Advanced Reynolds stress turbulence modeling of subsonic and transonic flows

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Abstract

In order to increase the performance of RANS modeling in complex flows, Reynolds-stress turbulence models (RSM) are investigated. With the direct calculation of Reynolds stresses using transport equations, several flow effects like anisotropy of the Reynolds-stress tensor or streamline curvature can be naturally captured, which are not considered by eddy-viscosity models. At the Institute of Fluid Mechanics of TU Braunschweig, a Reynolds-stress model by Jakirlić and Hanjalić was implemented into the DLR flow solver TAU which uses the homogeneous part of the dissipation rate ε^h as length-scale variable (ε^h-RSM).

Here are presented the simulation results of several typical validation cases, including the subsonic 2D flow around the horizontal stabilizer HGR-01, the transonic 2D test case RAE 2822 Case 9 as well as the transonic 3D flow around the airfoil ONERA M6. The performance of the ε^h-RSM in the HGR-01 test case is satisfying, the trailing edge stall at maximum lift is predicted in good agreement with experimental results. However, in the transonic test case RAE 2822 discrepancies to experimental data concerning the shock location on the upper surface arise. Therefore different configurations of the Reynolds-stress model are tested and compared.

Nomenclature

a_{ij}	Stress anisotropy tensor	c_p	Pressure coefficient
A	Stress flatness parameter	$C^*_{\varepsilon 4}$	Coefficient in $S_{\varepsilon 4}$
A_2	Stress anisotropy invariant	D^t_{ij}	Turbulent diffusion tensor
b	Wing span	D^ν_{ij}	Viscous diffusion tensor
c	Chord length	k	Turbulent kinetic energy
c_f	Skin friction coefficient	l	Turbulent length scale

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Ma	Mach number	$\tilde{W}_{ij}$	Mean vorticity
P_{ij}	Reynolds-stress production tensor	y^+	Non-dimensionalized wall distance
Re	Reynolds number	**Symbols:**	
$\tilde{S}_{ij}$	Mean rate of strain	α	Angle of attack
$S_{\varepsilon 4}$	Pressure gradient term	δ_{ij}	Kronecker symbol
S_l	Length scale limiting term	ε	Dissipation rate of k
Tu	Turbulent intensity	ε^h	Homogeneous part of ε
$u_i = u, v, w$	Velocity fluctuations	$\tilde{\varepsilon}^h$	Isotropic part of ε^h
$U_i = U, V, W$	Mean flow velocities	ε_{ij}	Dissipation rate tensor
U_e	Boundary-layer edge velocity	ν	Kinematic viscosity
U_∞	Freestream velocity	Φ_{ij}	Redistribution tensor
$\overline{u_i u_j}$	Reynolds-stress tensor	ω	Specific dissipation rate

1 Introduction

For the numerical simulation of relevant flows in aircraft aerodynamics, the statistical treatment of turbulence is an effective way to reduce calculation costs. This can be achieved by solving the Reynolds Averaged Navier-Stokes equations (RANS) which describe the time-averaged motion of fluid flow [1]. In these equations the turbulence appears as Reynolds-stress tensor containing six different Reynolds stresses, which need to be provided by a turbulence model. Most of the turbulence models currently used are based on the Boussinesq assumption. The Boussinesq assumption, as an analogue of the molecular viscosity, sets the Reynolds-stress tensor in relation to the mean strain rate tensor, with the scalar eddy viscosity μ_t as a constant of proportionality. These eddy-viscosity models (EVM) produce satisfying results for simple aerodynamic flows, but they fail in more complex flows including flow separation, streamline curvature or strong effects of stress anisotropy. In order to increase the performance of RANS modeling in more complex flows, Reynolds-stress turbulence models (RSM) are investigated.
RSMs directly calculate the Reynolds stresses via transport equations, hence the Boussinesq assumption is discarded. Consequently turbulent stress anisotropy can be captured naturally. Beside the Reynolds-stress equations, RSMs calculate an additional transport equation in order to evaluate the turbulent length scale, which is generally expressed by the dissipation rate ε or the specific dissipation rate ω. At the Institute of Fluid Mechanics of TU Braunschweig, a Reynolds-stress model by Jakirlić and Hanjalić [2] was implemented into the DLR flow solver TAU [3], which uses the homogeneous part of the dissipation rate as length scale variable [4]. This turbulence model has already shown its capabilities to predict trailing edge stall in subsonic flows during the implementation process.
Within the framework of the research project *ComFliTe*, different subsonic and transonic airfoil flows are simulated in order to check the industrial applicability of this turbulence model. Additionally, different variations of the length scale equation as well as variations of the redistribution term are investigated. The results of the subsonic 2D test case HGR-01 as well as the transonic test cases RAE 2822, Case 9 (2D) and ONERA M6 (3D) are presented in this paper.

