Magnetic field-induced anomalies and Kondo effect in lanthanide in heavy-electron systems

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We investigate the influence of magnetic fields on the heavy quasiparticles in the Kondo lattice system YbRh$_2$Si$_2$ and in filled skutterudites containing Pr ions. In the Kramers system YbRh$_2$Si$_2$ the Kondo singlets break up in an applied field which leads to a general reduction of the effective mass. The observed anomalous behavior of the specific heat is quantitatively described by the Renormalized Band method where the field-dependent quasi-particle parameters are deduced from Numerical Renormalization Group calculations for a single Anderson impurity. In filled skutterudites containing the non-Kramers Pr ions, on the other hand, the heavy quasi-particle mass can be explained by aspherical Coulomb scattering of conduction electrons off Crystalline Electric Field excitations. The field-induced splitting of the low-lying triplet should lead to a strongly field-dependent mass enhancement. It is suggested that a field-induced Kondo effect could occur in the dilute system La$_{1-x}$Pr$_x$Os$_4$Sb$_{12}$ in close analogy to the Kondo effect found in integer-spin quantum dots.

KEYWORDS: Heavy electrons, Kondo lattice, filled skutterudites, Crystalline Electric Field effects

1. Introduction

Magnetic fields may strongly affect the electronic properties of materials containing lanthanide or actinide ions. In these systems, the relevant energy scales of the electronic system are strongly reduced due to the strong correlations of the partially filled 4f shells (for recent reviews see2,3) and references therein). The small energy scales comparable to a Zeeman energy of ~0.1 eV at 50T arise from removing local degeneracies by the anisotropy of the crystalline electric field (CEF), from building-up long-range order among moments or from forming (local) singlets via the Kondo effect. Important examples for unusual behavior are the field-induced changes of carrier-concentration in CeB$_6$ and Ce-BiPt.4,7 In the heavy fermion system (HFS) CeRu$_2$Si$_2$, a deHaas-vanAlphen (dHvA) frequency changes abruptly at the metamagnetic transition.9 This reflects changes in the ground state from a Fermi liquid with f-derived itinerant quasiparticles and a “large” Fermi surface to a conventional metal with polarized local f-moments and, concomitantly, a “small” Fermi surface.9,10 Metamagnetic transitions in Ce-based heavy-fermion compounds are a topic of high current interest.13

The present paper focusses on the evolution of the heavy Fermi liquid state with an external magnetic field in the stochiometric compound YbRh$_2$Si$_2$ and in the dilute alloys La$_{1-x}$Pr$_x$Os$_4$Sb$_{12}$. Of particular interest is the field-dependence of the heavy fermion mass which is reflected in the coefficient of the linear specific heat at low temperatures. We anticipate qualitatively different behavior in these two systems since the free Yb ion has a Kramers-degenerate ground state while the degeneracy of the f-shell of the non-Kramers ion Pr is removed by CEF effects.

The temperature vs. magnetic-field phase diagram of YbRh$_2$Si$_2$ exhibits numerous anomalies.14 This heavy fermion compound which crystallizes in the tetragonal ThCr$_2$Si$_2$ structure has been in the focus of interest during the past decade because it has emerged as a prototypical system for investigation of quantum critical phenomena.15 In its ground state, YbRh$_2$Si$_2$ orders antiferromagnetically below the Néel temperature, $T_N=70$ mK.16 By applying a weak magnetic field of $B_z=60$ mT in the basal plane the magnetic order is suppressed17 and the characteristic features of a Fermi liquid are observed. The high effective masses are ascribed to a Kondo effect which removes the Kramers degeneracy of the partially filled Yb 4f shells. Here we discuss the anomalies observed at ~10T in various thermodynamic and transport properties.14,18 We show that the latter result from a combination of a coherence effect of the periodic Kondo lattice, i.e., a van-Hove-type peak in the quasiparticle density of states (DOS) and a local many-body effect related to breaking-up the Kondo singlets.

