Technische Universität Braunschweig

MUSEN

Sparse Grid Collocation for Uncertainty Quantification in Groundwater Flow

Lecture of

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In this talk we consider an elliptic partial differential equation (PDE) describing groundwater flow, but with an uncertain diffusion coefficient. This accounts for the fact that in practice physical properties of subsurface layers such as hydraulic conductivity are typically unknown and only finitely many local and noisy measurements of these properties are available. The objective is then to predict exit times of pollutants transported by the (uncertain) groundwater flow. We take an uncertainty quantifying (UQ) approach, model the uncertain coefficient as a lognormal random field based on the available data, and aim for computing the resulting probability distribution of the exit times, because the random coefficient yields a random solution of the PDE. Since a plain Monte Carlo approach can become prohibitively expensive we propose a surrogate approximating the random PDE solution. This surrogate is based on a Karhunen-Loeve expansion (KLE) of the random diffusion field and approximates the high-dimensional mapping from the KLE coefficients to the random PDE solution. In order to allow for a high-dimensional approximation we use sparse grid collocation methods. Sparse grid techniques are able to break the curse of dimensionality, i.e., the computational effort does not grow exponentially w.r.t. the dimension, in fact it only grows polynomially. The idea of sparse grids dates back to Russian mathematicians in the 1970s and they were again promoted for numerical simulation by Christoph Zenger in the early 1990s. We discuss their definition and properties, and how they can be used in an UQ context. Our results demonstrate that sparse grid collocation can reduce the computational work for our UQ application by a factor of 100 (compared to Monte Carlo) for a moderate number of uncertain parameters. In the last part of the talk, we discuss some recent developments in the theoretical analysis of sparse grid collocation in infinite dimensions and show that adaptive sparse grid collocation algorithms can achieve a dimension-independent convergence rate.