2 Numerical method

2.1 RANS Solver

The DLR solver TAU uses a finite-volume method to calculate the Reynolds Averaged Navier-Stokes (RANS) equations on unstructured or hybrid grids. It can be chosen from several central and upwind spatial discretization schemes, as well as from explicit and implicit time-stepping schemes. In order to accelerate the convergence local time stepping, residual smoothing and multigrid is implemented. Furthermore a low-Mach number preconditioning for computing incompressible flows is available.

2.2 Turbulence modeling

Reynolds-stress turbulence models determine the Reynolds-stress tensor by solving a transport equation for each element. An incompressible form of the Reynolds-stress equations can be written as:

$$\frac{D\overline{u_i u_j}}{Dt} = P_{ij} + \Phi_{ij} - \varepsilon_{ij} + D^{\nu}_{ij} + D^{t}_{ij} \quad (1)$$

Moreover, a transport equation for the length scale of turbulence is required. Usually either the dissipation rate ε or the specific dissipation rate ω is chosen as the additional variable. A major issue in Reynolds-stress turbulence modeling is to reproduce the behaviour of turbulence close to walls. Jakirlić and Hanjalić have shown that using a transport equation for the homogeneous dissipation rate ε^h for closing of the equation system can improve the reproduction of DNS results of ε in the near-wall region [2]. The non-homogeneous part of the dissipation rate is equal to one half of the molecular diffusion of the turbulent kinetic energy, hence:

$$\varepsilon = \varepsilon^h + \frac{1}{2}\nu\frac{\partial^2 k}{\partial x_l \partial x_l} \quad (2)$$

The anisotropic dissipation tensor ε_{ij}, which is required to solve the Reynolds-stress equations (eq. 1), is calculated with an algebraic relation from the dissipation rate ε.
Additionally, a source term S_l has been introduced into the ε^h-equation in order to limit unphysical growth of length scale in local non-equilibrium turbulence [5]:

$$S_l = max\left\{\left[\left(\frac{1}{C_l}\frac{\partial l}{\partial x_n}\right)^2 - 1\right]\left(\frac{1}{C_l}\frac{\partial l}{\partial x_n}\right)^2 ; 0\right\}\frac{\varepsilon^h\tilde{\varepsilon}^h}{k}A \quad \text{with} \quad C_l = 2.5 \quad (3)$$

A second additional source term has been inserted into the length scale equation to sensitize the flow to adverse pressure gradients [6], [7]:

$$S_{\varepsilon 4,2D} = -C^*_{\varepsilon 4}\frac{\varepsilon}{k}\left(\overline{u^2_s}\frac{\partial U_s}{\partial x_s} + \overline{u^2_n}\frac{\partial U_n}{\partial x_n}\right) \quad \text{with} \quad C^*_{\varepsilon 4} = 1.16 \quad \text{(simplified 2D form)} \quad (4)$$

The production term in equation 1 can be treated exactly as:

$$P_{ij} = -\left(\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k}\right) \quad (5)$$

For the redistribution term, a linear formulation

$$\Phi_{ij} = -C_1\varepsilon a_{ij} - C_2\left(P_{ij} - \frac{2}{3}P_k\delta_{ij}\right) + \phi^w_{ij} \quad (6)$$

as well as a quadratic formulation

$$\begin{aligned}\Phi_{ij} = -\varepsilon \left[C_1 a_{ij} + C_1' \left(a_{ik}a_{jk} - \frac{1}{3}\delta_{ij}A_2\right)\right] \\ + C_3 k\tilde{S}_{ij} + C_4 k \left(a_{ip}\tilde{S}_{pj} + a_{jp}\tilde{S}_{pi} - \frac{2}{3}a_{pq}\tilde{S}_{pq}\delta_{ij}\right) \\ + C_5 k \left(a_{ip}\tilde{W}_{pj} + a_{jp}\tilde{W}_{pi}\right) - C_2' a_{ij} P_k\end{aligned} \tag{7}$$

are implemented [8].
The diffusion is calculated with the diffusion model of Daly and Harlow [9].