The heavy fermion behavior of the filled-skutterudite compound PrOs$_4$Sb$_{12}$ is related to the presence of the partially filled 4f shells as can be seen by comparing its linear specific heat coefficient $\gamma_P \sim 350 - 500 mJ/molK^2$ to that of the isostructural non-f reference system LaOs$_4$Sb$_{12}$ $\gamma_{La} \sim 30 mJ/molK^2$.19,20 It has been recently shown that the key to the formation of heavy quasiparticles excitations lies in the crystalline electric field (CEF) splitting of the $J = 4$ multiplet and that the heavy quasiparticle mass is related to the inelastic scattering processes of the conduction electrons.22,23 This picture is supported by the observation of an anti-correlation between CEF splitting and effective mass. The concept of mass enhancement due to CEF excitations has been previously applied to Pr metal using the isotropic dipolar exchange interaction $H_{ex}^{iso}$.21 In the present communication we suggest to test the underlying hypothesis that the effective mass enhancement in PrOs$_4$Sb$_{12}$ is due to aspherical Coulomb scattering by investigating the variation with magnetic field of the specific heat coefficient in dilute alloys La$_{1-x}$Pr$_x$Os$_4$Sb$_{12}$ with $x \ll 1$ where the large Pr Pr distance prevents the formation of magnetic field-induced long-range order. It is an interesting speculation whether a Kondo effect can be induced by a magnetic field in a non-Kramers system in close analogy to recent findings in quantum dots.24 We show that a Kondo-like many-body ground state should exist.
The paper is organized as follows: In Section 2 we describe the calculation of the Renormalized Bands in YbRh$_2$Si$_2$. The variation with magnetic field of the quasiparticle DOS and the Fermi surface in high fields are discussed in Section 3. Section 4 is devoted to the model Hamiltonian for the low-energy excitations in filled skutterudites containing Pr. The mass renormalization due to aspherical Coulomb scattering is derived in Section 5 within (self-consistent) second-order perturbation theory. In Section 6 we discuss the influence of an external magnetic field. We show in Section 7 that the divergence of the effective mass at a critical magnetic field might indicate the formation of a novel Kondo-like many-body ground. A summary is given in Section 8.

2. Renormalized Bands for YbRh$_2$Si$_2$

The strongly renormalized heavy quasiparticle bands are determined by means of the Renormalized Band (RB) scheme which combines material-specific ab-initio methods and phenomenological considerations in the spirit of the Landau theory of Fermi liquids. The ansatz successfully describes the quasiparticle dispersion in Ce-based HFS$^{2,10,25-29,31}$ The Fermi surfaces and effective masses deduced from RB calculation reproduce the Hall effect data of the isoelectronic and isostructural HFS counterpart YbRh$_2$Si$_2$.$^{30}$ For a detailed description of the method we refer to $^{9}$

We introduce renormalized phase shifts for waves which have 4f-symmetry with respect to the rare-earth or actinide sites. Operationally, the renormalization procedure amounts to transforming the f-states of the spin-orbit ground state multiplet at the lanthanide site into the basis of CEF eigenstates $|m\rangle$ and introducing resonance-type phase shifts

$$\tilde{\eta}_{fm}(E) \approx \arctan \frac{\tilde{\Delta}_f}{E - \epsilon_{fm}}$$

where the resonance width $\tilde{\Delta}_f$ reflects the renormalized quasiparticle mass. The resonance energies $\epsilon_{fm} = \tilde{\epsilon}_f + \delta_m$ refer to the centers of gravity of the f-derived quasiparticle bands. Here $\tilde{\epsilon}_f$ denotes the position of the band center corresponding to the CEF ground state while the $\delta_m$ are the measured CEF excitation energies. One of the remaining two parameters, $\epsilon_{fm}$, is determined by imposing the condition that the charge distribution is not altered significantly by introducing the renormalization. This makes the RB method a single-parameter scheme. The free parameter, $\Delta_f$, is adjusted so as to reproduce the coefficient of the linear specific heat at low temperatures.

In the case of Yb-based heavy fermion compounds, we have to renormalize the 4f $j=7/2$ channels at the Yb sites. As the 4f hole count is slightly less than unity the center of gravity $\tilde{\epsilon}_f$ will lie below the Fermi energy. In addition, we have to reverse the hierarchy of the CEF scheme, i.e.

$$\tilde{\epsilon}_f < 0 ; \quad \epsilon_{fm} = \tilde{\epsilon}_f - \delta_m .$$

