LIST OF CORRECTIONS

LOCALLY PRESENTABLE AND ACCESSIBLE CATEGORIES

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The following is a list of corrections of all mistakes that have been to our knowledge discovered in our book. The authors are grateful to all who contributed to the list, most of all to D. Benson, M.Hébert, H.-E. Porst and Jan Jürjens.

Page 3 lines 5-6 (as well as Exercise 1.f(1)): This is incorrect and to be deleted.
Page 11 line -2: C should read ω^T
Page 12 line 10: a subset of D_n is open iff its intersection with A is M ... (and not open in A).
Page 14 line 17: all should read: all finite
Page 16 lines -3 to -1: To verify (2), observe that since $\mathcal{A}$ is a set and all categories are locally small, see 0.1, all split subobjects of objects of $\mathcal{A}$ form a set, up to isomorphism. It is sufficient to verify that every finitely presentable object K is a split subobject...
Page 21 line 16: 1.14(4) instead of 1.14(3)
Page 24 line 7 add: $n \geq 1$
Page 18 line 3: instead of "since .. contain $\overline{\mathcal{A}}$" the text should be: $\overline{\mathcal{A}}$ is a set because it the the union of the sets $\mathcal{A}_n, n \in N$, where

$\mathcal{A}_0 = \mathcal{A} \bigcup \{0\}$ (for an initial object 0)

and

$\mathcal{A}_{n+1} =$ the closure of $\mathcal{A}_n$ (under binary coproducts and coequalizers.)

Page 26 line -8: $\mathcal{A}^{mor}$ instead $\mathcal{A}^{obj}$
Page 30 line 1.33(7): the definition of m is to be corrected as follows:

$$m : \hom(A_1, -) + \hom(A_2, -) \to \hom(A_1 \times A_2, -)$$

has components $\hom(A_i, -) \to \hom(A_1 \times A_2, -)$ corresponding to the projections.
Page 31 line 1.33(8): analogously to page 30,

$$m_i : \mathrm{colim}_d \hom(D_i d, -) \to \hom(lim_d D_i d, -)$$

The same correction on **Page** 40 line 13
Page 31 line 18 at the end: $k_{t_0} \cdot f_1 \cdot m_i = k_{t_0,t} \cdot f_2 \cdot m_i$
Page 31 line 20: $k_{t_0} \cdot f_1 \cdot m_i = k_{t_0,t} \cdot f_2 \cdot m_i$ which implies, since K_t is orthogonal to m_i, that $k_{t_0,t} \cdot f_1 = k_{t_0,t} \cdot f_2$.
Page 33, line 3: The statement is incorrect: if the construction stops, the object X_{i_0} is orthogonal. The converse implication is not true in general (consider $M = \{m\}$ in Set for an embedding $m : 1 \to 1 + 1$: although 1 is orthogonal, the construction does not stop). However, if in the diagram on p.32 one adds for every pair $p, q : A' \to X_i$ the connecting morphism $m : A \to A'$ whose domain corresponds to f defined as $p.m(= q.m)$, then the resulting diagram makes the proposition true. This improved construction is used by Max Kelly in [Kelly 1980].
Pages 34-35: Theorem 1.39 is true for all uncountable cardinals λ, but the proof presented in our book is wrong. A correct proof can be found in the paper "Uncountable orthogonality is a closure property" by M. Hébert and J. Rosický, Bulletin of the London Mathematical Society 33 (2001), 685-688. For $\lambda = \omega$, the implication $(ii) \Longrightarrow (i)$ in Theorem 1.39 does not hold, and a more technical description of ω-orthogonality classes has been presented in the paper "More on injectivity in locally presentable categories" by M. Hébert, J. Adámek and J. Rosický (Cahiers de Topologie et Geometrie Diffèrentielle et Categorique 32(2001), 51-80). A counter-example to 1.39 is also presented in that paper, which is a modification of example by Hugo Volger, see Bibliography (Volger 1979). An independent counter-example has been found by Jan Jürjens in "On a Problem of Gabriel and Ulmer", Journal of Pure and Applied Algebra 158 (2001), 183-196.
Page 38 line 13: $d_{A,a} \cdot Ff = d_{A',Hf(a)}$
Page 38 In (ii) we need to add the argument why is $Cont_\lambda \mathcal{A}^{op}$ cocomplete. From 1.33(8) we know that is it a λ-orthogonality class in $Set^{\mathcal{A}^{op}}$, therefore, it is locally λ-presentable by Theorem 1.39.
Page 39 lines 10-18: It is not true that the categories c_λ-CAT and lp_λ-CAT are dually equivalent: the correct statement is that the corresponding 2-categories are biequivalent. See J. Adámek and H.-E. Porst: Algebraic theories of quasivarieties, Journal of Algebra 208 (1998), 379-398
Page 39 line 18: $Cont_\lambda\ \mathcal{A}$ instead of $Cont_\lambda\ \mathcal{A}^{op}$, and $F \mapsto F \cdot H$ instead of $F \mapsto F \cdot H^{op}$
Page 40 line 13: see the correction of page 30
Page 57 line 15: obvious that id_A does not factor through any k_n
Page 62 Exercise 1.1: The diagrams of $S_1 \otimes S_2$ are $\{A_1\} \times D_2$ and $D_1 \times \{A_2\}$ where A_i ranges through all objects of the underlying category of S_i, and