3 Test cases

3.1 HGR-01

The first test case that is investigated is the subsonic 2D horizontal stabilizer HGR-01. The onflow conditions are set to $\alpha = 12°$, Ma = 0.073 and Re = 656 000. Detailed experimental results for this airfoil flow were generated in the low speed wind tunnel MUB of the Institute of Fluid Mechanics, TU Braunschweig, including oil flow visualisation, pressure distribution and PIV measurements [10], [11]. Numerical simulations using the ε^h-RSM for angles of attack ranging from 0° to 14° were already conducted during the implementation phase of the turbulence model by Probst [4] on a grid with a high spatial resolution (105 000 pts.), especially in the trailing edge region.
For this investigation of industrial applicability, a coarser hybrid grid with 63 000 total points is used, containing a structured part of 390 x 104 points. The angle of attack $\alpha = 12°$ was chosen, this yields a flow near maximum lift with a trailing edge separation at approximately $x/c = 0.9$. Steady-state solutions were calculated using the ε^h-RSM and a central discretization scheme, low-Ma number preconditioning and scalar dissipation.
Eddy-viscosity models usually tend to underestimate the trailing edge separation, even the SSG/LRR-ω-RSM [12] predicts a lower momentum loss in the boundary layer and a smaller separation as can be seen in figure 1. However, the ε^h-RSM in its initial configuration slightly overestimates the momentum loss in the boundary layer. Using the quadratic redistribution formulation instead of the linear formulation produces no remarkable changing of the flow solution. A considerable influence on the trailing edge separation is exerted by the additional source term $S_{\varepsilon 4}$. Halving the coefficient $C^*_{\varepsilon 4}$ decreases the momentum loss in the boundary layer, especially close to the trailing edge.
Improved numerical settings, e.g. matrix dissipation instead of scalar dissipation, lead to an unsteady movement of the trailing edge separation. A time-discrete simulation with the ε^h-RSM shows an oscillation of the bubble, the averaged separation size decreases in comparison to the steady-state computation. The velocity distributions in the boundary layer show a good agreement to the experimental results, furthermore the RANS simulations of Probst [4] with a high spatial resolution could be reproduced.

3.2 RAE 2822

The simulation of the 2D airfoil RAE 2822, with the onflow conditions Ma = 0.73, Re = $6.5 \cdot 10^6$ and $\alpha = 2.8°$ (Case 9), is a transonic test case with a shock located on the upper surface at about $x/c = 0.55$. The calculations are conducted on a hybrid grid with totally 44000 points, the

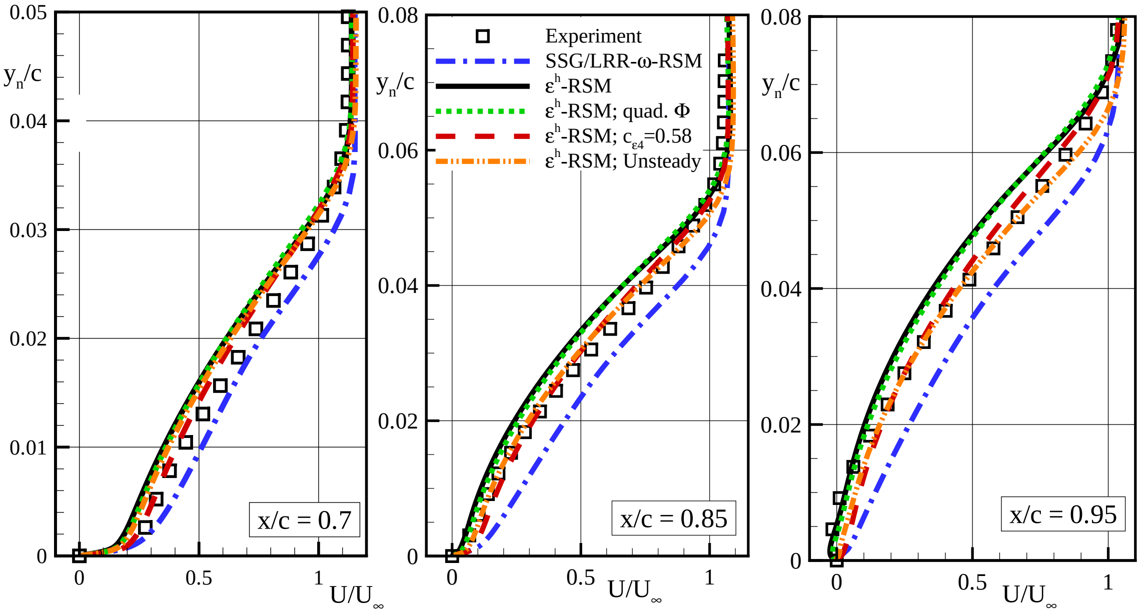

Figure 1: HGR-01: boundary layer profiles near the trailing edge for different RSM configurations