In the presence of a magnetic field, the quasiparticles have to be described by field-dependent parameters for the level $\epsilon_{fm}(h)$ and the resonance width $\tilde{\Delta}_f(h)$. The energy $h = \frac{1}{2} g_{\eta_f} \mu_B H$ denotes the (anisotropic) Zeeman splitting of the free ion CEF ground state which is related to that of an effective spin-1/2-system by introducing anisotropic effective g-factors. In the present calculation, we use field-dependent parameters $\epsilon_f(h)$ and $\tilde{\Delta}_f(h)$ whose highly non-trivial variation with field is derived from fits to field-dependent quasiparticle DOS of the single-impurity Anderson model.$^{32,35}$ The latter are calculated microscopically by means of the Numerical Renormalization Group (NRG). This procedure properly accounts for the progressive de-renormalization of the quasiparticles with increasing magnetic field and the correlation-enhanced Zeeman splitting. As will be shown below, it allows for a quantitative description of the observed field-induced anomalies as well as for the observed dHvA data in the high-field regime.

The band-structures were obtained by the fully relativistic formulation of the linear muffin-tin orbitals (LMTO) method.$^{36-38}$ The spin-orbit interaction is fully taken into account by solving the Dirac equation. We adopt the atomic sphere approximation (ASA) and include the combined correction term which contains the leading corrections to the ASA.$^{39}$ The calculations are done at the experimental lattice parameters $a=b=4.007\,\AA$, $c=9.858\,\AA$ for YbRh$_2$Si$_2$. For details of the calculation we refer to $^{30,39}$

The RB calculations reported here adopt a CEF scheme which is consistent with susceptibility and the inelastic neutron data.$^{40,41}$ The latter indicate that the 4f$^{13}$ states in YbRh$_2$Si$_2$ are split into 4 doublets with the energies $0.17-25.43$ meV. The parameters for the tetragonal CEF are $B_{20}=0.5246$ meV, $B_{22}=0.01195$ meV, $B_{40}=-0.0004725$ meV, $B_{44}=0.03598$ meV, $B_{46}=0.01206$ meV.$^{42,43}$ The low-energy properties are mainly determined by the CEF ground state which is a superposition of $|j=7/2; j_z=\pm5/2\rangle$ and $|j=7/2; j_z=\mp3/2\rangle$ and which is well separated from the excited states. We find that the coupling to the conduction states is rather weak and strongly anisotropic. The resulting g-factors are $g_{||}=0.26$ and $g_{\perp}=3.79$ for magnetic fields parallel to the tetragonal axis and in the basal plane.

We use a quasiparticle resonance width of $\tilde{\Delta}_f = 20$K as inferred from specific heat and thermopower measurements.$^{14,44}$

3. Magnetic field-dependent quasiparticles in YbRh$_2$Si$_2$

The RB calculation yields narrow bands with f-character in the vicinity of the Fermi energy while the dispersion of the broad non-f conduction bands remains essentially the same as in the local moment regime. The quasiparticle DOS displayed in Figure 1 is mainly due to the CEF-split 4f-states. The RB calculations yield a DOS of $\sim 290$ states/(eV unit cell) corresponding to specific heat coefficient $\sim 680mJ/(mole\,K^2)$. This value should be considered as theoretical zero-field limit of the specific heat coefficient measured in the Fermi liquid state at small finite magnetic fields. The actual zero-field specific heat data are strongly enhanced by the anomalous fluctuations associated with the quantum critical point.

There are three bands intersecting the Fermi energy. The overall topology qualitatively agrees with LDA results of Refs.$^{45-49}$ The dominant contribution to the quasiparticle DOS and, concomitantly, to the specific heat and magnetic susceptibility comes from the Z-centered hole surface which has predominantly f-character. The anisotropy of the CEF ground state results in a strongly anisotropic hybridization. As the heavy band is rather flat over a wide range of the Brillouin zone we find a very sharp van-Hove type feature in the quasi-
particle DOS displayed in Figure 1. It is this feature which leads to the anomalies at ~10T. The variation with magnetic field of the quasiparticle DOS at the Fermi energy is shown in Figure 1. It describes the experimental specific heat and susceptibility data.\textsuperscript{18}

The states forming the “jungle gym” sheet of the Fermi surface, on the other hand, are strongly hybridized and consequently, less affected by magnetic fields. Several groups reported a large extremal orbit corresponding to relatively light quasiparticles.\textsuperscript{49,50} The jungle gym sheet of the RB Fermi surface in Figure 2 is consistent with these findings. In a field of ~15T there is a closed orbit in the 110 plane with area F=13kT and m^* ~ 20m in the 110 plane.