D_i through all diagrams of S_i, with

$$\sigma(\{A_1\} \times D_2) = \{id_{A_1}\} \times \sigma_2(D_2)$$

and

$$\sigma(D_1 \times \{A_2\}) = \sigma_1(D_1) \times \{id_{A_2}\}$$

The proof of Proposition 1.53 must be corrected analogously.
Page 65: for 1.s see the correction of page 39.
Page 69 2.3(2): Scott domains should read: CPOs
Page 71: Proposition 2.6 is true, but the proof we present is wrong. A correct proof can be found in the book Makkai and Pare (1989), see their Proposition 2.2.1.
Page 85 line -1: $\lambda' \leq \lambda$
Page 87: reference in (ii) should be 1.55(1)
Page 87 line -6: 1.55(1) instead of 1.55(2)
Page 88 line -7: $b_i : B_i \longrightarrow B \, (i \leq j)$
Page 88 line -4: $b_i \cdot f_i \cdot \bar{f}_i$ instead of $f_j \cdot \ \bar{f}_j \cdot b_{i,j}$
Page 88 line -1 to **Page** 89 line 1: $b_i \cdot f_i \cdot \bar{f}_i \cdot f_i = b_i \cdot f_i$ (instead of the equations there)
Page 98 line -5: left-hand side: $d^*_{j_0,j_{0'}}$ instead of $d^*_{j,j_{0'}}$; right-hand side: $d^*_{j_0,j_{0'}}$ instead of $D^*(j_0 \to j_{0'})$
Page 99 line 14 to 15: $C_L \downarrow F$ instead of $F \downarrow C_L$
Page 101 line -11: equivalent instead of isomorphic
Page 109 lines 17-18: ...whose Kan extension $F^* : Set^{\mathcal{B}} \to Set$ has, due to **C**, the property that G is naturally isomorphic to the domain restriction $F^*/\mathcal{A}$. And F^* preserves λ-small limits of hom-functors since G does (due to **L**).
Page 125: Delete 2.b(2)
Page 145 line 14: For infinite $\lambda > \operatorname{card} \Sigma$
Page 150 line -12: This functor is an embedding. In fact, since T is finitary, it is easy to verify that G is one-to-one. Moreover, given an algebra $k : TK \to K$, then every element x of TK corresponds to an operation symbol k' in Σ such that $k(x)$ is the result of k' in the algebra $G(K, k)$. From this it follows easily that G is full.
Page 152 line -10: $(s(x) = s(y)) \wedge (t(x) = t(y)) \Rightarrow x = y$
Page 154 line -8: change reference to example 3.21(1)
Page 155 in 3.24: The third sentence of the proof ("It is easy...projective.") is incorrect. The theorem is valid, in fact, it can be strengthened as follows. It suffices to assume the existence of a set $\mathcal{B}$ of finitely presentable regular

projectives such that any object is a regular quotient of a coproduct of objects from $\mathcal{B}$ (i.e., $\mathcal{B}$ is a regular generator). A correct proof of the stronger theorem is as follows:
Conversely, let $\mathcal{K}$ to a cocomplete category with a regular generator $\mathcal{B}$ of finitely presentable regular projectives. It is easy to see that a finite coproduct of regular projectives is a regular projective. Hence $\mathcal{B}$ can be assumed to be closed under finite coproducts in $\mathcal{K}$ and, consequently, $\mathcal{B}^{op}$ can be regarded as an FP sketch. Following Remark 3.17, the category $\mathcal{V} = Mod_{FP}\,(\mathcal{B}^{op})$ of all finite-product preserving functors $\mathcal{B}^{op} \to Set$ is equivalent to a variety.
Let $E : \mathcal{K} \to \mathcal{V}$ be the codomain-restriction of the canonical functor (cf. 1.25). Following 1.26 and 1.27, E is a faithful right adjoint preserving directed colimits. Since all objects in $\mathcal{B}$ are regular projectives, E also preserves regular epimorphisms. We will prove that E preserves coproducts of objects from $\mathcal{B}$. Since E preserves directed colimits, it suffices to work with finite coproducts. We now simply follow the argument in H.Schubert, Kategorien, Akademie-Verlag, Berlin 1970 (see 10.3.5 there):
Let