structured part of 350x88 points is shown in figure 2. For different variations of the Reynolds-stress model, the c_p-distributions can be seen in figure 3 in comparison to the SSG/LRR-ω-RSM and to experimental results by Cook et al. [13]. The unmodified ε^h-RSM (solid line) shows

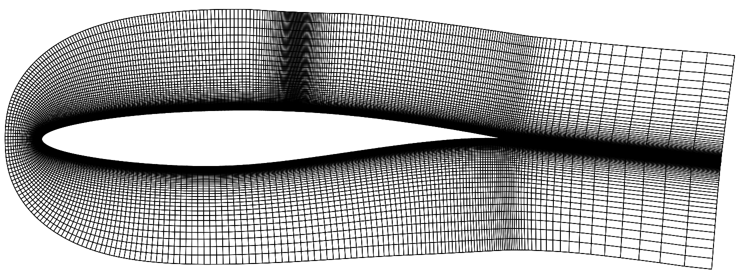

Figure 2: RAE 2822: structured grid with 350x88 points

a deviation of the shock location of about 5%, resulting from an overestimated displacement thickness of the boundary layer near the trailing edge. A slight improvement can be achieved by using the quadratic redistribution formulation instead of the linear formulation. Similarly to the HGR-01 test case, the additional source term $S_{\varepsilon 4}$ has noticable influence on the flow solution. When halving the coefficient $C^*_{\varepsilon 4}$, displacement thickness of the boundary layer is reduced and the shock is displaced about 2% downstream. Likewise the experimental shock location can be approximated by using ε instead of ε^h as variable for the length scale equation (ε-RSM).

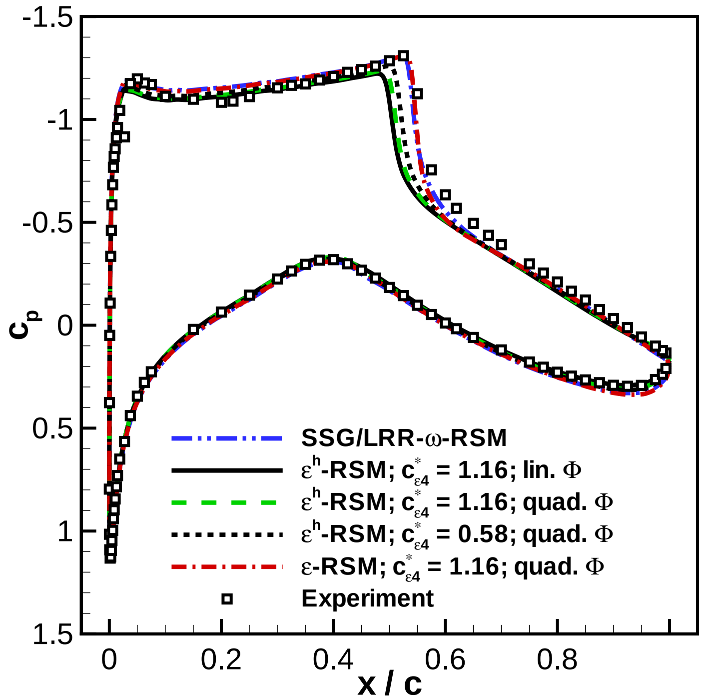

Figure 3: RAE 2822: c_p-distributions for different RSM

3.3 ONERA M6

The calculation of the flow around the 3D semi-span wing ONERA M6 [14] is conducted at $\mathrm{Ma} = 0.84$, $\mathrm{Re} = 11.72 \cdot 10^6$ and $\alpha = 3.06°$. The hexahedral grid was created at the DLR with a C-topology containing $1.76 \cdot 10^6$ points and an average y^+ around 0.8.
Figure 4 shows the pressure contours and the wall streamlines calculated with the ε^h-RSM, converged to a steady-state solution. The position of the transition line is fixed directly downstream of the suction peak on the upper surface, this corresponds approximately to $x/c = 0.05$. Near the symmetry plane, the transonic flow experiences two shocks. At about $y/b = 0.90$, the two shocks merge and induce a small separation. The pressure distribution at different spanwise positions are in good agreement with the experimental data (fig. 5).