4. Model Hamiltonian for La\textsubscript{1-x}Pr\textsubscript{x}Os\textsubscript{4}Sb\textsubscript{12}

The electronic part of the Hamiltonian for the system La\textsubscript{1-x}Pr\textsubscript{x}Os\textsubscript{4}Sb\textsubscript{12} is given by

\begin{equation}
H = H_\text{el} + H_{CEF} + H_Z + H_{AC} .
\end{equation}

Here \(H_\text{el}\) contains the conduction band dispersion which may be described by a n.n.n. tight binding model\textsuperscript{7} according to

\begin{equation}
\epsilon_\sigma = t \cos \frac{\pi}{2} k_x \cos \frac{\pi}{2} k_y \cos \frac{\pi}{2} k_z + t' \left( \cos k_x + \cos k_y + \cos k_z \right)
\end{equation}

with \(t=174\) meV and \(t'=27.84\) meV. It corresponds to a single band originating in Sb-4p states. The transfer integrals \(t\) and \(t'\) are chosen so as to reproduce the observed linear specific heat coefficient \(\gamma = 36\) meV/(mole K^2) of the non-f reference compound LaOs\textsubscript{12}Sb\textsubscript{12}.\textsuperscript{51} Aside from subtle effects the resulting Fermi surface is quite similar to the LDA FS in Ref.\textsuperscript{52}

The CEF and Zeeman Hamiltonian are

\begin{equation}
H_{CEF} + H_Z = \sum_{i} E_i \left| \Gamma_i(n) \right\rangle \left\langle \Gamma_i(n) \right| + g_J \mu_B \sum_i \mathbf{J}_i \mathbf{H} .
\end{equation}

The external magnetic field is denoted by \(\mathbf{H}\), \(g_J\) is the Landé factor and \(\mu_B\) is Bohr’s magneton. Furthermore \(i\) labels the Pr sites, and \(\left| \Gamma_i(n) \right\rangle\) denotes the CEF states with energies \(E_i\). The data are explained best by a \(\Gamma_4\) ground state with a low-lying triplet excited state at an energy of \(\Delta = 8K\).\textsuperscript{53,54} The other CEF levels are so high in energy that they can be neglected. The compound has tetrahedral \(T_h\) site symmetry for Pr which implies that the triplet state is a superposition of two triplets \(\Gamma_4\) and \(\Gamma_5\) of O_b symmetry:\textsuperscript{53,54}

\begin{equation}
\left| \Gamma_i^{m} \right\rangle = \sqrt{1 - d^2} \left| \Gamma_5, m \right\rangle + d \left| \Gamma_4, m \right\rangle , \quad m = \pm 0 .
\end{equation}

The conduction electrons interact with the CEF energy levels of the Pr\textsuperscript{3+} ions. We focus here on the aspherical Coulomb scattering\textsuperscript{55}

\begin{equation}
H_{AC} = g \sum_i \sum_{k\sigma} \sum_{\alpha \beta} O_i^{\alpha \beta} f_{\alpha \beta} (k, k') c^\dagger_{\alpha \sigma} c_{\beta \sigma} e^{ik \cdot R} ,
\end{equation}

where \(c^\dagger_{\alpha \sigma}(c_{\alpha \sigma})\) are the creation (annihilation) operators for conduction electron with momentum \(k\) and spin \(\sigma\) while
O_{\alpha\beta} = \frac{2}{3}(J_{1\alpha} J_{1\beta} + J_{3\alpha} J_{3\beta}), \quad \alpha\beta = yz, zx, xy \text{ denote the three quadrupole operators with } \Gamma_3 \text{ symmetry. The remaining } \Gamma_3 \text{ quadrupole terms are neglected since they do not couple to the excitations under consideration. The scattering anisotropy is accounted for by the quadrupole form factors}

\[ f_{\alpha\beta}(k, k') = \frac{1}{\langle |k - k'|^2 \rangle_{FS}} (k_\alpha - k'_\alpha) (k_\beta - k'_\beta) \]  

where the momentum transfer is measured in units of its Fermi surface average. The coupling constant \( g \) may in principle be determined by experiments. A derivation of Eq. (7) may be obtained from Ref. 36.

