz inequality

$$|\langle \mathbf{x}, \mathbf{y} \rangle| \leq ||\mathbf{x}|| \cdot ||\mathbf{y}||$$

Pythagorean theorem

$$\langle \mathbf{x}, \mathbf{y} \rangle = 0 \;\Leftrightarrow\; ||\mathbf{x} + \mathbf{y}||^2 = ||\mathbf{x}||^2 + ||\mathbf{y}||^2$$

Kronecker symbol

$$\delta_{k\ell} = \begin{cases} 0 & \text{for } k \neq \ell \\ 1 & \text{for } k = \ell \end{cases}$$

Surjective function $\varphi : V \to W$

$$\forall w \in W \; \exists v \in V : \; \varphi(v) = w$$

Injective function $\varphi : V \to W$

$$\forall v, v' \in V : \; \varphi(v) = \varphi(v') \;\Rightarrow v = v'$$

Inverse map

$$v = \varphi^{-1}(w) \;\Leftrightarrow\; \varphi(v) = w$$

Image of $\varphi : \mathbb{R}^n \to \mathbb{R}^m$ resp. $A \in \mathbb{R}^{m \times n}$

$$\operatorname{im} \varphi = \operatorname{im} A = \{\mathbf{w} \in \mathbb{R}^m : \mathbf{w} = \varphi(\mathbf{v}) = A\mathbf{v}, \mathbf{v} \in V\}$$

Null space or **kernel** of $\varphi : \mathbb{R}^n \to \mathbb{R}^m$ resp. $A \in \mathbb{R}^{m \times n}$

$$\ker \varphi = \ker A = \{\mathbf{v} \in \mathbb{R}^n : \varphi(\mathbf{v}) = A\mathbf{v} = \mathbf{0}\}$$

Rank of $\varphi : \mathbb{R}^n \to \mathbb{R}^m$ resp. $A \in \mathbb{R}^{m \times n}$

$$\operatorname{rg} A = \dim \operatorname{im} \varphi = \dim \operatorname{im} A$$

Inverse matrix

$$A^{-1}A = AA^{-1} = I$$
$$(AB)^{-1} = B^{-1}A^{-1}$$

Determinant, $A, B \in \mathbb{C}^{n \times n}$

$$\det(AB) = \det A \cdot \det B$$
$$\det A^{\mathrm{T}} = \det A$$
$$\det A^{-1} = \frac{1}{\det A}$$
$$\det(\mu A) = \mu^n \det A$$

Spectral radius of $A \in \mathbb{C}^{n \times n}$ with eigenvalues $\lambda_1, \ldots, \lambda_n \in \mathbb{C}$

$$\varrho(A) = \max_{k=1,\ldots,n} |\lambda_k| \in \mathbb{R}_{\geq 0}$$

Eigenspace

$$W = \ker(A - \lambda I)$$

Diagonalisable matrix

$$\exists V \text{ with } \det V \neq 0 \text{ and } V^{-1}AV = \Lambda$$

Algebraic multiplicity: multiplicity of the zero of the characteristic polynomial

Geometric multiplicity: number of respective Jordan boxes, dimension of the eigenspace

pq formula for $x^2 + px + q = 0$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$