In order to check the influence of the spatial discretisation on the solution, further calculations on a refined grid with $13.96 \cdot 10^6$ points were conducted. Using the ε^h-RSM and a steady-state solver on this grid, an oscillating interaction between the shock and the separation occurs leading to an unphysical flow solution. The Reynolds stresses in the separated near-wall region almost vanish, consequently the separation grows and moves upstream into the transitional region. Simultaneously the shock is shifted upstream.
When simulating steady-state with the ε-RSM, the flow solution remains stable without converging (fig. 6, top). An unsteady calculation of 1.5 convective runs restarted from the steady-state solution shows no changing in streamlines or pressure distributions (fig. 6, bottom). According to the RAE 2822 test case, the shocks are situated further downstream when using the ε-RSM in comparison to the ε^h-RSM. The pressure distributions are in a good agreement with the experimental results on the inner wing for both RSM configurations, near the wing tip the discrepancy of shock locations simulated with the ε^h-RSM grows.

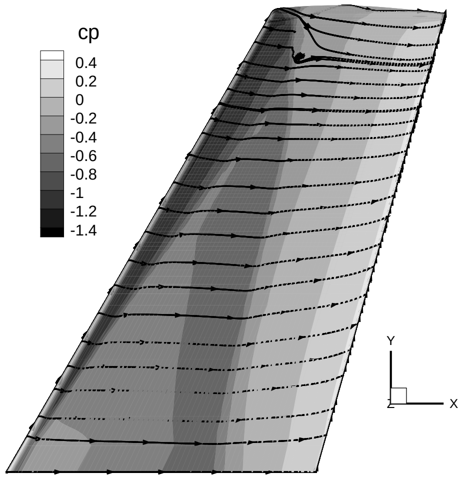

Figure 4: ONERA M6: pressure contours and wall streamlines

4 Conclusion

Subsonic and transonic test cases have been simulated with a near-wall Reynolds stress turbulence model, which uses the homogeneous part of the dissipation rate ε^h as variable for the length scale equation. Previous investigations have shown the good performance of this turbulence model in subsonic flows with trailing edge separation. These results could be reproduced for the test case HGR-01.

Unsatisfying results are achieved when simulating the transonic 2D test case RAE 2822, Case 9. The boundary layer thickness at the trailing edge is overestimated, leading to a shock position upstream of the experimental results. The performance can be improved by decreasing the additional source term $S_{\varepsilon 4}$.

First calculations using the ε^h-RSM for the 3D test case ONERA M6 show reasonable results in comparison to experimental pressure distributions, while the unphysical disappearance of Reynolds stresses when using the steady-state solver on the fine grid needs further investigation. The influence of the additional source term $S_{\varepsilon 4}$, which is supposed to sensitize the flow to adverse pressure gradients, has been discussed, as especially in the RAE 2822 test case the displacement thickness is overestimated in adverse pressure gradient regions. In order to figure out a valid coefficient for the pressure gradient source term, further investigations of the ε^h-RSM at transonic flow conditions are recommended.