5. Mass renormalization due to aspherical Coulomb scattering

The effective mass enhancement due to interactions of the conduction electrons follows from the self-energy due to \( H_{AC} \). Neglecting vertex corrections, it is given by

\[ \Sigma(k, \omega) = i \sum_{\alpha_1, n} \frac{1}{V} \sum_k |\Lambda_{\alpha_\beta}^n(k, k')|^2 \int \frac{d\omega'}{2\omega'} D_n(k - k', \omega + \omega') G(k', \omega') \]  

where the momentum dependence of the bare vertex \( \Lambda_{\alpha_\beta}^n(k, k') \) follows from the quadrupolar \( \Gamma_3 \) form factors in Eq. (8). Here \( D_n(k, \omega) \) denotes the boson propagator of CEF excitations. It is related to the dynamical quadrupolar susceptibility of the CEF system. In the present case we will neglect effective RKKY type interactions between CEF states on different sites assuming a local \( \mathbf{q} \)-independent singlet-triplet boson propagator

\[ D_n(q, \omega) = D_n(\omega) = \frac{2\delta_n}{\delta_n^2 - \omega^2} \]  

with the field dependent singlet-triplet excitation energies

\[ \delta_n(H) = \epsilon_n^s(H) - \epsilon_n^t(H) (n = +0,-) \text{.} \]

The self-energy becomes \( k \)-independent. In the wide-band limit, the result for \( \Sigma(\omega) \) agrees with that of non-selfconsistent second order perturbation theory. After differentiation with respect to \( \omega \) under the integral and using integration by parts one finally gets

\[ \frac{\delta m^*}{m} = \frac{m^*}{m} - 1 = - \frac{\partial \Sigma(\omega)}{\partial \omega} \bigg|_{\omega=0} = g^2 N(0) \int \chi_Q(H) \]  

with

\[ \chi_Q(H) = \sum_{\alpha_\beta, n} \frac{2 |\langle \Gamma_{\alpha_\beta}^S(H) | O_{\alpha_\beta} | \Gamma_{\alpha_\beta}^T(H) \rangle|^2}{\delta_n(H)} \]  

The reference mass \( m \) includes band effects and effects of electron-phonon coupling. The averaged quadrupolar form factors \( f = \langle |f_{\alpha\beta}(k, k')|^2 \rangle_{FS} \) is a constant. Furthermore \( \chi_Q(H) \) in Eq. (12) is the field-dependent static uniform quadrupolar susceptibility. Explicit expressions for the quadrupolar matrix elements in Eq. (12) are given in. 22, 23

6. Magnetic field-dependent mass enhancement in \( \text{La}_{1-x} \text{Pr}_x \text{Os}_4 \text{Sb}_{12} \)

It has been shown that aspherical Coulomb scattering can consistently and quantitatively explain the zero-field effective mass enhancement in Pr telluridutes. 22, 23 When a magnetic field is applied to the sample the field dependence of the effective mass is completely determined by that of the quadrupolar susceptibility in Eq. (12). To calculate this quantity we use the singlet-triplet excitation energies \( \delta_n(H) \) and the eigenstates and matrix elements in applied field which were given by Shina et al 23, 54 in closed form for field applied along cubic symmetry directions. The field dependence of \( \delta_n(H) \) has recently been determined by INS experiments. 57

The two triplet states \( |\Gamma_{\pm}^T \rangle \) have a linear Zeeman splitting independent of the tetrahedral CEF contribution \( d \). When \( d = 0 \), the energies of \( |\Gamma_{+}^T \rangle \) and \( |\Gamma_{-}^T \rangle \) will be independent of the field \( H \). For nonzero tetrahedral contribution these two levels will repel with increasing field \( H \). For \( d^2 < 0.42 \) the singlet ground state level \( E_s \) and lowest triplet level \( E_{t0}' \) cross at a critical field \( H_c \) defining \( \delta_s(H_c) = 0 \). In the case of weak tetrahedral CEF such as realised in \( \text{Pr}_{1-x} \text{La}_x \text{Os}_4 \text{Sb}_{12} \) the level repulsion of \( \Gamma_{+}^T \) and \( \Gamma_{-}^T \) is also weak and therefore the \( \Gamma_{+}^T \) level crosses the \( \Gamma_{+}^T \) ground state at a critical field \( H_c \) in the dispersionless case.

The decrease in the excitation gap for \( H < H_c \) and the field dependence of matrix elements leads to a field dependence of \( \delta m^*/m \) which is shown in Fig. 3 for \( d = 0.26 \). For larger \( d^2 \) the increase is diminished and eventually for \( d^2 > 0.42 \) the level repulsion due to the tetrahedral CEF is strong enough to lead to an increase in excitation energy and hence to a decreasing effective mass.