$$B_1 \xrightarrow{i_1} B_1 + B_2 \xleftarrow{i_2} B_2$$

be a coproduct in $\mathcal{B}$, and

$$EB_1 \xrightarrow{l_1} EB_1 + EB_2 \xleftarrow{l_2} EB_2$$

be a coproduct in $\mathcal{V}$. Let

$$f : EB_1 + EB_2 \longrightarrow E(B_1 + B_2)$$

be the induced morphism. By the argument from the implication $(i) \Rightarrow (ii)$ in the proof of 1.46, we get a morphism $g : E(B_1 + B_2) \longrightarrow EB_1 + EB_2$ satisfying $g.E\,i_k = l_k$, $k = 1, 2$. We have $g \cdot f = id_{EB_1+EB_2}$ and, since E is full on $\mathcal{B}$, $f \cdot g = id_{E(B_1+B_2)}$. Therefore $E(B_1 + B_2) \cong EB_1 + EB_2$.
Next we prove that E is full. Consider a morphism $f : EK \longrightarrow EK'$ in $\mathcal{V}$. There exists a regular epimorphism $p : \coprod_{j\in J} B_j \longrightarrow K$ where $w_i : B_i \longrightarrow \coprod_{j\in J} B_j$ is a coproduct of objects from $\mathcal{B}$. There are $h_j : P_j \longrightarrow K'$ such that

$$f\ .\ Ep\ .\ Ew_j = Eh_j$$

for any $j \in J$. Since E preserves coproducts of $\mathcal{B}$-objects, we get $h : \coprod_{j\in J} B_j \longrightarrow K'$ with $f \cdot Ep = Eh$.
Let

$$L \overset{u}{\underset{v}{\rightrightarrows}} \coprod_{j\in J} B_j \xrightarrow{p} K$$

be a kernel pair. Then

$$EL \overset{Eu}{\underset{Ev}{\rightrightarrows}} E\coprod_{j\in J} B_j \xrightarrow{Ep} EK$$

is a kernel pair. Since

$$Eh \cdot Eu = f \cdot Ep \cdot Eu = f \cdot Ep \cdot Ev = Eh \cdot Ev,$$

we get $hu = hv$ and, therefore, we obtain a morphism $t : K \longrightarrow K'$ with $t \cdot p = h$. Hence $Et \cdot Ep = Eh = f \cdot Ep$, thus $f = Et$.

We have proved that $\mathcal{K}$ is equivalent to a full reflective subcategory of $\mathcal{V}$ closed under directed colimits. Following Theorem 3.22, it remains to prove that $E(\mathcal{K})$ is epireflective in $\mathcal{V}$. Any A in $\mathcal{V}$ can be presented by a coequalizer

$$E\coprod_{j\in J} B_j \overset{Eu}{\underset{Ev}{\rightrightarrows}} E\coprod_{i\in I} C_i \xrightarrow{p} A$$

in $\mathcal{V}$ where B_j, C_i belong to $\mathcal{B}$ for $i \in I$ and $j \in J$. Then a reflection $r_A : A \longrightarrow EA^*$ of A to $E(\mathcal{K})$ is given by a coequalizer

$$\coprod_{j\in J} B_j \overset{u}{\underset{v}{\rightrightarrows}} \coprod_{i\in I} C_i \xrightarrow{q} A^*$$

in $\mathcal{K}$ via $r_A \cdot p = Eq$. Since Eq is a regular epimorphism in $\mathcal{V}$ and $\mathcal{V}$ is a variety, r_A is a regular epimorphism in $\mathcal{V}$.
Page 157: Delete "colimit of... is full" from lines 6 to 7. Insert: To prove that H is full, let $p : HK_1 \to HK_2$ be a homomorphism. The canonical diagram of K_1 w.r.t. $\mathcal{B}$ has the following compatible cocone:

$$\text{to } f : P_s \to K_1 \text{ assign } (U^*p)_s(f) : P_s \to K_2.$$

The compatibility, i.e., $(U^*p)_s(f) \cdot h = (U^*p)_{s'}(f \cdot h)$ for all $h : P_{s'} \to P_s$, follows from the commutation of p with the Σ-operation $\sigma = \sigma_{X_s,h} : s \to s'$: we have

$$(U^*p)_{s'}(f \cdot h) = [(U^*p)_s \cdot \sigma_{HK_1}](f) = [\sigma_{HK_2}(U^*p)_s](f) = (U^*p)$$