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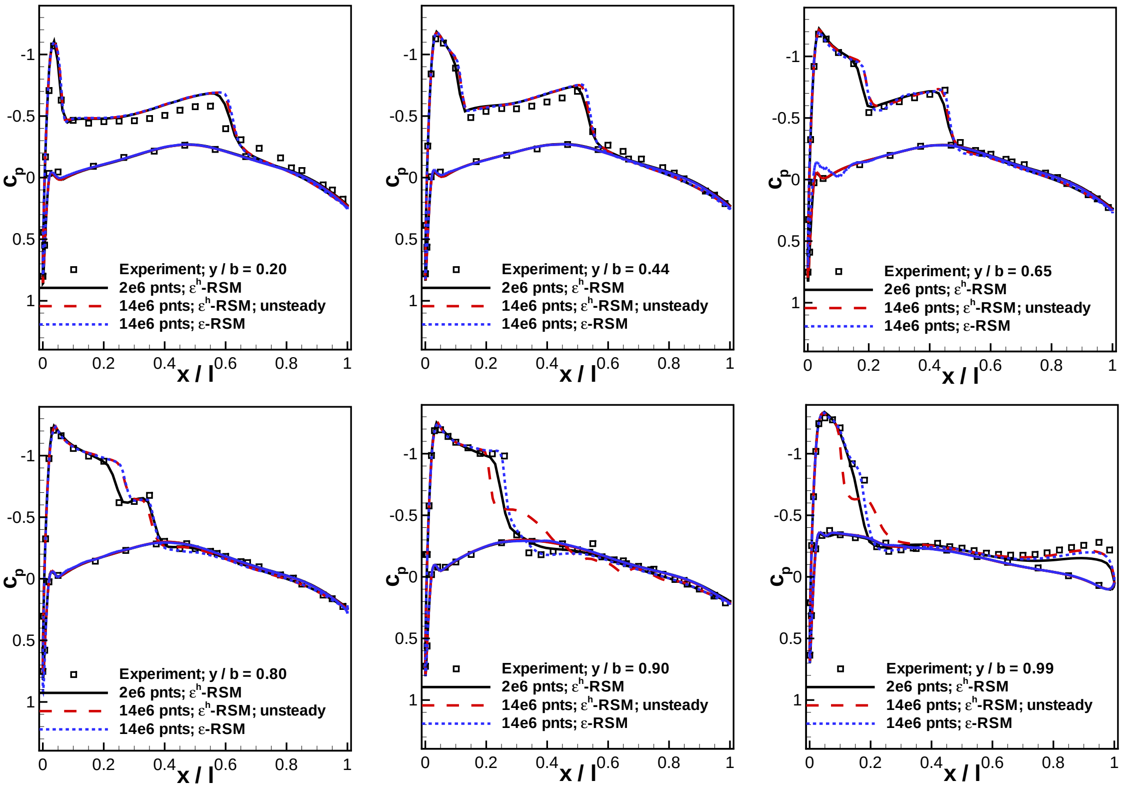

Figure 5: ONERA M6: pressure coefficient distribution at different spanwise locations, using different grids and RSM configurations

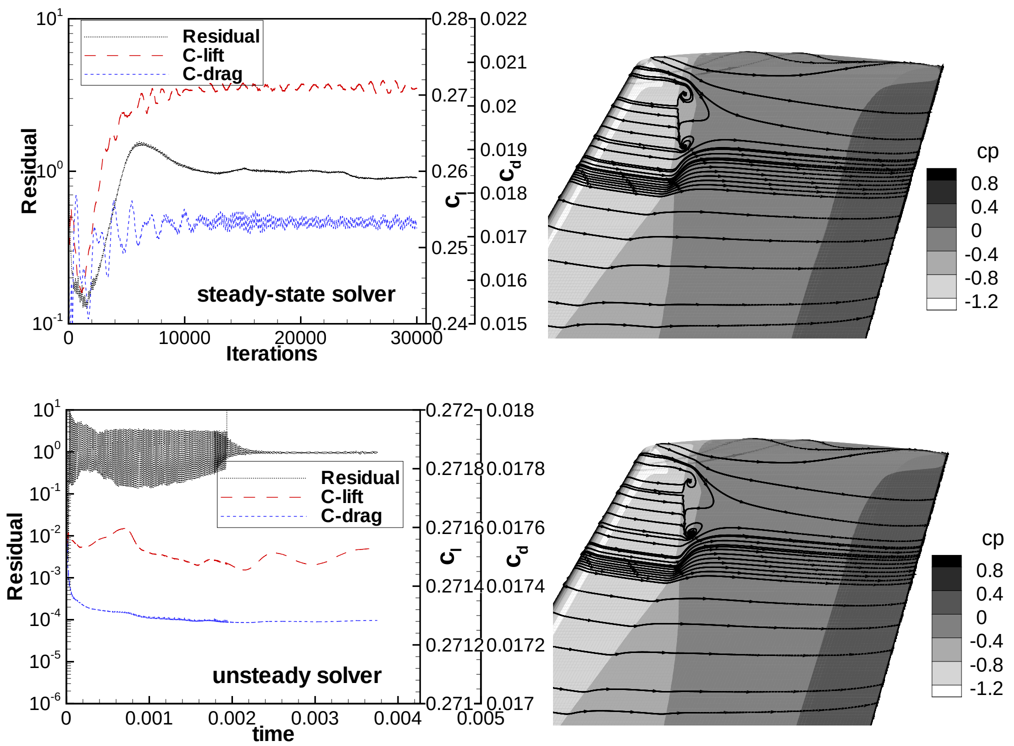

Figure 6: ONERA M6: developement of density residual, c_l and c_d (left) and shock-induced separation at the wing tip, simulated with the ε-RSM

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