7. Is there a field-induced Kondo effect for non-Kramers \( \text{Pr} \) ions?

Let us next turn to the (unphysical) divergent mass renormalization which is predicted for dispersionless undamped CEF excitations when the triplet level approaches the singlet
It describes an electron interacting with a Pr ion in the presence of a filled Fermi sea \(|\nu\rangle\). The creation (annihilation) operators for \(c_{\nu\sigma}^\dagger\) (\(c_{\nu\sigma}\)) refer to conduction electron states introduced above. The functions \(a(\nu; n)\) are to be determined variationally while \(A\) is an overall normalization constant and \(D\) is the width of the conduction band. The matrix elements of \(H_{\text{AC}}\) vary weakly with energy and hence can be evaluated at the Fermi energy. Minimizing the energy with respect to the energy-dependent amplitudes \(a(\nu; n)\) yields the self-consistency equation for \(\tilde{a}(\nu; n) = N(0) \int_0^D d\epsilon(\nu; n)\)

\[
\tilde{a}(\nu; n) = N(0) \ln \left| \frac{E - E_n}{D} \right| \sum_{\nu' n'} \langle 0\nu; n | H_{\text{AC}} | 0\nu'; n' \rangle \tilde{a}(\nu'; n')
\]

(15)

which has a non-trivial solution provided an eigenvalue of the matrix

\[
K(\nu n; \nu n') = -N(0) \sqrt{\lambda(n; E)} \sum_{\nu' n'} \langle 0\nu; n | H_{\text{AC}} | 0\nu'; n' \rangle \sqrt{\lambda(n'; E)}
\]

(16)

with \(\lambda(E; n) = \ln \left| \frac{D}{E - E_n} \right|\) is unity. This condition is always satisfied for an energy \(E < E_n(H_c) = E_n^+(H_c)\) since the hermitian matrix \(K\) with vanishing trace has at least one positive eigenvalue.

From these considerations we conclude that a field-induced Kondo effect should exist in the system under consideration. In this state the orbital degrees of freedom of the f- and conduction states are entangled as can be seen from Figure 4 which schematically displays the density distributions of the ground state obtained for the present model. A quantitative estimate of the Kondo scale requires a more refined theoretical treatment with more detailed information on the (effective) coupling constant \(g\).25

8. Summary and outlook

We calculated the variation with magnetic field of the quasiparticle bands in YbRh\(_2\)Si\(_2\) by means of the RB method where the field-dependent quasiparticle parameters extracted from a NRG treatment of the single-impurity Anderson model account for the correlation enhanced Zeeman splitting and the reduced effective mass of the CEF ground state doublet. The calculated DOS reproduces the observed variation with magnetic field of the linear specific heat coefficient and the magnetic susceptibility. The “large” Fermi surface consistently explains the quantum oscillation frequency observed in the high-field limit.

For the filled skutterudites Pr\(_{1-x}\)La\(_x\)Os\(_4\)Sb\(_{12}\) we have studied in detail the quasiparticle mass enhancement originating in the aspherical Coulomb scattering of conduction electrons from singlet triplet CEF excitations. For small enough tetrahedral CEF characterised by the parameter \(d^2 \ll 1\) the lowest triplet component crosses the singlet ground state at a critical field \(H_c\). In second order perturbation theory the mass enhancement increases with field and becomes singular at \(H_c\). For larger tetrahedral CEF \((d^2 > 0.42)\) the excitation energy between singlet ground state increases with field leading to a decrease of the mass enhancement, similar as has been observed in Pr metal where the mass renormalisation is due to exchange scattering from a singlet-singlet CEF level scheme. The singular mass enhancement close to the critical field of level crossing is an artefact of the model. Any dispersion of the singlet-triplet excitations due to effective in-
tertise quadrupolar interactions will lead to a finite effective quasiparticle mass. In addition, we showed that a Kondo-type ground state can be induced by applying a magnetic field.

Therefore we propose that the field dependence of the electronic specific heat in mixed crystals of Pr$_{1-x}$La$_x$Os$_4$Sb$_{12}$ is systematically investigated and analysed. It may hold important clues to the microscopic nature of the heavy-electron state in PrOs$_4$Sb$_{12}$.

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