Since $\mathcal{B}$ is dense, there exists a morphism $q : K_1 \to K_2$ with $(U^*p)_s(f) = qf$ for all f, thus $(U^*p)_s = (Uq)_s = (U^*Hq)$, which proves $p = Hq$.
Page 159 line 16 Add:
$(x_0 \vee x_1 \vee x_2 \vee ...) \vee y_1 \vee y_2 \vee ... = x_0 \vee y_1 \vee x_1 \vee y_2 \vee x_2 \vee ...$
Page 163 line 12: $(s(x) = s(y)) \wedge (t(x) = t(y)) \Rightarrow x = y$

Page 163 line 18: Def $(\sigma) = \{s(x) = s(y), t(x) = t(y)\}$
Page 163 line -14: t(x) instead of t(s)
Pages 164 to 167: The proof of Theorem 3.36 is not correct. A correct proof is presented in J.Adámek, M.Hébert and J.Rosický: "On essentially algebraic theories and their generalizations", Algebra Universalis 41 (1999) 213-227.
Page 179 line -6: is weakly
Page 180: 4.10(3) is wrong
Page 184: In 4.15 the cone should be changed to the cone of *all* torsion quotients of $\mathbb{Z}$, i.e.:

$$
\begin{tikzcd}
\mathbb{Z} \arrow[r] \arrow[dr] \arrow[ddr] & \{0\} \\
& \mathbb{Z}_2 \\
& \mathbb{Z}_3 \\
& \vdots
\end{tikzcd}
$$

Page 190 Example: (1) From "Given ... " on the text should be changed as follows:
Given a strictly ordered poset $(X, <)$, a multireflection is formed by all canonical maps $c : (X, <) \to (X/\sim, \sqsubset)$ where

1. $\sim$ is an equivalence relation on X such that the derived relation $<^*$ on $X/\sim$ (with $M <^* N$ iff $m < n$ for some $m \in M, n \in N$) is irreflexive

and

2. $\sqsubset$ is a linear order extending $<^*$.

There is a missing arrow in the concrete example: the canonical arrow w.r.t. the equivalence with classes $\{a\}$ and $\{b, c\}$.
Page 192, Theorem 4.29: This theorem is wrong. See our Correction of pages 34-35: Given a reflective subcategory $\mathcal{A}$ of a locally finitely presentable category $\mathcal{K}$ closed under directed colimits, then if $\mathcal{A}$ is not an ω-orthogonality class, then it is not an ω-cone-orthogonality-class either. In fact, given a cone to which all objects of $\mathcal{A}$ are orthogonal, then there exists a member of that cone to which, again, all objects of $\mathcal{A}$ are orthogonal (this is an easy consequence of the fact that $\mathcal{A}$ is closed under products of pairs in $\mathcal{K}$). Thus, a presentation of $\mathcal{A}$ as a cone-orthogonality class reduces to a presentation as

an orthogonality class.
Page 195 line -6 and -3: $Set^{\mathcal{A}}$ should read Set^S
Page 197 line -6: regular epimorphisms instead of monomorphisms
Page 197 line -1 should read: projective w.r.t. every epimorphism with kernel A iff $\mathrm{Ext}(G, A) = 0$
Page 198 line 4: multicolimits
Page 201 lines -7 to -6: A has less then λ edges, i.e., card $\bigcup_{\sigma\in\Sigma_{rel}} \sigma_A < \lambda$ (instead of the equations there)
Page 207 in 5.8(3): the last axiom should be replaced by the following:
$(\forall x_1, ..., x_6)(comp(x_1, x_2, x_3) \wedge comp(x_2, x_4, x_6) \wedge comp(x_3, x_4, x_5) \Rightarrow comp(x_1, x_6, x_5))$, and
$(\forall x_1, ..., x_6)(comp(x_1, x_2, x_3) \wedge comp(x_2, x_4, x_6) \wedge comp(x_1, x_6, x_5) \Rightarrow comp(x_3, x_4, x_5))$
Page 208 line 8: 2.46 should read 2.48
Page 210 line 15: $id_X : (X, \emptyset) \to (X, X \times X)$
Page 213, second line under 5.17(3): of subsets of I such that the intersection of any finite subcollection is non-epmty, there exists...
Page 238 line 6: instead of "positive-primitive" write "conjunctions of atomic formuals"
Page 264 line 8: accessible iff it satisfies
Page 297: Problem 9, this is a repetition of Problem 5